Radford, L. (2020). Davydov's concept of the concept and its dialectical materialist background. Educational Studies in Mathematics, Online First. https://doi.org/10.1007/s10649-020-09959-y

Davydov's concept of the concept and its dialectical materialist background

Luis Radford

Abstract
The goal of this article is to investigate Davydov's concept of the concept against the backdrop of its philosophical system, namely, dialectical materialism. In the first part, after briefly sketching the context of Davydov's work, I consider some ontological and epistemological ideas on which Davydov bases his concept of the concept. I pay particular attention to Hegel's and Marx's contributions. Then, I discuss Davydov's concept of the concept and the relationship between the *logical* and the *historical*—a relationship that proved to be crucial in the making of the educational curricular program he and El'konin launched in the 1960s in Russia. I argue that, in tune with the dominant epistemology of the twentieth century, Davydov's concept of the concept is based on a scientific outlook of the world, one in which theoretical scientific thought is considered the pinnacle of human cognition. I conclude with a critique that intends to place the notion of the concept in a broader dialectical materialist perspective.

Keywords Davydov · Hegel · Marx · Concept · Dialectical materialism · Conceptual and nonconceptual

1 Introduction

One of the best-known works of Vasily Vasilyevich Davydov, at least in the Western world, is *Types of Generalization in Instruction* (Davydov, 1990). In this book, Davydov provides a definition of what a *concept* is. He says:

> A *concept* functions here as a form of mental activity by means of which an idealized object and the system of its connections, which reflect in their unity the *generality* or *essence* of movement of the material object, are reproduced. (Davydov, 1990, p. 116; emphasis in the original)

This article is a critical commentary on Davydov's functionalist concept of the concept. My critical commentary—which is part of an increasing interest in better understanding the nature of mathematical concepts (de Freitas, Sinclair, & Coles, 2017)—is an attempt to investigate Davydov's concept of the concept against the backdrop of its philosophical system, namely dialectical materialism. My contention is that it is only through an examination of the philosophical system that underpins Davydov's work that we can fully appreciate both his concept of the concept and its educational implications. I start by offering a short sketch of the historical context of Davydov's work and the general vision that oriented his curricular research. Then, I discuss some key ideas of Davydov's dialectical materialist background. The rest of the article revolves around Davydov's concept of the concept and the problem of the historical and the logical, a problem that proved to be crucial in the making of the educational curricular program Davydov and Daniil El'konin launched in the 1960s in Russia. Drawing on Heidegger's view of Being and Adorno's negative dialectics, I conclude with a critique that intends to place the notion of the concept in a broader dialectical materialist perspective.

2 The context

In 1959, 2 years after having defended his doctoral thesis under the guidance of Piotor Gal'perin, Davydov started working at the Institute of General and Pedagogical Psychology of the Academy of Medical Sciences of the USSR (Rubtsov, 2015). In collaboration with El'konin, he started developing a "modeling experiment," which came later to be known as the El'konin-Davydov curriculum. Davydov's modeling experiment was embedded in a Soviet educational reform initiated in 1958 by Nikita Khrushchev's government. The reform sought to overcome the poor "academic level of students and their [in]ability to apply their knowledge to practical tasks and workplace-related activities" (Boyko, 2019, p. 83). In mathematics, the reform was led by the famous mathematician Andrey Kolmogorov. It is in this context that governmental efforts were made to rebuild the curriculum and to produce new textbooks (Abramov, 2010). Commenting on the inadequacy of the mathematics curriculum of the time, Davydov (1975a) notes that "Deficiencies in the traditional mathematics curricula for the school are being discussed frequently both here and abroad" (p. 55).

Davydov cites Kolmogorov, saying that mathematics

> studies the material world from a particular point of view, that *its immediate subject is the spatial forms and quantitative relationships of the real world*. These forms and relationships themselves, in their pure form, rather than specific material bodies, are the reality which mathematics studies. (Kolmogorov, cited in Davydov, 1975a p. 68; emphasis as in Davydov's passage)

From this viewpoint, Davydov notes:

> The curriculum should provide the child with work in which he (sic) will be able to "move away" from concrete bodies accurately and at the proper moment, *after having distinguished* their spatial forms and quantitative relations and having given them their 'pure form.' Only on the basis of this can he (sic) develop an accurate understanding of mathematics. (Davydov, 1975a, p. 68)

Naturally, the child cannot start from the pure forms of mathematics. "What the research mathematician has before him (sic) in its 'pure form' has to be constructed in the child's head.

This 'form' is not given to him (sic) at the start" (Davydov, 1975a, p. 68). The fundamental question was then as follows: "What organization of the course and what method of introducing concepts contributes best to the solution of this problem?" (p. 68). Davydov's experimental research, carried out for about 25 years and mainly conducted at the Experimental School N. 91 in Moscow, was an attempt at answering this question. Davydov argued that the curriculum should be based on clear *logical* principles to structure its content. He claimed that it is "particularly necessary to determine the most appropriate concepts with which to begin mathematics instruction in school" (1975a, p. 56). Structuring the curriculum through logical principles was, however, not enough. An appropriate *psychological* approach was also needed. As he put it, the outdated curriculum of his time was failing to "provide for the necessary development of children's mathematical thought" (1975a, p. 55).

But the distinctiveness of Davydov's approach is not to be found in his resorting to logical and psychological principles to structure the mathematics curriculum. What makes it really distinctive is Davydov's understanding of the logical and psychological principles. Drawing on dialectical materialism, he derived the overarching goal of his whole enterprise. The goal was not about developing a curriculum that would allow the child to simply acquire mathematical knowledge. To think so is to miss entirely Davydov's whole point. As Schmittau and Morris aptly put it, "Davydov's curriculum has as its overriding goal the development of the ability to think theoretically" (2004, p. 61; see also Ivashova, 2011, p. 59) or, in the words of Libâneo and Freitas, "the formation of theoretical-scientific thought" (2013, p. 318). Now, for Davydov, the attainment of theoretical-scientific thought is not the result of a speculative mind getting hold of concepts through ruminative cogitations. It is rather a sensuous developmental process that, in tune with Vygotsky's school of thought, privileges the process of generalization. Starting from material objects and "the *immediate* character of empirical knowledge" (Davydov, 1990, p. 115; emphasis in the original), generalization allows one to recognize the inner structure of *scientific* concepts and the system of their constitutive connections.

Thus, when Davydov intends to understand what a number is, he purposely moves beyond the immediacy of empirical knowledge and tries hard to find the theoretical connections that would constitute the scientific concept of number. After a theoretical archeological excavation (of which I will have more to say below), rather than counting aggregates of objects—rather than cardinality *as such*—he finds *relations*: relations like "more than," "less than," and "equal to" (Davydov, 1975a, 1975b, 2008). From here he devises a curricular organization of arithmetic and algebra that might look puzzling when considered from traditional epistemologies like those offered by idealism and empiricism.

Davydov presented a schema of the main curricular ideas in various papers (see, e.g., Davydov, 1975b; see also Schmittau & Morris, 2004). I will limit myself here to merely mention that the curriculum starts with a focus on a quantitative comparison of quantities (length, width, volume, and area), first on instructional material, then on representations through line segments and letters, in order to assert whether $A = B$, $A \neq B$, $A > B$, or $A < B$, without reference to numbers. In Davydov's view, the use of symbols is required to help children move away from specific objects and to pay attention to the connections at the heart of theoretical concepts. Symbols, indeed, help the children to divorce themselves from using objects so that they can focus on "verbal and logical evaluations (constructions of the type: 'if…and…then…')" (Davydov, 1975b, p. 140). As a result of plunging into the theoretical realm through symbols, "What becomes central for the

children is the relationship itself, its type, rather than the [material] objects" (Davydov, 1975b, p. 164).[1]

In a similar vein, the children use letters to represent unknown parts of a whole (e.g., h + r = t) and reflect on their mutual connections (see Schmittau & Morris, 2004, p. 70; see also Freiman and Fellus, this Special Issue). The use of letters, however, should not be seen as something that makes the child enter automatically into the realm of algebra. For Davydov, algebra is not about using letters. To understand the use of letters in Davydov's approach, we have to bear in mind that Davydov is following the Vygotskian tradition where theoretical thinking is *mediated*. And it is this mediation—a theoretical mediation—that signs are accomplishing. Signs, in this account, allow the child to perceive, deal with, and reproduce the essence of the object under study. This is why it is crucial, for example, that in the reproduction of the essence of number, children shift from equality (A = B) to inequality (A + K > B) and back (A + K = B + K) (Davydov, 1975b, p. 137).

Davydov's logical approach to the concept of number and its "shift from 'techniques of calculation' to the study of the structural characteristics of mathematical 'objects'" (Davydov, 1975b, p. 141) is a reminiscence of Piaget's epistemological analyses. Yet, while Piaget talks about invariants, one-to-one correspondence, and abstraction (Piaget, 1964), Davydov (1990), by contrast, talks about generalization—the generalization of material objects into idealized objects, which, as it proceeds in practical, sensuous activity, discloses the *generality* or *essence* of the object. The general that this *general*-ity produces should not to be confounded with the general of empiricist generalizations. The general (or essence) that Davydov refers to has to be understood in its dialectical materialist sense, that is, as "the genetically initial 'cell' of some *developing integral system* which generates all its particular manifestations" (Davydov, 1967, p. 50; emphasis in the original).

The goal of the two following sections is to offer an overview of Davydov's dialectical materialist background. Although the content is theoretical, I hope that it will be helpful in better appreciating Davydov's approach and in particular his concept of the concept, which I see as the "cell" of his curricular enterprise. My overview is organized in two parts, one dealing with ontology (which explains, for instance, Davydov's insistence that a concept is made up of internal connections), and one dealing with epistemology (which explains *how* things are known).

3 The ontology of dialectical materialism

3.1 The principle of universal connection

Davydov's dialectical materialism starts from a Spinozist, systemic, ontological premise about the nature of the world (Spinoza, 1989). According to this premise, the world is an "aggregate

[1] Here is an example:

> The students are shown a picture of two balloons. The volume of one balloon is labeled L; this balloon is completely drawn. The other balloon of volume P is only partially drawn. The problem says: If L = T and T > P, then L__P. The students are unable to directly compare the volumes (one is only partially drawn) so they have to make an inference about the relationship between L and P. (Schmittau & Morris, 2004, pp. 63–64)

of objects and phenomena linked with one another by extremely diverse relations and connections" (Spirkin, 1990, p. 119).

The idea that objects are linked to other objects through systems of relations and connections is articulated in the work of Hegel, upon which dialectical materialism draws. In the *Encyclopaedia*, Hegel notes that "A determinate content . . . contains a manifold *connection* within itself and is the basis for connections with many other objects" (Hegel, 1991, p. 89; emphasis in the original). As a result, in this view, material and ideational objects are not merely substantial things. Objects are *relational* through and through. The *essence* of an object—and in the previous section we saw this idea in Davydov's account of number—consists indeed in the connections that link such an object to other objects.

3.2 Motion and development

Dialectical materialism considers the world in constant motion. All is in flux, "on the path toward something else" (Spirkin, 1990, p. 123), pushed, so to speak, in one direction or another by connections and relations. In social life, for instance, the needs and interests of people constitute basic connections that underlie the forms of production of a community.

Among the different types of connection in the universe, there is one that plays a significant role in dialectical materialism: it is the law-governed connection, which serves to explain development: "*Development is an irreversible, definitely oriented and law-governed change of material and ideal objects resulting in the emergence of new qualities*" (Spirkin, 1990, p. 123; emphasis in the original).

In general terms, law-governed connections can be conceived of in several different ways. They can be conceived of as *laws of the world—objective* laws that have nothing to do with the individuals. These laws are not human-made. The discovery of the laws that govern the universe is indeed, in this line of thought, the task of *scientific* research. Dialectical materialism adopts here a kind of Galilean view of the universe, which provides it with scientific-rationalist ontology.

Law-governed connections can also be understood as laws created by the individuals—*subjective* laws that individuals produce to make sense of their world. So, while the first Galilean view gives the primacy to the object (truth lies in the *object*), the second one gives the primacy to the subject (truth lies in the *subject*).

Davydov draws on a different dialectics—a Hegelian dialectics as articulated further by Marx. This materialist dialectics rests on a view where subject and object *become entangled*, so that law-governed connections always contain the imprint of human activity, which makes them neither transcendental vis-à-vis the individuals nor merely subjective.

There is another question that we need to discuss—the meaning of the *essence* that, as we saw in the Introduction, Davydov brings to the fore in his understanding of what a concept is.

3.3 Essence and phenomena

Dialectical materialism makes from the outset a distinction between essence and phenomena. Essence and phenomena are part of two different ontological realms. A phenomenon (or appearance) is what is observable, sensible. Essence, by contrast, is inaccessible to human perception. In the words of Davydov, essences are "internal, essential relationships [that] *cannot* be observed directly by the senses" (Davydov, 1990, p. 119; emphasis in the original).

While in Kant's (2003) dualistic theory of knowledge, the realm of the essence (the realm of *things-in-themselves*) remains beyond human cognition, for Hegelian dialectical materialism this is not the case. There is a clear and explicit relationship between them: within the dynamic ontological view of the universe, essences are conceived of as continuously *passing* into appearances.

Essence, Spirkin says, "*is the basis of all the forms of their external manifestation . . .* Essence is in this sense something internal, a certain organizing principle of the object's existence in the forms of its external expression" (Spirkin, 1990, p. 155; emphasis in the original).

The problem of theoretical thought is precisely to reveal the connections and relations of objects, as these connections manifest externally or actualize themselves in "the realm of objectively interconnected phenomena" (Davydov, 1990, p. 119).[2]

Within the dialectical materialist framework, essences lose the metaphysical meaning with which idealism endowed them. Essences as a system of internal connections manifest themselves in motion, in practical activity—as when children compare lengths of visible, non-visible or partially visible objects (as in footnote 1) or add fractions (Swanson & Williams, 2014).

4 The epistemology of dialectical materialism

The previous section offered a short account of the ontology of dialectical materialism. Now we turn to its epistemology. The question is: What does it mean to know? The answer is in Hegel's work. As previously mentioned, for Hegel, an object of knowledge is a determinate content; that is, a manifold of connections within itself and the basis for connections with other objects. In this context, "To be cognizant," Hegel says, "means nothing else but the knowing of an object according to its determinate content" (Hegel, 1991, p. 89). In other words, I know something if I go beyond the superficial aspect of the object and know its essence (internal relations and connections). The question that naturally arises in this context is the question of the kind of knowing that we, as humans, are capable of. Hegel notes that

> if we inquire into the truth of knowledge, it seems that we are asking what knowledge is *in itself*. Yet in this inquiry knowledge is *our* object, something that exists *for us*; and the *in-itself* that would supposedly result from it would rather be the being of knowledge *for us*. What we asserted to be its essence would be not so much its truth but rather just our knowledge of it. (Hegel, 1977, p. 53; emphasis in the original)

Knowledge, hence, would not be knowledge of the object itself, but *our* knowledge of it. "Consciousness provides its own criterion from within itself, so that the investigation becomes a comparison of consciousness with itself" (Hegel, 1977, p. 53).

> The object, it is true, seems only to be for consciousness in the way that consciousness knows it; it seems that consciousness cannot, as it were, get behind the object as it exists for consciousness so as to examine what the object is *in itself*. (Hegel, 1977, p. 54; emphasis in the original)

[2] The adjective "objective" should not be understood as independent of the individual. As mentioned previously, in dialectical materialism, the connections between things and phenomena refract the *entanglement* between subject and object.

Imagining the object as concealed behind a curtain, Hegel says that "there is nothing to be seen unless *we* go behind [the curtain] ourselves" (Hegel, 1977, p. 103; emphasis in the original). However, what we would see, were we to walk behind the curtain, is ourselves at work. "The inside of things is a construction of the mind. If we try to lift the veil that covers the real, what we will find is only ourselves, the universalizing activity of the mind that we call understanding" (Andler, 1931, p. 332).

It seems, then, that we have reached an impasse and that, as humans, we are limited to have a subjective knowledge of things, knowledge of appearances only. This was Kant's (2003) conclusion. In Kant's account, this was the limit of pure human reason. But it is precisely at this point that Hegel went further. He reasoned that if the comparison between the object and our knowledge about it (the object *in itself* and the object *for us* or for consciousness) do not correspond, we still can alter our knowledge "to make it conform to the object" (Hegel, 1977, p. 54).

However, Hegel's crucial innovation is not to be found in the adjusting endeavor of our knowledge to make it come *closer* and *closer* to the object, to its essence, to Truth. What Hegel is articulating in these passages is a new theory of knowing that moves epistemology beyond the classical objectivist and subjectivist paradigms of Western thought.

> We see that consciousness now has two objects: one is the first *in-itself*, the second is the *being for-consciousness of this in-itself* . . . the first object, in being known, is altered for consciousness; it ceases to be the in-itself, and becomes something that is the *in-itself* only for *consciousness*. (Hegel, 1977, p. 55; emphasis in the original)

As a result, there is an entanglement between object and subject, world and consciousness. They co-produce each other mutually.

Hegel's theoretical position gained great popularity during the nineteenth century in German intellectual circles. German philosophers engaged in discussions about his work one way or the other. And Karl Marx was certainly not the exception. He found Hegel's account very idealistic. For Marx, the entanglement between object and subject is not the result of a majestic flight of consciousness in its march to perfection. In the first of the *Theses on Feuerbach* (Marx, 1998), the co-creation of subjects and objects and their entanglement are explained as occurring in something terrestrial: in *praxis*, or sensuous collective human practical activity. Through *praxis* human affection and cognition become "inscribed in matter" (Aron, 1981, p. 91). This inscription constitutes the essential structure of *reality—Wirklichkeit*, which for Marx signifies *effective reality;* that is to say, "everything that offers itself first of all to consciousness as constituting reality" (Macherey, 2008, p. 47). In this way, *effective reality* embraces much more than the products of human activity (i.e., les *oeuvres* humaines): it also embraces the embodiments of social needs built culturally and historically and the ideas and values of a given culture at a certain moment (Descombes, 1996). As a result, objects (material or ideal) do not present to us as untouched by humans but as something already carved by history. In this line of thought, truth is neither on the side of the object (as *a*historical materialism suggested) nor on the side of the subject (as idealism and empiricism contended). In articulating the epistemological question of the subject and object in this way, by going beyond the classical dichotomy objective/subjective, Marx opened up a fresh avenue to look at human cognition in a new anthropological way (Dupré, 1983; Henry, 1976). To know an object amounts to putting into action "a dynamic of reflection and explanation that reproduces the very movement of reality without . . . fixing it in the consideration of some of its isolated results, cut off from the complex conditions of their elaboration"(Macherey, 2008, p. 74).

It is from this anthropological perspective on human cognition that Davydov—and before him, Vygotsky, Luria, and Leont'ev—explored a new path to the study of human psychology. And it is in this sense that consciousness and objects appear in Davydov's work. For Davydov, objects are objects of need. He quotes Rubinstein: "Objects of needs and actions, rather than objects of contemplation, are given initially" (Davydov, 1990, p. 111). He also quotes G. A. Kursanov: "For a thing to function for human consciousness, it should function as an *object of activity*" (p. 190; emphasis in the original).

Bearing in mind the previous dialectical materialist background, we can now turn to Davydov's concept of the concept.

5 Davydov's concept of the concept

As mentioned in the introduction, Davydov defined a concept as follows:

> A *concept* functions here as a form of mental activity by means of which an idealized object and the system of its connections, which reflect in their unity the *generality* or *essence* of movement of the material object, are reproduced. A concept simultaneously also functions as a form for reflecting the material object and as a means of mentally reproducing, constructing it—that is, as a particular *mental action.* (Davydov, 1990, p. 116; emphasis in the original)

He explains that, ontogenetically speaking, this object appears first as existing independently of the individual. There is, hence, an element of *passivity* where the object appears having an objective content vis-à-vis the individual and an element of *activity* since "to have a concept of an object means to reproduce or construct it mentally" (Davydov, 1990, p. 116). Davydov continues: "This action of constructing and transforming a mental object is an act of understanding and explaining it, of discovering its essence" (p. 116).

He sympathetically cites Kant who "has astutely noted that 'thinking' means 'acting'" (Davydov, 1990, p. 116). In this context, Davydov mentions Kant's concept of the schema and reminds us that Kant's key terms such as "drawing" or "describing" in the formation of schemas are "none other than reproducing or constructing an object on an ideal level" (p. 117). He ends up approvingly citing Y. M. Borodii saying that a concept "is a *rule* for reproducing an object, or, expressed in Hegel's language, a *measure*" (p. 190).

There are three things in Davydov's concept of the concept:

(1) A concept is a form of mental activity.
(2) It reproduces or constructs on an ideal level the essence of the object.
(3) It works normatively.

I will dwell on these points below.

(1) A concept is a form of mental activity.

The first point indicates that a concept is a psychological entity. It is an activity carried out by an individual. This activity is of a certain type. It is *mental*.

(2) A concept reproduces or constructs the essence of the object.

The second point states that, in the concept of an object, the essence of the object can be captured *in its mental construction or reproduction.* Not any reproduction will fit here. The construction has to satisfy a condition: it is supposed to *reproduce* the object's essence. We

can ask the question: is such a thing possible? For Davydov, it is. And, as we can see, this is a very strong epistemological assumption.

Of course, Davydov was very well aware of the epistemological commitment he was making. In fact, on this point, as we saw above, he drew on Kant's concept of the schema and the role imagination plays therein. He wrote: "Kant was right in indicating its [the imagination's] role in constructing a concept" (Davydov, 1990, p. 118). But he disagreed with Kant on two counts:

First, he disagreed with Kant's assumption that schema-making and imagination are both powers of the mind (something that is built into the architectonics of the human mind). On this point, Davydov and Kant are far away from each other. As a good dialectical materialist thinker, Davydov understood human cognition as something deeply related to the cultural-historical context. We find the following statement at the very beginning of his very dense chapter "Basic Propositions in the Dialectical Materialist Theory of Thought": "An individual person's thought is the functioning of historically developed forms of society's activity which have been *conferred on* him" (Davydov, 1990, p. 108; emphasis in the original). And then, this other statement: "An analysis of the origin and development of thought must begin with a clarification of the features of human labor activity" (Davydov, 1990, p. 108). These statements, which derive from Marx's ideas of *effective reality* (*Wirklichkeit*) and human cognition discussed above, are antithetic to Kant's conception of human thought. Kant's philosophy and dialectical materialism parted ways here and ended up informing two radically different pedagogies—one articulated around the work of Piaget and the other articulated around the work of Vygotsky, one of which is the El'konin-Davydov Program.

Second, Davydov is in disagreement with Kant on the role of material culture in the constitution of the schema. For Davydov, the question of material culture is crucial as it is through the practical activity with concrete objects, and more specifically with their conceptual *transformation*, that the essence of an object can be disclosed. Davydov wrote that through the transformation of material things "the learning person comes to understand the relation between a material's external appearances and changes in appearances" (Davydov, 1999, p. 126).

So, there are important disagreements in the ways Davydov and Kant understood the human mind. Both thinkers recognize the active side of the mind in concept formation, but they end up taking different routes.

The differences may be better appreciated if we bear in mind the fact that Davydov's concept of the concept draws from an epistemological shift that occurred in sixteenth century Europe. Before this shift, to know a thing was generally understood as to know the properties of such a thing. This conception of knowing was epitomized by classical Greek geometry. Then, with the arrival of new forms of production in the late Middle Ages and early Renaissance, with the invention of Western capitalism, the conception of knowing changed. As Arendt put it, within the new epistemology of early modernity, to know something became associated with knowing the manner of its production: "I 'know' a thing whenever I understand how it has come into being" (Arendt, 1958, p. 585). We find this conception clearly articulated in Spinoza's 1667 text (modern edition in Spinoza, 1989), *De Intellectus Emendatione* (*Improvement of the Understanding*). Spinoza says that a true idea "shows how and why something is or has been made" (1989, p. 29). Thus,

> If a circle be defined as a figure, such that all straight lines drawn from the center to the circumference are equal, every one (sic) can see that such a definition does not in the

> least explain the essence of a circle, but solely one of its properties. (Spinoza, 1989, p. 32)

Within the new epistemological paradigm, the "how and why something is or has been made" have to be revealed in what Spinoza calls its "proximate cause." Spinoza then gives the proper definition: "the figure described by any line whereof one end is fixed and the other free. This definition clearly comprehends the proximate cause" (Spinoza, 1989, p. 32).

Spinoza's concept of proximate cause is what appeared later thematized as a schema or a rule, as we have seen in Davydov's concept of the concept. In his commentary on Spinoza's definition of the circle, Davydov says:

> Spinoza perceived the essence of a circle in the act of its emergence or construction ("creation"). Its definition should express the reason why the given thing *arose*, the method of *constructing* it . . . Here a method of obtaining any and infinitely varied circles is given. (Davydov, 1990, p. 117)

Spinoza's definition of a circle is cited again and again by dialectician thinkers—Davydov himself, as we have seen, but also other dialecticians such as Ilyenkov (1977). Its importance does not reside only in that it provides us with a neat and short example of a concept (i.e., as something rule-based). The example can also be thought of in terms of an activity with a concrete object—the compass—through which a transformation of material marks on a paper may acquire a theoretical content.

(3) A concept works normatively.

The third point about Davydov's concept of the concept refers to the *regulatory* feature of concepts. As Davydov put it, individuals "act and produce things according to the concepts which exist as norms in the society *beforehand*" (1990, p. 118). From an ontogenetic viewpoint, in this perspective, individuals do not create concepts; they encounter them. This point has to do with one of the main tenets of dialectical materialism that I mentioned before, according to which individuals find the conditions of possibility of their thinking in the historically developed forms of activities of their society.

I shall come back to the implications of Davydov's ontological position in the concluding section. For the time being, I want to discuss the question of the historical and the logical aspects of concepts.

6 The historical and the logical

Davydov did not embrace a recapitulationist idea of concept development. That is, he did not accept the idea that ontogenesis is a recapitulation of phylogenesis, or, more simply put, that human cognitive development follows the same path as the historical path in the development of ideas. To accept a *full repetition* would mean that knowledge would repeat itself regardless of time and culture. On the other hand, to accept a *full independence* between ontogenesis and phylogenesis would amount to reducing the production of knowledge to what individuals do right now, independent of historical influences. To accept a full independence between ontogenesis and phylogenesis could easily lead to the kind of epistemological subjectivism that we find in Kant's theory of knowledge, for instance. These two extreme points (full recapitulation and full independence) are at odds with the tenets of Hegel's dialectics and its ensuing dialectical materialism. Davydov offers an intermediate position. Ontogenesis repeats

phylogenesis in some qualified sense only. As he put it, "Ontogenesis . . . does not repeat phylogenesis *in totality*" (Davydov, 1975a, p. 65; my emphasis). There must be some kind of relationship between them. But what is it?

Davydov's educational work is, in practice, articulated around this question. As mentioned in Section 2, Davydov believed that the school curriculum should start from the basic or essential connections that make up the essence of the conceptual objects to be learned. There is an important ontological assumption here: according to this assumption, objects of knowledge *can* be broken down into a few relations or connections. The problem, then, is to find those small relations or connections. This *ontological* assumption is supplemented with a *methodological* one: history, on the one hand, and current mathematics on the other, can provide us with the clues to find them. In other words, we need to look at the *historical* and the *logical* dimensions of knowledge, and their relationship.

In dialectical materialism, such a relationship is often considered in the following terms. The logical is associated with the movement of *essences*. The historical is associated with the actual movement of the *phenomena* of effective reality. In this context, the logical is considered as the *theoretical reflection* of the historical (see Kopnin, 1966, p. 84). In terms of the concepts discussed in the previous sections, history finds its material in the ontological category of Appearance or Phenomena (the *Concrete*)—not in the category of essences (the Abstract). But, as mentioned before, in dialectical materialism, these categories are deeply interconnected. History, it turns out, is but another name for the theoretical principle of the unity between the Abstract and the Concrete, Essence, and Appearance. This is why the logical aspect of an object, "besides reflecting the history of the object itself, also reflects the history of its knowledge" (Kopnin, 1966, p. 188). It does not come as a surprise, then, that the problem of the relation between the logical and the historical constitutes "the most important [problem] of dialectical logic" (Kopnin, 1966, p. 84).

From a methodological viewpoint, "To reveal the essence of an object it is necessary to reproduce the actual *historical process* of its development" (Kopnin, 1966, p. 186; emphasis in the original). But at the same time, the reproduction of the actual historical process of the object's development "is only possible if we know the essence of the given object" (Kopnin, 1966, p. 186). It seems that we run into a vicious circle. Marx (1993) agonized in front of this problem in the *Grundrisse*, where he was trying to find the proper categories to understand production and consumption. So, how do we get out of this vicious circle? The dialectical answer is this: We can grasp the essence by studying its more *mature forms* as they appear in the phenomenological realm. This is why, "The researcher must begin the study of the object by the end, by the most mature form; for in this most mature form, [the] essential aspects [of the object] appear in a more developed way" (Kopnin, 1966, p. 186). Hence, it does not come as a surprise that Vygotsky cites Marx arguing that "the anatomy of man (sic) is the key to the anatomy of the ape" (Vygotsky, 1978, p. 94). It does not come as a surprise either that in his investigations Davydov turned to the most evolved mathematical forms of his time. What did he find? Bourbaki. The Modern Mathematics of the twentieth century: Bourbaki, but also Lebesgue, Kolmogorov, and others. Through his habitual sharp, penetrating, incisive, and brilliant multidisciplinary approach, he embarked in a theoretical archeological excavation to understand these most mature mathematical forms. What he found was a *structural conception* of mathematics. Davydov cites Bourbaki extensively and his idea that the fundamental mathematical notion is *not* the number. In mathematics—like in life, as understood by the Marxist structuralists of Davydov's time, such as Althusser (1965)—there are *structures* rather than objects. Davydov hence searched hard to find those essential connections and relations

out of which mathematics as a school subject should be erected. He found that it is not numbers we should start with, it is not counting, but other more *primitive* or *fundamental* relations that underpinned, in the twentieth century modern conception of mathematics, the notion of number, namely, the *relations of order* (less than, more than, equal to) and *equivalence*. For him, the primitive notion is *quantity* (understood as a collection of objects endowed with a total order relation). He wrote:

> the concept of number arises within the context of measurement of a continuous quantity so that a multiple relationship is established between that quantity and a part of it that is used as a unit of measure. According to this it is possible to consider counting as the measurement of a set of discrete objects. (Davydov, 1982, p. 228)

Thus, in this view, the origin of arithmetic thinking is not to be found in cardinality. Cardinality is here a derivative concept. Given the evanescence of a Bourbakist approach to mathematics today, we can imagine that if Davydov were to engage in the same archeological digging, he would come up with a different result—and, likely, with a different curriculum proposal as well.

7 A critique of Davydov's concept of the concept

By way of conclusion, let me summarize some of the ideas that I have discussed about Davydov's concept of the concept and outline a brief critique. I hope that my critique may spark some reflections on the theoretical premises, entailments, and commitments embedded in the notions of the mathematical concept that we, as mathematics educators, use implicitly or explicitly in our research.

I mentioned that Davydov drew on the dialectical distinction between the two chief categories of dialectical materialism: essence and appearance, and he assumed a dialectical Hegelian-Marxist ontology that *links* these two categories (as opposed to Kant's ontology that strives to keep them apart). From there he articulated a dialectical concept of the concept: a concept is a form of mental activity derived from human activity, a reproductive rule that works ontogenetically speaking in a normative way. From an epistemological viewpoint, to know an object is to reproduce it mentally in its theoretical movement.

Davydov adopted a view of the concept that is grounded on the model of scientific concepts. For him, scientific concepts are the basis "of theoretical thought" (Davydov, 1990, p. 116), and as we saw in Section 2, it was precisely the child's acquisition of theoretical thought that he strived to promote through his curricular enterprise. One of the main features of Davydov's epistemological stance is the belief that the essence of objects is graspable through the revolutionary concept of modern science, namely *law* (see Section 3.2. above). Davydov says: theoretical thought "is an idealization of the basic aspect of practical activity involving objects, and of the reproduction in that activity of the universal forms of things, their measures, and their laws" (1990, p. 116).

We cannot fail to see that Davydov's concept of the concept is imbued with a general outlook of the world that favors a certain form of knowability—the one that predominated throughout the twentieth century and considered *scientific* theoretical thought as the summit of human cognition. *Scientific* theoretical thought was perceived as the model *par excellence* by which to understand the world. Certainly, Davydov was not the only educational psychologist to embrace such a view. We only need to think of Vygotsky's famous distinction between everyday and scientific concepts.

One of my reviewers argues that Vygotsky's (and Davydov's) use of the adjective "scientific" to designate a specific type of concept does not amount to any commitment with a scientific outlook of the world. I would like very much to agree, perhaps more than anyone, but I do not think that the use of the adjective "scientific" is merely circumstantial. I side here with van der Veer (1991, 1996), who has shown that Vygotsky's work was deeply influenced by the modern idea of culture that was built around the model of science, a model that transpires in Vygotsky's marked interest in *tools* as means to master nature (and oneself). If we remove the concept of tool from Vygotsky's work, internalization and Vygotsky's laws of cultural development vanish into thin air. In *Ape, Primitive Man, and Child*, Vygotsky and Luria argued that the first phase in the child's cultural development lies precisely in the use of tools:

> We have seen how the small child, for whom the world of external objects was initially alien, gradually comes closer to it, and begins to master those objects and make functional use of them as tools. This is the first phase in cultural development. (Luria & Vygotsky, 1998, p. 117)

Vygotsky's commitment to a scientific outlook of culture also transpires in his emphasis on abstract thinking. So, when Luria (1931, 1934) carried out the "Psychological Expeditions to Central Asia" (that he prepared with Vygotsky), what were the questions he posed to the Uzbekistan peasants? Aristotelian syllogisms—i.e., the embodiment of decontextualized thinking.

Undoubtedly, there were very powerful historical reasons in the twentieth century that led people to see the world through the lenses of science. First, there was the modern general "belief in technology and the promise it held for cultural and social progress" (van der Veer, 1996, p. 258). Then, there were the two world wars; then the Cold War (i.e., the time in which Davydov developed his work). The military successes of these wars were absolutely dependent on scientific and technological development. The famous founding document of Khrushchev's reform, "Law on Strengthening the Links Between School and Life, and Further Development of the System of Public Education in the USSR" conveyed in an explicit manner the idea of scientific knowability as the true one: "The Soviet school educates the younger generation in the spirit of the most progressive ideas, the ideas of communism, and forms in young people a materialistic worldview—*the basis of truly scientific knowledge of the world*" (Government of the USSR, 1958; cited in Boyko, 2019, pp. 81–82; my emphasis).

Within this scientific outlook, law, measure, and calculation became the key concepts to understand the world. Commenting on this epistemological scientific outlook specific to the modern period of the Western world, Heidegger (2002, p. 48) says, "Beings became [conceived of as] transparent objects capable of being mastered by calculation." Heidegger gives the example of a stone. "The stone presses downwards and manifests its heaviness" (p. 24). But can heaviness be reduced to its numerical value?

> If we try to grasp the stone's heaviness . . . by placing it on a pair of scales, then we bring its heaviness into the calculable form of weight. This perhaps very precise determination of the stone is a number. (Heidegger, 2002, p. 25)

Now, can we say that we have grasped the stone in this way? Heidegger's answer is no. Can we say at least that we have grasped the stone's heaviness? Again, Heidegger's answer is no: "the heaviness of the weight has escaped us" (Heidegger, 2002, p. 25). The stone "shows itself only when it remains undisclosed and unexplained" (p. 25). We could say the same of the

circle and its roundness: the process of its construction does not exhaust the essence of the object (notwithstanding Spinoza and the Spinozist dialecticians).

Davydov's epistemology is articulated around this modern view where truth is of the order of the logical and the scientifically conceptual. Certainly, the focus on this order of reality is the strength of scientific thought. But it is also its weakness. The reason is that scientific thought and its scientific laws necessarily marginalize the centrality of the singular, the concrete, the sensible, and the complex ever-changing feature of cultural contexts. In doing so, scientific thought marginalizes a part of the whole complex conditions of the object's cultural-historical elaboration (e.g., aesthetical, ethical, political, and economic conditions).

But I want to go a bit further. So far, I have been concerned with, and critical of, the totalizing stance with which scientific thought has been endowed in the understanding of reality. My argument has been *epistemological*. Now I want to articulate an *ontological* argument. What Heidegger is saying in the citations above is that there is a substrate in any object that is *nonconceptual*. In dialectical terms, the nonconceptual of an object is precisely the object's own negation—the non-identical-with-itself—that which, in the object's historical movement, is what the object is *not*. It is the unsettling and inexhaustible realm of the object that resists consciousness. As a result, there is always a partial inadequacy of the concept and the real. "Knowledge does not wholly possess any of its objects" (Adorno, 2008, p. 187). In other words, what individuals produce through practical sensible activity—through *praxis*—is both conceptual and nonconceptual, the latter being "the sheer heterogeneity of thought" (Huhn, 2004, p. 17).

In the creative historical movement of effective reality objects emerge. In their emergence, each one of them brings to the fore a conceptual dimension, but also a nonconceptual one. Now, it is in the nature of the nonconceptual to resist translation into the symbolic—scientific laws, painting, poetry, music, linguistic expression, etc. It resists encapsulation:

> no matter how hard we try for linguistic expression of such a history congealed in things [the movement of the real—LR], the words we use will remain concepts. Their precision substitutes for the thing itself, without quite bringing its selfhood to mind; there is a gap between words and the thing they conjure. Hence, the residue of arbitrariness and relativity in the choice of words as well as in the presentation as a whole. (Adorno, 1973, pp. 52–53)

Thus, "The cognitive utopia would be to use concepts to unseal the nonconceptual with concepts" (Adorno, 1973, p. 10). It is only through the reflexive and critical praxis that Marx (1998) talks about in the third thesis on Feuerbach (see analysis in Macherey, 2008) that humans have the possibility to change their circumstances and their ideas. Thus, only through such praxis, by going to the heart of contradictions, a dim light may be shed on the nonconceptual. In the course of this movement, human consciousness and the object have been transformed, but a new nonconceptual has emerged. It is impossible to get rid of the nonconceptual. It is what propels consciousness *forward* again and again. This is why the nonconceptual is the vitality of life. To cite Heidegger again, it is what "remains undisclosed and unexplained" (2002, p. 25). In the case of numbers, to come back to Davydov's work, the nonconceptual is that which resists being encapsulated by the concept of quantity and the structure of total order. In the case of the Euclidean circle, the nonconceptual is that which is more than the traces left by a radius in motion. Were there a total coincidence between concept and object, we would find ourselves in a closed universe, a bound reality. Dialectics—i.e., the movement of life, "the consistent sense of nonidentity" (Adorno, 1973, p. 5)—would collapse: life would become movement without movement in a tautological silent identity.

Like Davydov, I do think that "An individual person's thought is the functioning of historically developed forms of society's activity which have been *conferred on* him" (sic) (Davydov, 1990, p. 108; emphasis in the original). I concur with him that labor—what I call in my work *joint-labor* (Radford, 2019)—"is the basis of all human cognition," and that it is "Only within historically developing modes of . . . activity [that] all forms of thought [are] formed" (Davydov, 1990, p. 108). Yet, I prefer not to conceive of a concept as a mental rule, but as something more *poetic*, something that brings together the cultural rationality and worldview of the contexts where the object has emerged and evolved with all its historical and political tensions. In this view a concept would be what we *enact* with *others* in joint activity—a cultural-historical enactive experience that is not merely conceptual and theoretical, but also esthetic, ethical, political, and emotional; something that *questions* us.

Acknowledgments I would like to deeply thank Anna Sfard and two anonymous reviewers of the journal for their precious comments.

Funding information This article is a result of a research program funded by the Social Sciences and Humanities Research Council of Canada/Le conseil de recherches en sciences humaines du Canada (SSHRC/CRSH)

References

Abramov, A. (2010). Toward a history of mathematics education reform in Soviet schools (1960s−1980s). In A. Karp & B. Vogeli (Eds.), *Russian mathematics education. History and world significance* (pp. 87−140). Singapore: World Scientific Publishing.

Adorno, T. (2008). *Lectures on negative dialectics*. Cambridge: Polity.

Adorno, T. W. (1973). *Negative dialectics*. New York: The Seabury Press.

Althusser, L. (1965). *Pour Marx* [for Marx] Paris: Maspero.

Andler, C. (1931). Le fondement du savoir dans la "Phénomenologie de l'esprit" de Hegel [the foundation of knowledge in Hegel's "phenomenology of Spirit"]. *Revue de Métaphysique et de Morale, 38*(3), 317−340.

Arendt, H. (1958). The modern concept of history. *The Review of Politics, 20*(4), 570−590.

Aron, R. (1981). *Introduction à la philosophie de l'histoire. [introduction to the philosophy of history]*. Paris: Gallimard.

Boyko, M. (2019). *Soviet mathematics curriculum reforms (1958−1985): Redefining the purpose of mathematics education*. Toronto, Canada: Ph. D. Thesis. University of Toronto.

Davydov, V. (1967). The problem of generalization in the works of L. S. Vygotsky. *Soviet Psychology, 5*(3), 42−52.

Davydov, V. (1975a). Logical and psychological problems of elementary mathematics as an academic subject. In L. Steffe (Ed.), *Children's capacity for learning mathematics. Soviet studies in the psychology of learning and teaching mathematics, vol. 7* (pp. 55−107). School Mathematics Study Group: Stanford, CA (Original work published 1966).

Davydov, V. (1975b). The psychological characteristics of the "prenumerical" period of mathematical instruction. In L. Steffe (Ed.), *Children's capacity for learning mathematics. Soviet studies in the psychology of learning and teaching mathematics, Vol. 7* (pp. 109−205). School Mathematics Study Group: Stanford, CA.

Davydov, V. (1982). Psychological characteristics of the formation of elementary mathematical operations in children. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: cognitive perspective (pp. 224−238)*. Lawrence Erlbaum Associates.

Davydov, V. (1990). *Types of generalization in instruction: Logical and psychological problems in the structuring of school curricula. Soviet studies in mathematics education* (vol. 2). Reston, Virginia: NCTM.

Davydov, V. (1999). What is real learning activity? In J. Lompscher & M. Hedegaard (Eds.), *Learning activity and development* (pp. 123−138). Aarhus: Aarhus University Press.

Davydov, V. (2008). *Problems of developmental instruction: A theoretical and experimental psychological study*. (P. Moxhay, Trans. New York: Nova Science Publishers.

de Freitas, E., Sinclair, N., & Coles, A. (2017). *What is a mathematical concept?* New York: Cambridge University Press.

Descombes, V. (1996). *Les institutions du sens [the institutions of meaning]*. Paris: Les Éditions de Minuit.
Dupré, L. (1983). *Marx's social critique of culture*. New Haven: Yale University Press.
Hegel, G. (1977). *Hegel's phenomenology of spirit (A. V. Miller, Trans.)*. Oxford: Oxford University Press (First edition, 1807).
Hegel, G. (1991). *The encyclopaedia logic. Part I of the encyclopaedia of philosophical sciences (T, F. Geraets, W. A. Suchting, & H. S. Harris, trans.)*. Indianapolis / Cambridge: Hackett Publishing Company, Inc..
Heidegger, M. (2002). *Off the beaten track (J. Young & K. Haynes, trans.)*. Cambridge: Cambridge University Press.
Henry, M. (1976). *Marx*. Paris: Gallimard.
Huhn, T. (2004). *The Cambridge companion to Adorno*. Cambridge, UK: Cambridge University Press.
Ilyenkov, E. V. (1977). *Dialectical logic*. Moscow: Progress Publishers.
Ivashova, O. (2011). The history and the present state of elementary mathematical education in Russia. In A. Karp & B. Vogeli (Eds.), *Russian mathematics education. Programs and practices* (pp. 37–80). Singapore: World Scientific Publishing.
Kant, I. (2003). *Critique of pure reason (N. K. Smith, Trans.)*. New York: St. Martin's Press.
Kopnin, P. (1966). *Lógica dialectica [Dialectical logic]*. Mexico: Editorial Grijalbo.
Libâneo, J., & Freitas, R. (2013). Vasily Vasilyevich Davidov: A Escola e a formacão do pensamento teórico-científico [Vasily Vasilyevich Davidov: The school and the formation of theoretical-scientific thinking]. In A. Maturano Longarezi & R. Valdés Puentes (Eds.), *Ensino Desenvolvimental. Vida, pensamento e obra dos principais representantes russos* (pp. 315–350). Uberlândia, Minas Gerais: Editora da Universidade Federal de Uberlândia.
Luria, A. R. (1931). Psychological expedition to Central Asia. *Science, 74*, 383–384.
Luria, A. R. (1934). The second psychological expedition to Central Asia. *Journal of Genetic Psychology, 41*, 255–259.
Luria, A. R., & Vygotsky, L. S. (1998). *Ape, primitive man, and child: Essays in the history of behavior*. Boca Raton, Fl: CRC Press LLC.
Macherey, P. (2008). *Marx* 1845. *Les thèses sur Feuerbach. [Marx 1845.* The theses on Feuerbach]. Paris: Éditions Amsterdam.
Marx, K. (1993). *Grundrisse: Foundations of the critique of political economy*. London, England: Penguin Books.
Marx, K. (1998). *The German ideology, including theses on Feuerbach and introduction to the critique of political economy*. New York: Prometheus Books.
Piaget, J. (1964). *The child's conception of number*. London: Routledge & Kegan Paul Ltd..
Radford, L. (2019). On the epistemology of the theory of objectification. In U. T. Jankvist, M. V. D. Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11, February 6–10, 2019)* (pp. 3062–3069). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.
Rubtsov, V. V. (2015). V. V. Davydov – The founder of a significant scientific school and director of the psychological institute. *Psychological Science and Education, 20*(3), 182–196.
Schmittau, J., & Morris, A. (2004). The development of algebra in the elementary mathematics curriculum of V. *V. Davydov. The Mathematics Educator, 8*(1), 60–87.
Spinoza, B. (1989). *Ethics including the improvement of the understanding. (R. Elwes, Trans.)*. Buffalo: Prometheus (Original work published 1667).
Spirkin, A. (1990). *Fundamentals of philosophy*. Moscow: Progress Publishers.
Swanson, D., & Williams, J. (2014). Making abstract mathematics concrete in and out of school. *Educational Studies in Mathematics, 86*, 193–209.
van der Veer, R. (1991). The anthropological underpinning of Vygotsky's thinking. *Studies in Soviet Thought, 42*, 73–91.
van der Veer, R. (1996). The concept of culture in Vygotsky's thinking. *Culture and Psychology, 2*, 247–263.
Vygotsky, L. S. (1978). *Mind in society*. Cambridge, Ma: Harvard University Press.