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TOWARDS AN EMBODIED, CULTURAL, AND MATERIAL CONCEPTION OF MATHEMATICS COGNITION

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In this article I sketch an embodied, cultural, and material conception of cognition and discuss some of the implications for mathematics education. The sketched approach, which I term sensuous cognition, rests on a cultural and historical dialectical materialist understanding of the senses, sensation and the material and conceptual worlds. Sensation and matter are considered to be the substrate of mind, and of all psychic activity (cognitive, affective, volitional, etc.). I argue that human cognition can only be understood as a culturally and historically constituted multimodal sentient form of creatively responding, acting, feeling, transforming, and making sense of the world. To illustrate the aforementioned ideas I briefly refer to a classroom episode involving 7–8-year-old students dealing with pattern generalization.

Key Words: Cognition, sensation, senses, materiality, concepts, multimodality.

> *Am I so dependent on the body and the senses that I could not be without them?* Descartes (1641/1982, p. 19)

1. INTRODUCTION

In a Grade 10 class, working in small groups of three, 15–16-year-old students discuss how to draw a distance-time graph related to two moving individuals. The individuals (Mireille, to the left, and Nicholas, to the right; see Figure 1) start walking at the same time on a straight line in opposite directions. Knowing that Nicholas starts from point R and stops at point S, while Mireille starts at point P and stops at point Q, the students are invited to come up with a graph conveying information about the distance between Mireille and Nicholas as time elapses,

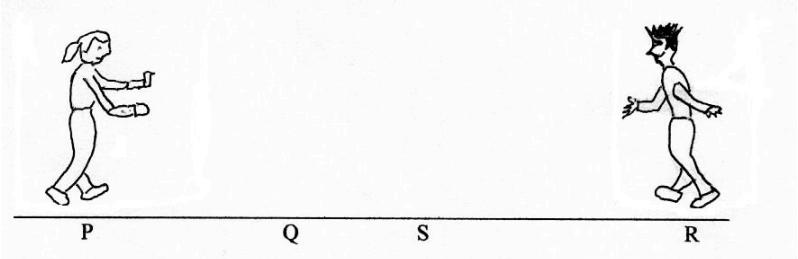


Figure 1. Mireille (left) and Nicholas (right) move towards each other.

Since no indication is given about speeds, in one of the groups, Zacko, Carla, and Jane start considering the simpler case—both individuals travel at the same speed. Distance, of course, decreases, but how? Carla suggests a kind of parabolic shape; Zacko suggests a straight line:

Carla: So she [Mireille] goes like this (*referring to the drawing (Figure 1) she moves her pen from the Mireille's place towards Nicholas*) … because she… she goes towards…

Zacko: Yeah, but he [Nicholas] is going towards her too… It's going to be weird…

Carla: OK, minute… a parabola!

Zacko: Its not a parabola! it would be a… steep [line] (*making an almost vertical gesture*)

Carla: Why?

Zacko then opens his arms and simulates the individuals' motion (see Figure 2):

Zacko: I am just coming closer and closer in less time (*while talking, he moves slowly her hands one towards the other*).

Jane: Yes, like… it takes less time to arrive… (*Articulating once more her thought*) because it has taken half time to arrive.

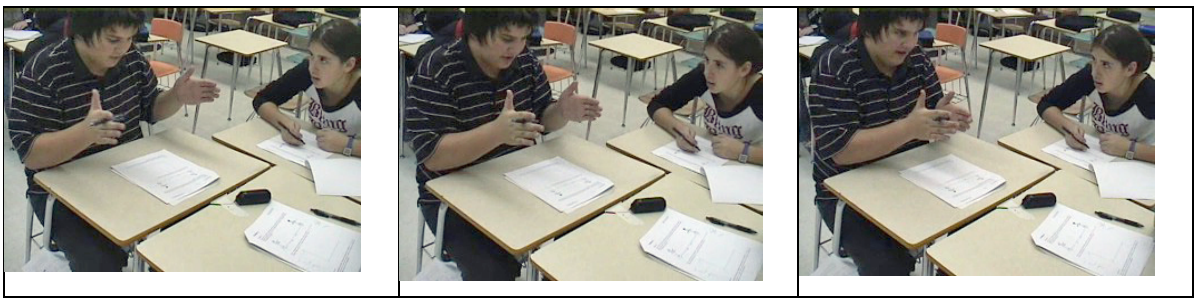


Figure 2. Zacko makes a dynamic gesture to signify the decreasing space between Mireille and Nicholas.

About two decades ago, an analysis of the previous episode for a mathematics education audience would have been unlikely to include an analysis of gestures, body posture, and other embodied signs. Indeed, most of the psychological traditions that inspired our field at the time conceptualized the body and material culture as playing a secondary role in cognition. And if the body played some role, as in Piaget's (1970) influential genetic epistemology, it was as a mere transitory step towards abstract, genuine thinking. That was the fate of the sensory-motor stage in Piaget's account of conceptual development.

New research trends, however, offer a different approach to the understanding of human cognition. They consider our tactile-kinesthetic bodily experience of the world and our interaction with artifacts to be much more than transitory or merely secondary aspects of cognition (Bautista & Roth, 2011; Borba & Villareal, 2006; Edwards, Radford, & Arzarello, 2009; Lakoff & Núñez, 2000; Sheets-Johnstone, 2009). Within these research trends, an account of the students' understanding of the problem and the production of their graph in the previous episode would require an analysis of the students' embodied and discursive activity. Many interpretations and accounts are possible. For instance, some approaches inspired by cognitive linguistics (e.g., Fauconnier and Turner 2002; McNeill, 2005) emphasize the metaphoric dimension of language and the integrative constitution of embodied mental spaces (see e.g., Edwards, 2009; Lakoff & Núñez, 2000; Yoon, Thomas, & Dreyfus, 2011). Other approaches inspired by research in phenomenology emphasize the fleshy nature of thought (Thom & Roth, 2011), while others stress its material dimension (de Freitas & Sinclair, 2013).

Embodied and materialist approaches to cognition, however, face the difficult problem of offering a cogent account of the theoretical categories of the conceptual and the embodied, and their relationships. The apparent absence of compelling accounts of these categories has led some critics, such as Terry Eagleton, to see in contemporary embodied perspectives of the mind no more than "the return in a more sophisticated register of the old organicism" promoted by 17th century empiricists and a token of "the post-modern cult of pleasure" and love for the concrete (Eagleton, 1998, pp. 157-58). What Eagleton fears is that contemporary embodied perspectives remain trapped in the sensing subject, and hence in a curious form of radical subjectivism with unclear possibilities to regain contact with culture and history (Eagleton, 1996).

Dwelling upon Vygotsky's (1987-99) and Leont'ev's (1978, 2009) work, and Hegelian dialectic materialism (Ilyenkov, 1977), in this article I articulate a cultural-historical theoretical perspective on cognition. Briefly put, in the perspective on cognition that I outline here—that I have previously termed *sensuous cognitio*n (Radford, 2009a)—mind, body, and world are conceived of as intertwined entities. Sensuous cognition stresses the idea that our thinking, feelings, deeds, and in fact all our relations to the world (hearing, perceiving, smelling, sensing, etc.), are an *entanglement* of our body and material and ideational culture.

The idea of sensuous cognition rests on a specific historical understanding of sense, sensation, materiality, and the conceptual realm. Within this theoretical perspective, our cognitive domain can only be understood as a culturally and historically constituted sentient form of creatively responding, acting, feeling, imagining, transforming, and making sense of the world. I articulate this idea in the second part of the article. In the first part, I discuss how idealist and empiricist epistemologies framed the problem of the sensual and the conceptual. In the third part of the article, to illustrate in a concrete manner the idea of sensuous cognition I

briefly refer to a classroom episode involving 7–8-year-old students dealing with pattern generalization.

2. THE SENSUAL AND THE CONCEPTUAL

The insouciance that classical psychology has shown towards the body in the investigation of human thinking is related to a great extent to the idealist theories of knowing that have been influential since Plato's time.

Plato considered the body to be a nuisance or obstacle in the attainment of true knowledge (Radford, 2005; Radford, Edwards, Arzarello, 2009). The 17th and 18th century idealists continued this tradition. When, in *Second Mediati*on, Descartes asks the question: "What am I?" he answers: "A thinking substance" (Descartes, 1641/1982, p. 22). "I am anything but mind" (p. 25). For Descartes, to know something amounted to having a distinct apprehension of the thing to be known. "I cannot be deceived in judgments of the grounds of which I possess a clear knowledge." (p. 56). And apprehension and the distinctiveness of things were not ensured by the senses. Thus, to explain how bodies and external things become known, Descartes says that "bodies themselves are not properly perceived by the senses nor by the faculty of imagination" (Descartes, 1641/1982, p. 26). True knowledge is ensured, Descartes continues, "by the intellect alone … [things] are not perceived because they are seen and touched, but only because they are rightly comprehended by the mind" (p. 26). Knowledge was not to be sought in the materiality of things or in the feelings of the sentient body. Knowledge and ideas, the idealist philosophers supposed, are in us. This is what Leibniz contended: "our ideas, even those of sensible things, come from within our own soul" (Leibniz 1949, p. 15).

However, not all philosophical traditions followed a disembodied view of thinking and knowing. The empiricists offered an opposing view. Hume, for instance, argued that ideas are impressions or combination of impressions that external things cause on us. Kant tried to articulate a theory of knowledge that combines the idealist and the empiricist tenets, resulting in a kind of compromised rationalism traversed by unresolvable tensions.

In the *Critique of Pure Reason* Kant says:

> A new light flashed upon the mind of the first man (be he Thales or some other) who demonstrated the properties of the isosceles triangle. The true method, so he found, was not to inspect what he discerned either in the figure, or in the bare concept of it, and from this, as it were, to read off its properties; but to bring out what was necessarily implied in the concepts that he had himself formed *a priori*, and had put into the figure in the construction by which he presented it to himself. If he is to know anything with *a priori* certainty he must not ascribe to the figure anything save what necessarily follows from what he has himself set into it in accordance with his concept. (Kant, 1781/1929; p. 19; B xi-xii)

The first part of the passage reveals Kant's acknowledgment and dismissal of the British empiricist approach to knowledge formation (as elaborated, in particular, by Hume and Berkeley). The second part ("but to bring out…") reveals Kant's alignment with the rationalist tradition (as epitomized by Descartes, Leibniz and others). In fact, Kant's epistemology is a desperate battle to reconcile sensual empiricism and conceptual idealism. As the example of the triangle suggests, according to Kant, in coming to know something, we resort to a *sensuous construction or representation*. However, what we represent is not *the* thing to be known (for instance, the concept of triangle) but *a* particular one. Furthermore, what we learn of the thing to be known is not read from the particular representation thus drawn, but from something *general* that, for him, is beyond the sensuous realm: something not sensual but intellectual.

Kant's epistemology rests indeed on the claim that human cognition will always need *both* a sensible component and an intellectual one. He theorizes the sensible part through the concept of *sensibility*: that is, our capacity for being affected by material things. He theorizes the intellectual component through the concept of *understanding*. He says:

> Without sensibility no object would be given to us, without understanding no object would be thought. Thoughts without content are empty, intuitions [i.e., representations] without concepts are blind . . . These two powers or capacities [sensibility and understanding] cannot exchange their functions. The understanding can intuit nothing, the senses can think nothing. Only through their union can knowledge arise. (Kant, 1781/1929; p. 193; B75-76)

To remain faithful to the rationalist project, Kant hence ended up portraying a theory where reason appears supported by the concrete realm of the senses and where, in the end, the rationalist component nevertheless predominates. He can offer a view of reason that, while requiring a sensual support, obeys to universal logical mechanisms regardless of culture and time. Piaget followed the same path—removing nevertheless all the aprioristic stance of Kant's theory of knowledge. The sensory-motor stage yields the developmental road of embodied actions to disembodied intellectual thought. The ephemeral concrete realm of action and materiality remains the carrier and the expression of a thinking measured by its rational structural features.

The previous discussion provides us with a short overview of the manner in which idealist epistemologies envisioned the realm of the conceptual and the sensual. Certainly, Kant offers an interesting and profound account. His concepts of sensibility and understanding are an attempt to respond to the sensual exigencies of empiricism and the logical demands of rationalism, respectively. In doing so, however, he adopts an intrinsic dualism that keeps the conceptual and the sensual separately.

Hegel (1830/2009) criticized Kant's dualistic stance. The problem, Hegel considered, was that Kant adopts a weak concept of the concrete world and subjectivity, leading to an impoverished idea of the conceptual and the sensual, and to an unnecessary distinction between them.

In the next section I sketch a view of the human mind that draws from a different philosophical perspective—the Hegelian historical-materialist dialectic paradigm—that posit mind and matter as consubstantial entities. Such a view provides, I argue, new possibilities to understand the role of senses, sensation and materiality in the teaching and learning of mathematics.

3. SENSUOUS COGNITION

In dualistic accounts, the mind is conceived of as operating through two distinctive planes, one internal and one external. The internal plane is usually considered to include consciousness, thought, ideas, intentions, etc., while the external plane refers to the material world—which includes concrete objects, our body, its movements, and so on. The idea of sensuous cognition that I advocate here rests on a non-dualistic view of the mind. Within this theoretical view, mind is considered to be a property of matter. More specifically, mind is conceptualized as a feature of living material bodies characterized by a capacity for *responsive sensation.*

Sensation is a phylogenetically evolved feature of living organisms through which they *reflect* and *respond to* or *act on* their environment. Since the organism is itself a part of the material world, any reflection of and action on reality is strictly a function of a material, corporeal organism (Leont'ev, 2009). As a result, reflection and action do not occur in two *separate* planes. They occur in the same plane—the plane of life.

Now, reflection cannot be considered a passive act of receiving sensorial impressions, as 17th and 18th century empiricists hold. Reflection involves both: (1) something that transcends the organism as such (something that, in order to differentiate it from the subject itself, we can call *objective,* namely *the object of reflection*), and (2) the *reflected object*, something that is *subjective* (in the sense that reflection depends on the *specific* organism reflecting the environment). In this account, reflection is a *relational category* between subject and object that keeps them entangled with each other. Reflection's phylogenetic evolution gives rise to a range of sensuous relationships between the organism and the world (thermal and other forms of sensation, movement, and so on).

In the case of humans, the entanglement of subject and object, mind and world, acquires a specific dimension by virtue of their historically evolved capacity to interact with the world and among species' members. The explanations of such a capacity is beyond the scope of this article (see, e.g., Greenspan & Shanker, 2004; Leont'ev, 2009; Mithen, 1996). Suffice it to say that this evolution culminated with a progressive refinement of the senses and sensation. This is why the forming of the human senses is not a natural process but the "labor of the entire history of the world

down to the present" (Marx, 1932/1988, pp. 108-109). The human eye, for instance, "has become a *human* eye, just as its *object* has become a social, human object" (p. 107), that is an object of labor, emanating from social sensuous interaction. In the course of labour and interaction, the human senses appeared hence transformed and became what they are now, highly sensitive cultural organs—"*theoreticians*" as Marx put it (1932/1988, p. 107, emphasis as in the original).

The historical nature of sense and sensation and the entanglement of cognition and the world make it impossible to reduce the material world to pure stuff. Matter, notwithstanding the empiricists, is not merely the concrete stuff that we touch with or hands or perceive with our eyes. Matter—all that is revealed to us in our dealing with the world—is cultural-historical in the sense that it bears in itself the traces of human labour and intellectual activity. To see matter subjectively only is to make the mistake that Marx reproached to materialist philosopher Ludwig Feuerbach (1843/1986):

> [Feuerbach] does not see that the sensuous world around him is not a thing given direct from all eternity, remaining ever the same, but the product of industry and of the state of society; and, indeed, [a product] in the sense that it is an historical product, the result of the activity of a whole succession of generations . . . Even the objects of the simplest "sensuous certainty" are only given him through social development, industry and commercial intercourse. (Marx, 1846/1998, p. 45)

What Feuerbach does not grasp, Marx contends, is that what we access through our senses is not the world in its intact materiality, Adam and Eve's intact world, but a world of labour replete with historical and cultural sensuous activity:

> The cherry-tree, like almost all fruit-trees, was, as is well known, only a few centuries ago transplanted by *commerce* into our zone, and therefore only *by* this action of a definite society in a definite age it has become "sensuous certainty" for Feuerbach. (Marx, 1846/1998, p. 45; emphasis in the original)

The concept of sensuous cognition rests on this understanding of sense, sensation, and the material world as historical and cultural products.

4. REVISITING THE CONCEPTUAL

The aforementioned emphasis on sensation and matter does not amount, however, to reducing the mind to the realm of pure senses or the materiality of the world. The chief idea of sensuous cognition is that our thinking, feelings, deeds, in fact all of relations to the world (hearing, perceiving, smelling, sensing, etc.), are an entanglement of both the *material* world and *ideational* culture.

Now, the notion of ideational culture rests on a dialectical understanding of the conceptual. In classical rationalist and idealist epistemologies the conceptual is reduced to something already given —concepts are mind-independent entities (e.g.,

Kant's "things-in-themselves")— or are the products of the individual's cogitations. Western philosophy is to an important extent the tormented effort to explain how we can have access to mind-independent entities or, conversely, how the subject's self-produced ideas relate to the concrete world. In one case or the other, concepts are considered susceptible to be *represented*. They are somewhere "there" (in a transcendental world of ideas or inside the subject's mind). To the static ontology of Kant, Platonists and Idealists, Hegel opposed an ontology of *movement*. Concepts (which belong to Hegel's theoretical category of the *general*) are pure virtuality or possibility. They come into existence when we set them in motion, in practical embodied and material activity, and through which they appear in sensuous singularity. When Vygotsky (1987) argues that all conceptual phenomena need to be studied as processes in motion and change, he is drawing on this Hegelian feature of concepts.

Concepts considered as pure possibility are not to be confused with their idealist or rationalist counterparts. Concepts are *crystalized historical forms of human action* continuously affected and transformed by social practices. The concept of number, for instance, is a pure possibility (e.g., to count things or to carry out complex calculations). To come into existence, concepts have to be endowed with particular determinations. Their coming into existence is the event of their appearance—an always new and unrepeatable sensuous and material event, simultaneously historical and actual, in which the abstract and the concrete, the sensual and the conceptual, come together and stand to each other in a relation of mutual determination. Both are different while at the same time being part of a same organic whole-and-part unity. Through their particular sensuous instances concepts are not transcendentally inaccessible. They are entities that move and are continuously transformed by the activity through which they appear in their sensuous and material form. They cannot appear by themselves: they are mediated entities and their mediation is concrete human practice (Radford, 2013).

5. THE MULTIMODAL NATURE OF CONCEPTS

Human action is multimodal. As codified historical forms of human action, concepts are multimodal too. But they are also multimodal in their actualization, in the passage from the virtual to the actual. Indeed, in their movement into existence, in which they become objects of thought and consciousness, concepts are endowed with particular determinations. They have to be actualized in sensuous multimodal and material activity.

So, in our introductory example, when Zacko opens his arms and puts his hands at a certain distance to simulate Mireille's and Nicholas' walk, he draws on, and puts into motion, cultural-historical concepts of space and time. And he does so according to his own understanding of those concepts—understanding that is built on his prior formal and informal acquaintance with them. The distance between Mirelle and Nicholas that Zacko simulates is not accurate, nor is their speed. Yet, through the sensuous enactment of the walk, Zacko feels the distance decrease. His eyes (and his

teammates' eyes too) are fixated on the hands. Not on the hands as such, but on the hands as means to sensuously *imagine* the phenomenon under consideration. Zacko follows perceptually and viscerally each hand moving towards the other, resorting at the same time to something that apes, insects, and other species lack: a historically constituted and highly categorial language. Through language Zacko can *qualify* what the body in itself can feel but cannot objectify beyond the lived present and kinesthetic memory (Freitag, 2002): an experience that attends in a categorial manner to the *quality* of proximity of the moving individuals and the effect of time. While moving and feeling his hands, Zacko says: "Because I am just coming closer and closer in less time." In the enactment of the concept, during the concept's transition from pure possibility to actuality, the concept appears in its multimodal actuality. The sensual and the conceptual become entangled. No distinction between them is possible. Distance and time—two culturally and historically evolved conceptual categories (Radford, 2008)—become entangled with Zacko's sensuous bodily multimodal actions.

6. THE ONTOGENETIC DEVELOPMENT OF THE SENSES

In the previous sections I outlined a reconceptualization of the sensual and the conceptual. This theoretical reconceptualization offers new avenues to understand cognition as sensuous through and through. It also allows us to posit the problem of the *development* of cognition as a problem intimately related to the cultural development of the senses and multimodality. Indeed, when Zacko opens his arms and moves his hands, one towards the other, he is resorting to an ontogenetically evolved form of multimodality in which the various modes of sensation have become specialized and integrated. Research with newborns shows that, at birth, sensorial modalities are not equally active (Lickliter & Bahrick, 2000). While tactual sensitivity, for instance, is highly developed in the newborn around the mouth-nose cavity, spreading throughout other areas of the body surface later on, olfactory sensitivity is minimal (Zaporozhets & Elkonin, 1971). The third month of life is crucial, for it is in this period that intersensory connections are formed and that the feeling movements of the hand appear. In the fifth and sixth months of life an enlargement and specialization of the development of intermodal connections happens in the visual-kinesthetic and visual-tactual spheres. Thus, "turning the head toward the point of contact [of object and body] and visually fixating upon that point, are expressive of visual-tactual connections" (Yendovitskaya, Zinchenko, & Ruzskaya, 1971, p. 15; see also Lewkowicz & Lickliter, 1994). The specialization of the senses and the acquisition of language will ensure later on that, through a tactile experience, the human hand does not only feel the trace of the object. We can say that the hand also "perceives its colour, its volume, its weight" (Le Breton 2007, p. 151).

In short, like tactility, cognitive functions like thinking, memory, and imagination remain directly and indirectly related to the materiality and conceptuality of the world (as conveyed, e.g., through linguistic conceptual categories, such as

'distance' and 'space' in our example) and to a large range of sensorimotor evolved functions expressed through the organism's movement, perception, sound reception and production, etc. Our cognitive functions unfold and evolve in correlation with the cultural transformation of our sense and our multi-modal sensorial experience of the world.

The ontogenetic process of the cultural transformation of the senses has been investigated in great detail in the past few years. To mention but one example, Zaporozhets (2002) reports research with three- to five-year old preschoolers who were learning to discriminate between variants of two geometric figures: triangles and quadrilaterals. At first, the preschoolers were making a substantial number of errors. Then, they were invited to trace systematically with a finger the outline of the figure, paying attention to directional changes of the motions at angles, and accompanying the tactile exploration with side counting (one, two three…). The investigator reports that at this stage perception was accomplished through the tactile experience, while the eye performed an auxiliary role. "Later," Zaporozhets says, "the eye developed the ability to solve these types of perceptual tasks independently, consecutively tracing the outline of a figure, as it was earlier done by a touching hand" (2002, p. 31). During this process, the eye undergoes a transformative change: "initially, the eye motions have an extremely extensive nature, consecutively tracing the entire outline of the perceived figure and simulating its specifics in all details" (p. 32). In a subsequent stage, the eye's motions "gradually begin to decrease and to focus on the individual, most informative attributes of the object" (p. 32).

7. THE ARTIFACTUAL DIMENSION OF SENSUOUS COGNITION

A closer look at the previous example shows that the new cultural forms of sensation are deeply interrelated with the use of *artifacts*. In Zaporozhets' (2002) example, preschool children develop a mathematical form of perception that allows them to distinguish between cultural categories of geometrical figures. In the course of this developmental process, the children have recourse to the material objects whose contours they cover with a finger while using numbers to count aloud. What this example shows is that our individual senses evolve intertwined not only one with the other senses, but also with the *materiality* of the objects in our surroundings. The materiality that shapes our senses is not, however, reduced to inert matter, but, as the example shows, matter already endowed with meaning (e.g., 'triangularity,' 'quadrilarity,' etc.).

It is this key role of artifacts in the constitution and evolution of forms of sensing and reflecting that Luria and Vygotsky underlined in their work. The use of artifacts, they contended, constitutes the first phase in cultural development (Luria & Vygotsky, 1998; Vygotsky, & Luria, 1994). Such a phase marks the emergence of new forms of actions and reflection and the concomitant appearance of psychic functions.

All in all, the previous discussion amounts to making a point about the embedded nature of artifacts in the evolution of our ways of sensing and reflecting. Luria and Vygotsky stress the fundamental cognitive role artifacts and material culture play in the ways we come to know. The claim that I am making hence goes beyond the conceptualization of artifacts as merely *mediators* of human thinking and experience, or as prostheses of the body. Artifacts do much more than mediate: they are a *constitutive part* of thinking. Behind this view lies, of course, the general concept of mind as a property of matter stated at the beginning of Section 3. This property expresses the enactive relationship between materiality and mind that inspired Vygotsky's, Luria's, and Leont'ev's work and that Bateson (1973) illustrates so nicely in his example of the blind person's stick. It is in this context that anthropologists Malafouris and Renfrew (2010) claim that we can speak of things as having a cognitive life. They say: "things have a cognitive life because minds have a material life" (p. 4).

Sensuous cognition is thus a perspective that highlights the role of sensation and materiality as the substrate of mind and of all psychic activity. But in contrast to other approaches where the focus remains on the individual's body, sensuous cognition offers a perspective where sensation and its cultural transformation in sensing forms of action and reflection are understood to be interwoven with history, cultural artifacts, and materiality at large. Sensuous cognition calls into question the usual divide between mind and matter and casts in new terms the classical boundaries of mind. It offers a new perspective in which to conceive of students' and teachers' actions in teaching-learning processes. In particular, sensuous cognition invites us to pay attention to perception, gestures, kinesthetic actions, sign- and artifact-use in different ways.

8. A CLASSROOM EXAMPLE

In what follows, I would like to discuss an example from a Grade 2 class (7–8-year-old students) involving the generalization of an elementary figural sequence (see Figure 3).

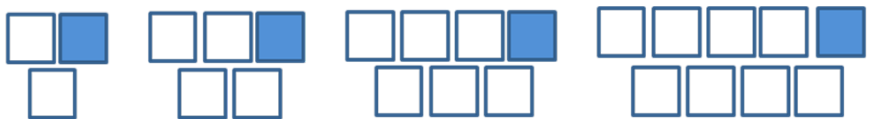


Fig. 3. The first terms of a sequence that 7–8-year-old students investigate in a Grade 2 class.

To become sensitive to the cultural-historical algebraic forms of perceiving terms in sequences like the one discussed here, students engage in processes that are far from mental. They engage with the task of exploring the sequence in a sensuous manner. I would like to illustrate this point by discussing the way in which the

teacher and a group of students reflect on Term 8 of the sequence. The first question of the mathematical activity consisted in extending the terms of the sequence up to Term 6. Then, in questions 2 and 3, the students were asked to find out the number of squares in terms 12 and 25. In question 4, the students were given a term that looked like Term 8 of the sequence (see Figure 4). They were told that this term was drawn by Monique (an imaginary Grade 2 student) and encouraged to discuss in small groups to decide whether or not Monique's term was Term 8. The trained eye would not have difficulties in noticing the missing white square on the top row. The untrained eye, by contrast, may be satisfied with the apparent spatial resemblance of these terms with the other terms of the sequence and might consequently fail to note the missing square.

Fig. 4. The students were requested to discuss whether Monique's term is Term 8 of the given sequence.

Let me focus on the discussion that a group of students had with the teacher—a group formed by James, Sandra and Carla. When the teacher came to see their work the students had already worked for about 32 minutes together. They had finished drawing Terms 5 and 6, tried (unsuccessfully) to find the number of squares in Term 12 and 25, and answered the question about Term 8 (which they considered to be Term 8 of the sequence). The teacher engaged in collaborative actions to create the conditions of possibility for the students to perceive a general structure behind the sequence:

1. Teacher: We will just look at the squares that are on the bottom (*while saying this, the teacher makes three consecutive sliding gestures, each one going from bottom row of Term 1 to bottom row of Term 4; Pics 1-2 in Fig. 5 show the beginning and end of the first sliding gesture*). Only the ones on the bottom. Not the ones that are on the top. In Term 1 (*she points with her two index fingers to the bottom row of Term 1; see Pic. 3*), how many [squares] are there?
2. Students: 1!
3. Teacher: (*Pointing with her two-finger indexical gesture to the bottom row of Term 2*) Term 2?
4. Students: 2! (*James points to the bottom row of Term 2; see Pic 4*).
5. Teacher: (*Pointing with her two-finger indexical gesture to the bottom row of Term 3*) Term 3?
6. Students: 3!
7. Teacher: (*Pointing with her two-finger indexical gesture to the bottom row of Term 4; see Pic 5*) Term 4?
8. Students: 4!

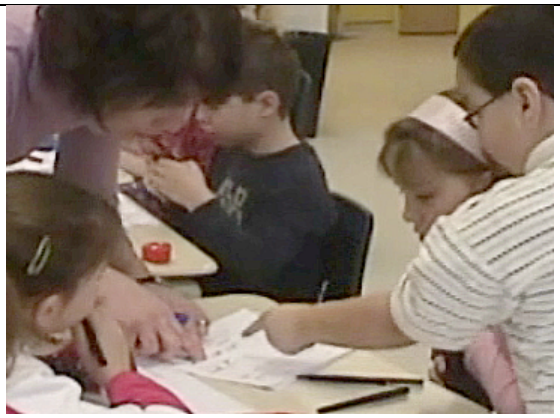
9. Teacher: (*Making a short pause and breaking the rhythmic count of the previous terms, as if starting a new theme in the counting process, she moves the hand far away from Term 4 and points with a two-finger indexical gesture to the hypothetical place where one would expect to find Term 8; see Pic 6*) How many squares would Term 8 have on the bottom?

10. Sandra: (*Hesitantly, after a relatively long pause*) 4?

Pic 1

Pic 2

Pic 3



Pic 4

Pic 5

Pic 6

Figure 5. The teacher's and students' sensuous (perceptual, gestural, tactile, aural, vocal) engagement in the task.

In Line 1, the teacher makes three sliding gestures to emphasize the fact that they will count the bottom row of the four given terms. The gestural dimension of the teacher-students' joint activity is somehow similar to the tactile experience of the students who, in the aforementioned experiment reported by Zaporozhets (2002), follow the contour of shapes with their fingers. Here, the tactile dimension is carried out instead with gestures through which the teacher suggests a cultural form of perceiving the terms of the sequence—one in which the mathematical ideas of variable and the relationship between variables are emphasized.

Now, the teacher does not gesture silently. Gestures are coordinated with utterances. This is why it might be more useful to consider the teacher's utterance as a *multimodal utterance*, that is to say as a bodily expression that resorts to various sensorial channels and different semiotic registers (Arzarello, 2006). In this case, the teacher coordinates eye, hand, and speech through a series of organized simultaneous actions that orient the students' perception and emergent understanding of the target mathematical ideas. In our previous work we have termed *semiotic node* this complex coordination of various sensorial and semiotic registers (Radford, 2009b). The investigation of semiotic nodes in classroom activity is a crucial point in understanding the students' learning processes. The concept of semiotic node rests indeed on the idea that the understanding of multi-modal action does not consist in making an inventory of signs and sensorial channels at work in a certain context. From a methodological viewpoint, the problem is to understand how the diverse sensorial channels and semiotic signs (linguistic, written symbols, diagrams, etc.) are *related, coordinated,* and *subsumed* into a new thinking or psychic *unity* (Radford, 2012). Such a methodological problem makes sense only against the background of a conception of the mind that overcomes the dualistic view of internal-external processes. In our case, the methodological problem makes sense against the background of a concept of the human mind as *sensuous* through and through.

Yet, as Line 10 intimates, the passage from Term 4 to Term 8 was not successful. The objectification (that is, the becoming aware; see Radford, 2010) of the algebraic manner in which sequences can be algebraically perceived has not occurred yet. The teacher hence decided to restart the process, with some important modifications, as we shall see.

As mentioned previously, Term 8 of the sequence was not materially drawn. In the previous excerpt, the teacher *pretends* that Term 8 is on the empty space of the sheet, somewhere to the right of Term 4. She points to the empty space, as she pointed to the other terms, to help the students imagine the term under consideration. During the second attempt, the teacher does not go from Term 4 to Term 8; this time she goes term after term until Term 8.

11. Teacher: We will do it again...
12. Teacher: (*Pointing to Term 1 with a two-finger indexical gesture*) Term 1, has how many?

13. Carla: (*Pointing with her pen to the bottom row*) 1, (*without talking to the teacher points to Term 2 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term 2*) 2, (*again without talking to the teacher points to Term 3 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term 3*), 3, (*same as abov*e) 4, (*now moving to the hypothetical place where Term 5 is expected to be and doing as above*) 5.
14. Teacher: Now it's Term 8! (*The teacher comes back to Term 1. She points again with a two-finger indexical gesture to the bottom row of Term 1*) Term 1, has how many [squares] on the bottom?
15. Students: 1.
16. Teacher: (*Pointing with a two-finger indexical gesture to the bottom row of Term 2*) Term 2?
17. Students: 2!
18. Teacher: (*Pointing with a two-finger indexical gesture to the bottom row of Term 3*) Term 3?
19. Students: 3!
20. Teacher: (*Pointing with a two-finger indexical gesture to the hypothetical place where bottom row of Term 4 would be*) Term 4?
21. Students: 4!
22. Teacher: (*Pointing as above*) Term 6?
23. Students: 6!
24. Teacher: (*Pointing as above*) Term 7?
25. Students: 7!
26. Teacher: (*Pointing as above*) Term 8?
27. Students: 8!
28. Sandra: There would be 8 on the bottom!

The teacher and the students counted together the squares on the bottom row of Monique's term and realized that the number was indeed 8. At this point the relationship between variables started becoming apparent for the students. The relationship started being objectified. The teacher then moved to a joint process of counting the squares on the top row. The students were perplexed to see that contrary to what they believed, Monique's Term 8 did not fit into the sequence (for details, see Radford, 2010).

Later on in the lesson the students were able to quickly answer questions about remote terms, such as term 12 and Term 25, which were not perceptually accessible. They refined the manner in which the terms of the sequence could be perceived. The number of squares on the bottom row was equated to the number of the term in the sequence, while the number of squares on the top row was equated to the number of

the term plus one. Here is an excerpt from the dialogue of Sandra's group as they discuss without the teacher:

29. Sandra: (*Referring to Term 12*) 12 plus 12, plus 1.
30. Carla: (*Using a calculator*) 12 plus 12 … plus 1 equal to …
31. James: (*Interrupting*) 25.
32. Sandra: Yeah!
33. Carla: (looking at the calculator) 25!

Through an intense interplay between various sensorial modalities and different signs, the students' perception and the concomitant mathematical thinking have gained a theoretical dimension that they did not have before. The interplay of the various senses has become contracted, subsumed and reorganized into a new complex psychic unity where no reference is made to top or bottom rows. Relational aspects of the variables in play are now emphasized. As a result, the students are now capable of dealing in a quick manner with remote terms.

The transformation of the students' initial perception into a new theoretical form of perception and thinking rests on a historically constituted manner of perceiving things that started with the Pythagorean and Neo-Platonist investigations of figural numbers and patterns in Antiquity and that was continuously refined during cultural evolution (see Radford, 2006, 2013 for details). As an ideal form of seeing, this theoretical perception appeared, from the students' viewpoint, as mere potentiality or mere virtuality. Through classroom activity and its pedagogical design, the ideal perceptive form was set into motion. Actualized through sensuous and material activity, it became an object of sensuous material thought and consciousness for the students.

9. SYNTHESIS AND CONCLUDING REMARKS

In this paper, I argued that one of the challenges to contemporary embodied and materialist approaches to human cognition consists in overcoming the dualism between the sensual and the conceptual that is a chief characteristic of empiricist and rationalist epistemologies. I started the article by briefly discussing some features of Descartes' and Kant's work. I noted that, by maintaining an irreducible gap between the sensual and the conceptual, Kant's concepts remain phenomenologically inaccessible. It is only by an allegedly aprioristic architecture of the human mind that we can subsume the material and empirical into the conceptual. In order to move beyond the transcendental idealism that characterizes Kant's theory of knowing, I claimed, it is necessary to rethink the conceptual and the sensual and their relationships. I sought support in Hegel's philosophy. In his criticism of Kant's work and idealism in general, Hegel argued that a paramount problem is the *heterogeneity* idealists assume concerning the conceptual and the sensual. Hegel contended that we need to resist considering that the conceptual and the sensual "are mutually external

to each other" (Hegel, 1801/1977, p. 165) and suggested that between them there is rather a mutual —or dialectical—constitution.

Drawing on Vygotsky's (1987-99) and Leont'ev's (1978, 2009) work and Hegel's (1830/2009) dialectic materialism, I sketched a theoretical perspective on cognition. This theoretical perspective rests on a cultural-historical understanding of sense, sensation, and the material world. It considers matter and sensation as the substrate of mind and all psychic activity. From this theoretical perspective emerges a sensuous concept of cognition where, instead of being something purely "mental," thinking, reflecting, imagining, etc. remain intertwined with the material and conceptual aspects of the world and with the organism's capacities for sensation.

However, the concept of sensuous cognition goes beyond asserting that mind, body, and world are dialectically intertwined entities. It also asserts that *cognition is transformed by human labour and social practice*. That is, human cognition is not a natural phenomenon, but a cultural-historical one. To take into consideration the role of the material in an account of human cognition is not enough. The material, as a theoretical category, cannot be taken for granted. We need to consider it from a historical viewpoint. As Marx noted in a critique of Feuerbach's materialism, "As far as Feuerbach is a materialist, he does not deal with history, and as far as he considers history he is not a materialist" (Marx, 1846/1998, location 849, Kindle edition).

As argued in this article, the raw range of orienting-adjusting biological reactions we are born with is transformed into complex, historically constituted forms of sensing that are correlated with the historical material dimension of the world. As we live in society, interact with others, and participate in more or less specialized forms of training, the biological orienting-adjusting reactions undergo cultural transformation and are converted into complex historically constituted forms of sensation, leading to specific features of human development and the concomitant forms of cultural reflection. Thus, instead of being something that evolves naturally, cognition is considered to be a culturally and historically constituted embodied and material sentient form of creatively responding to, acting, feeling, imagining, transforming, and making sense of the world.

The theoretical approach to cognition that I have outlined in this article offers also a cultural-historical understanding of the conceptual. Concepts, I contended, are *crystalized human labour* and, like all human labour, they are intrinsically multimodal. Their crystalization is an institutional process of generalization by which they become endowed with what philosopher Evald Ilyenkov called *ideality*. In its dialectical materialist sense the *ideal*, Ilyenkov says, "is the [inter]subjective image of objective reality, i.e. reflection of the external world in the forms of man's (*sic*) activity, in the forms of his consciousness and will" (1977, p. 252). Concepts as ideal entities are not transcendental Kantian "things in themselves," nor are they ideas *in* the head. Ilyenkov continues:

> The ideal is not an individual, psychological fact, much less a physiological fact, but a socio-historical one...It exists in a variety of forms of man's social consciousness and will as the subject of the social production of material and spiritual life. (Ilyenkov, 1977, p. 252)

In acquiring their ideality, concepts, as intersubjective generalized forms of action, keep their multimodal roots. But concepts are also multimodal in another sense. Since concepts are pure virtuality or pure possibility, their ontological nature is such that to become objects of thought and consciousness concepts have to be set into motion. They have to be actualized. Their multimodal nature reappears here, in their actualization in sensuous and material activity.

In the last part of the article, I presented a short example that, I hope, gives an idea of the manner in which sensuous cognition may help us understand teaching-learning activity. Sensuous cognition, I argued, does not amount to claiming that our various senses come into play in classroom interaction. At this point, this is no more than a banal statement. The real question, I argued, is about understanding how, through classroom activity, our forms of sensing and reflecting are culturally transformed. The episode suggests how the target cultural knowledge is progressively objectified as a new ideational-material psychic unity is forged. The students no longer need to *see* the terms of the sequence to reflect on "remote" terms. What could only be made apparent through an intense interplay between various sensorial modalities and different signs is later on contracted, subsumed and reorganized in a new complex psychic unity where no reference is made to top or bottom rows.

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