

THE THEORY OF OBJECTIFICATION AND ITS PLACE AMONG SOCIOCULTURAL RESEARCH IN MATHEMATICS EDUCATION

A TEORIA DA OBJETIVAÇÃO E SEU LUGAR NA PESQUISA SOCIOCULTURAL EM EDUCAÇÃO MATEMÁTICA

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ABSTRACT

This article is an attempt at locating the theory of objectification (TO) among sociocultural research in mathematics education. The first part contains a summary of the emergence of sociocultural perspectives in mathematics education research. The second part deals with some of the central ideas that underpin the TO. It begins with a discussion of the concepts of teachers and students. Then, the general trend of language-centered types of theorizing in mathematics teaching and learning is discussed. The discussion is followed by a brief presentation of the concept of *activity* as understood in dialectic materialism. Such a concept is central to the TO. This concept, however, is reformulated as *joint labor*, which works in tandem with a dialect materialist concept of knowledge and the knower. Through these concepts the TO reformulates teaching-and-learning as an ethical cultural-historical phenomenon, providing the TO with a distinct orientation among sociocultural research.

Keywords: activity; ethics; joint labor; language and discourse; sociocultural research.

RESUMO

Este artigo é uma tentativa de inserir a teoria da objetivação (TO) entre as pesquisas socioculturais na educação matemática. A primeira parte contém um resumo do surgimento de perspectivas socioculturais na pesquisa em educação matemática. A segunda parte aborda algumas das ideias centrais que sustentam a TO, partindo de uma discussão sobre os conceitos de professores e alunos. Em seguida, discute-se a tendência geral das teorias no ensino e aprendizagem da matemática centradas na linguagem. A discussão é seguida por uma breve apresentação do conceito de atividade tal como entendido no materialismo dialético. Tal conceito é central para a TO. Este conceito, no entanto, é reformulado como o trabalho conjunto, o qual trabalha em conjunto com um conceito materialista dialético do conhecimento e o conhecedor. Através desses conceitos a TO reformula o ensino-e-aprendizagem como um fenômeno ético histórico-cultural, fornecendo à TO uma orientação distinta entre a pesquisa sociocultural.

Palavras-Chave: atividade; ética; trabalho conjunto; linguagem e discurso; pesquisa sociocultural.

1. Sociocultural research in mathematics education

The theory of objectification (TO) is an educational theory that focuses on the problems of teaching and learning, and to do so draws on Hegel's (1991) philosophy and the ensuing dialectical materialism developed by philosophers such as Marx (1998) and Ilyenkov (1977). The dialectical philosophical underpinning of the theory of objectification means, among other things, that the TO is embedded in a line of thought in which human beings cannot be conceived of as set apart from the world and their cultures. In this sense, the TO is part of the increasing field of contemporary educational socio-cultural theories (Atweh, Forgasz, & Nebres, 2001; Cantoral, 2013; Lave, 1988; Sfard, 2008; Lave & Wenger, 1991) that draw from general socio-cultural theories developed in the fields of sociology and anthropology (e.g., Bourdieu, 1998; Lévi-Strauss, 1962). The common denominator of such theories is not the principle that human beings are somehow "related" to their culture. Their common denominator is rather the claim that human beings are *consubstantial* with the culture in which they live their lives. In other words, what human beings think, do, feel, imagine, hope, and dream is deeply entangled in their culture. To a great extent, differences between socio-cultural theories in sociology and anthropology appear according to the manner in which the aforementioned consubstantiality is theoretically thematized and understood. In the evolution of socio-cultural theories, these differences did not appear all of a sudden (Durkheim, 1965; Lévi-Strauss, 1962). Nor did they appear clearly formulated (Shweder and LeVine, 1984).

In the field of mathematics education, during the 1980s and 1990s, there was a pressing need to offer alternatives to:

- (1) the individualist learning approaches that have dominated the field (see, Lerman, 1996a; 1996b), and
- (2) the Eurocentric conception of mathematics (see e.g., Bishop, 1988; D'Ambrosio, 1985).

To respond to these general needs, some mathematics education researchers tried to come up with socially, historically, and culturally oriented conceptions of the mind and a broader conception of psychology than the narrow idealist subject-centered one inspired by the natural sciences. These mathematics education researchers tackled, in different ways and from different perspectives, the question of the consubstantiality of the individuals and their culture, and ended up focusing on different problems—e.g., language and discourse, material culture and its effect on the individual's cognition, the socio-cultural and political context of mathematics teaching and learning, and the ensuing manners of doing mathematics and thinking mathematically.

This range of problem is, indeed, what transpires in an overview of socio-cultural research published in the first 29 meetings of the International Group for the Psychology of Mathematics Education (PME)—that is, the PME meetings ranging from 1978 to 2005. In this overview, Lerman identified four crucial categories:

1. Cultural psychology, including work based on Vygotsky, activity theory, situated cognition, communities of practice, social interactions, and semiotic mediation
2. Ethnomathematics

3. Sociology, sociology of education, poststructuralism, hermeneutics, and critical theory

4. Discourse, to include psychoanalytic perspectives, social linguistics, and semiotics (Lerman, 2006, p. 351)

Although each one of these categories cannot be considered as strictly separated or independent from the others, we can recognize early representatives of the first category in the works of Bartolini Bussi (1991), Bishop (1985), and Lerman (1992). Two representatives of the Ethnomathematics category are Bishop and Pompeu (1991), and Lea (1990), while Atweh and Cooper (1991), on the one hand, and Ellerton (1991), on the other hand, are representatives of the third and fourth Lerman's categories, respectively.

To a greater or lesser extent, these researchers were interested in understanding the role of culture, history, and society in conceptions of mathematics and in ideas about the students' learning—conceptions and ideas that we are still trying to understand and that are far from being settled. Bartolini Bussi (1991) and Bartolini Bussi and Mariotti (1999, 2008), for instance, resorted to Vygotsky's work to investigate the role of the interaction and the role of cultural artifacts in the classroom; they called attention to the problem of semiotic mediation. Arzarello and collaborators focused on artifacts too, and more specifically, on mathematical signs and their evolution (Arzarello, Bazzini, & Chiappini, 1994). Inspired also by the work of Vygotsky (1987) and discursive psychology (Harré & Gillet, 1994), Lerman (1996b) studied the role of language in the constitution of intersubjectivity. Boero, Pedemonte, and Robotti (1997) investigated the students' emerging mathematics conceptualization as a phenomenon at the crossroad of phylogenesis and ontogenesis through the historicity of speech.

These pioneering studies brought a different conception of language, interaction, and material culture. The work of Boero and his team, for example, turned to the work of Bakhtin (1986) and his concepts of language and intersubjectivity. Bakhtin's concepts of language and intersubjectivity are certainly at odds with traditional psychology's concept of language and its idea of the individual as an autonomous entity who comes into the world somehow endowed with the internal cognitive mechanism necessary for its intellectual development (Martin, 2004). More recently, Sfard (2001), drawing also on Vygotsky, has suggested that thinking—one of those human features traditionally considered to be the most private and personal—has its roots in the social plane of communication: "thinking," Sfard (p. 26) says, "arises as a modified private version of interpersonal communication."

Where does the TO stand vis-à-vis those socio-cultural trends? In the next sections I sketch the theoretical underpinnings and practical orientation of the TO.

2. The theoretical underpinnings of the TO

I begin this section with a discussion of the concepts of teachers and students as conveyed by the TO. Then, I move to language-centered types of theorizing in mathematics teaching and learning. This is followed by a discussion of the concept of activity as articulated in dialectic materialism. Such a concept appears as the central concept in the TO. The concept, however, is reformulated as joint labor for reasons that are explained below.

2.1 Teachers and students

Let me refer to a mathematics lesson in a Grade 4 class (9–10-year-old students) where the teacher and the students are dealing with the following piggy bank problem.

For his birthday, Marc receives a piggy bank with one dollar. He saves two dollars each week. At the end the first week he has three dollars; at the end of the second week he has five dollars, and so on.

The teacher provided the students with bingo chips of two colors (blue and red) and numbered plastic goblets intended to represent week 1, week 2, etc., and invited the students to model the saving process until week 5. Then, drawing on the model, the teacher invited the students to find the amount of money saved at the end of weeks 10, 15, and 25.

Here is an excerpt from the discussion of a three-student group (Albert to the left; Krysta in the middle, and Manuel to the right).

14. Krysta: So, we should do . . . That (see pic 1 in Fig. 1) times two. So 11 . . .

15. Albert: 11 plus 11 . . . 22.

16. Krysta: 22.

17. Albert: (He laughs.)

18. Krysta: At the end of . . ., okay at the end . . .

19. Albert: Well, wait . . . No. It would be 11 plus 10 because . . .

20. Krysta: (She points to week 5.) 5.

21. Albert: (He points to the blue bingo chip.) We always start with the . . . [blue chip] (see pic 2 in Fig. 1).

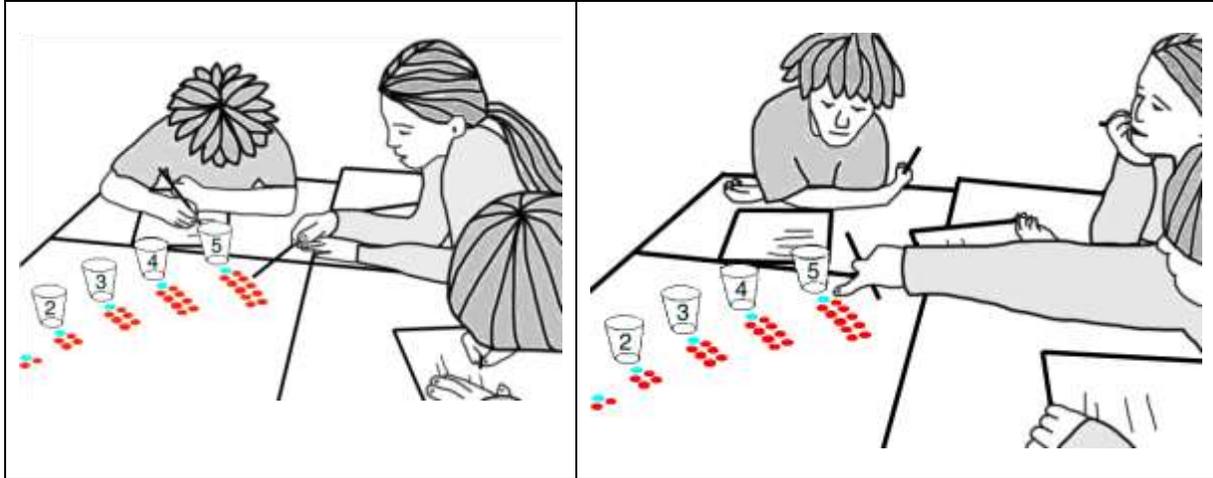


Figure 1. Left (pic 1), Krista points to the bingo chips in front of week 5. Right (pic 2), Albert points to the blue bingo chip

As we can see, to answer the question about week 10, the students resort to a “doubling strategy.” They add the amount of dollars saved until week 5, double it and proceed to remove one from that amount, which corresponds to a blue bingo chip, since there is always one and only one blue bingo chip in a week (see Figure 2).

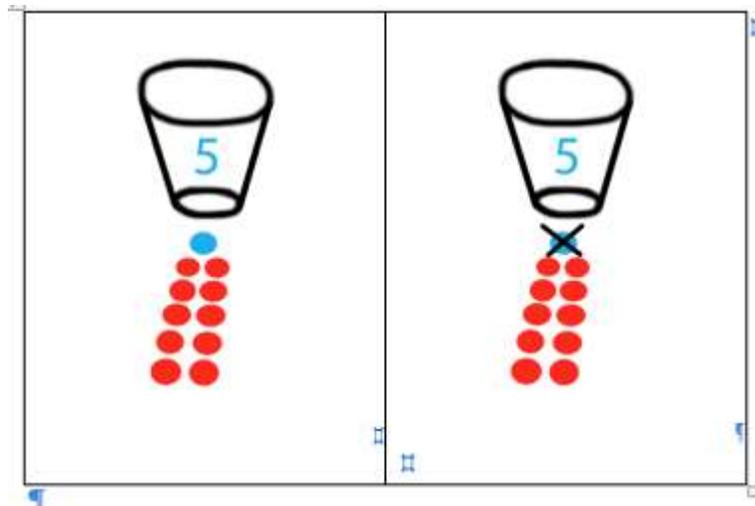


Figure 2. The double strategy, with the corresponding removal of one bingo chip. In Week 10, the savings amount to 11+10 dollars.

The reason is not perfectly articulated by Albert in lines 19 and 21, but his clear pointing gesture helps illustrate the intended explanation.

The “doubling strategy” can be applied to other cases, as the students noted. Hence, to find the amount of saved money at the end of, say week 25, we observed some groups start from week 5;

by doubling, they get the amount of saved money in week 10. They double again and add the amount of week 5, remembering to remove the extra blue chips that were added in the process. The strategy works just fine, but is cumbersome to determine the amount of savings in “remote” weeks such as week 78 or 103.

But let us continue with the piggy bank problem. When the teacher went to see Krysta and her teammates’ work, she realized that the students were resorting to the “doubling strategy.” The discussion went as follows:

22. Mrs. Giroux: What I find interesting here is that [in your model] you have bingo chips of two colors. What does it mean?

23. Krysta: Because blue was what he already had.

24. Manuel: Yeah, because it [the story problem] says that the piggy bank had one dollar.

25. Albert: The piggy bank had one dollar, so those (he points in sequence to all the blue bingo chips from weeks 1 to 5) are all the one dollars that he had already (now, he points to the red bingo chips from weeks 1 to 5) added to 2, 4, 6, 8, 10.

26. Mrs. Giroux: Okay, okay. What would happen if it was week 10?

27. Albert: Well (he points to week 5.), we added all this again (he makes a sweeping gesture: see Figure 3, pics 1 and 2), because we know that $5 + 5 = 10$, so . . .

28. Krysta: (Interrupting) Plus . . . we added . . . We added all this (she points to the red bingo chips in week 5; see Figure 3, pic 3) not the blue (she points to the blue bingo chip) . . .

29. Mrs. Giroux: (Trying to make noticeable to the students the co-variational structure) What do you remark about week 5 (she shows the glass corresponding to week 5) and (she points to the red bingo chips; see Figure 3, pic 4) the number of bingo chips? (She makes the same actions) The fourth week and the number of bingo chips?

30. Albert: It’s always twice . . .

31. Mrs. Giroux: (Repeating) It’s always twice.

32. Krysta: It’s the double of what you . . . No! (She watches the artifacts intensely for a while) I am confused!

33. Albert: Yeah! It’s twice, look! (He counts the red chips) $1+1, 2; 2+2, 4; 3+3, 6; 4+4$

34. Krysta: (Interrupting) 8.

35. Albert: (At the same time) 8.

36. Krysta: $5+5, 10$.

37. Albert: (He points to the bingo chips in week 5) $5+5, 10$.

38. Krysta: Cool. It's twice the week . . .

39. Mrs. Giroux: So, if the reds are twice [the number of the week], what happens to the [blue] bingo chip . . . (She points to the blue bingo chip in week 5)?

40. Krysta: Plus 1.

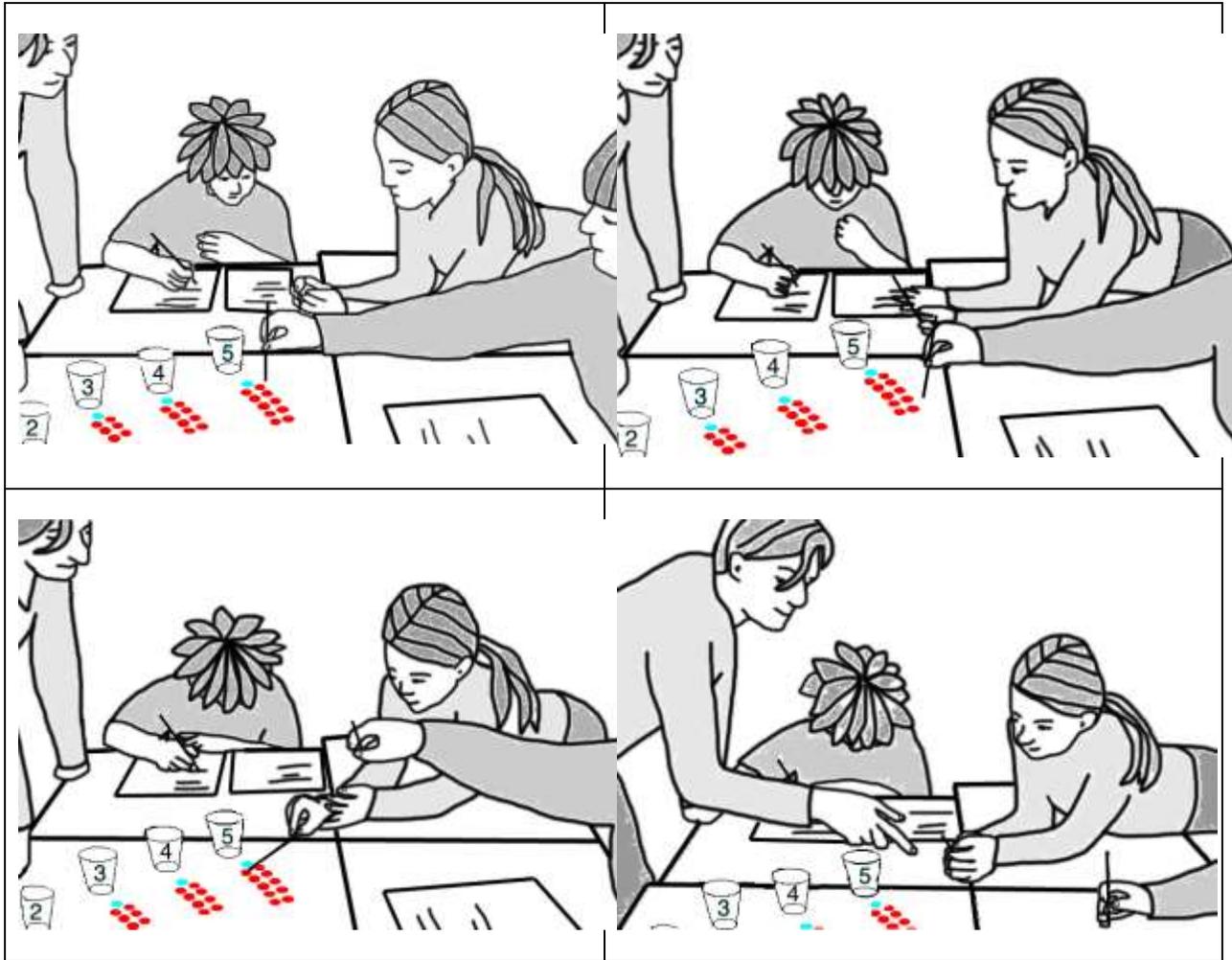


Figure 3. The students and the teacher discuss the strategy to find the number of bingo chips in week 10.

From lines 22 to 28 the students explain their strategy to the teacher. The strategy is not identified with a label (such as the “doubling strategy” or something else); rather, the strategy is explained in action, through words and gestures. In line 29, something outstanding happens. Indeed, in line 29 the teacher tries to introduce what seems to be a new approach to perceiving things: She says: “What do you remark about week 5 and the number of bingo chips? The fourth week and the number of bingo chips?” At the end of the passage, the students seem to start noticing a co-variational relationship between the number of the week and the red and blue

chips.

We can ask several questions from the previous passage. For instance: What are the students learning? How are they learning it? We can also ask questions about the teacher, such as: What is the nature of her intervention? Is her intervention appropriate?

If we are interested in providing accounts of the students' learning, what exactly should we attend to? Within a Piagetian pedagogical paradigm, the answer would be: the students. More precisely, the students' actions, for in such a pedagogical paradigm, learning is precisely that which emanates from what the students do. Action secretes knowledge. The Piagetian educational paradigm is perhaps the most elaborate and sophisticated account of child-centered pedagogy. The TO takes a different route.

Does it mean that the TO does not attend to the students? The TO does attend to the students, but in a manner that is radically different from the individualist accounts of the child-centered pedagogy. What that means is that the concept of the student put forward by the TO is different from what we have been generally exposed to in accounts of learning.

I admit that the problem of moving to a different conception of the student is not easy, as we come from a long historical tradition that considers student learning as the outcome of the student's own deeds. We find this idea clearly articulated in the "Universal teaching" pedagogic program of the French educator Joseph Jacotot (1770–1840) in the nineteenth century. Jacotot's fundamental idea was that any individual can learn anything by herself. We do not need to teach the students. They will learn by themselves. As Jacotot (1828, p. 25) tells his students, "You do not know integral calculus, but you can learn it without explanations." This conception of learning as the outcome of the student's own deeds, which as we saw in the previous chapter, was endorsed by constructivist pedagogies, has become a kind of second nature in mathematics education.

This is why Mrs. Giroux's intervention in line 29 may feel disturbing. Indeed, Mrs. Giroux's intervention may seem intrusive and as interfering with the students' conceptualizations and learning.

What if we put on hold for a second the idea that concepts, meaning, and intentionality do not necessarily have to emanate from the student in order for her to genuinely understand and learn mathematics? I am not advocating — and I should rush to clarify it — for a pedagogy in which the teacher tells the students how to solve the problems and think mathematically. What I am saying is that, unfortunately, education research and practice have narrowed the teacher/students relationship to two possibilities: (A) either learning emanates from the students or (B) the students receive knowledge from the teacher. This theoretical "either (A) or (B)" position is what Lerman (1996b) calls in his critique of constructivism the absolutist alternative. Following Lerman, I claim that the absolutist position leads to considering the teacher/students relationship in a too simplistic and narrow manner. The idea that students receive knowledge supposes that knowledge can be transmitted from an individual to another (from a sender to a receiver). This is the main idea of the transmissive pedagogical model — and is indeed a very naive one. It resorts to a very simplistic idea of knowledge (knowledge is conceptualized as something that individuals possess or not, hence to a merchandized concept of knowledge), and a very

superficial and unproblematic idea of the teacher and the student. Within the absolutist position, the only possible solution seems then to be that knowledge has to start and result from the students' deeds. This is the position that Williams (2016) adopts in his critique of Roth and Radford's (2011) book. Underpinned by an idealist theoretical position and armed with superficial Marxist jargon, Williams ends up adopting the ideas of knowledge and the student of constructivism and Piagetian pedagogy.

William's critique is perhaps a token of the difficulties that result from trying to overcome the absolutist position. Already in his early work, Vygotsky claimed a crucial role for the student in her own learning, without, however, reducing learning to the students' deeds. Thus, the old pedagogy, Vygotsky said, "treated the student like a sponge which absorbs new knowledge." Elaborating on the idea in more detail, he claimed that "the assumption that the student is simply passive . . . is the greatest of sins, since it takes as its foundation the false rule that the teacher is everything and the student nothing" (Vygotsky, 1997a, p. 48).

The problem is not to deny the students their role in learning. And the solution is not to embrace the other option of the absolutist position. If we agree to put on hold the idea that learning has to come from the students' deeds, the classroom episode referred to in the previous section appears in a new light. It appears much more complex. It requires us to see Mrs. Giroux's intervention through new lenses. In other words, putting on hold the idea that the student is the origin of knowledge and that her environment is merely a "facilitative grooming to become more fully socialized and intellectually engaged" (Martin, 2004, p. 197), demands to see teaching and learning in a different way. Not only does the concept of the student have to be revisited, but also the concept of the teacher. The teacher, Vygotsky argued, cannot be pictured as injecting knowledge into the student's mind. Indeed,

Just as a gardener would be acting foolishly if he were to try to affect the growth of a plant by directly tugging at its roots with his hands from underneath the plant, so is the teacher in contradiction with the essential nature of education if he bends all his efforts at directly influencing the student. (Vygotsky, 1997a, p. 49)

Recently, there has been a great deal of research about the teacher. Yet, to a large extent, the teacher is often pictured along the lines of a technical agent whose mission is to deliver a curriculum (Brown, 2010). This is a very restrictive view of the teacher, one that Canadian educator Ted Aoki criticized by reducing the teacher to a "competent teacher-implementer," that is

one who has skills and techniques oriented toward efficient control. Such a know-how-to-do view of implementation is embedded in scientific and technological thought/action framework that reduces human competence to instrumental reason and instrumental action. Here, the teacher is seen as a rule-oriented, rule-governed being cast within a manipulative ethos, an ethos in which even his [or her] future is conceived in terms of rules. (Aoki, in Pinar & Irwin, 2005, p. 113)

In short, the concept of the teacher and the student that the TO brings with it is not the concept of self-sufficient and self-made beings who already know their business. Teachers and students are conceptualized as subjectivities in the making, or as projects of life. Instead of being considered as something already given, as sources of knowledge and intentionality, they are considered as openness towards the world. The TO conceives of the teachers and students as human beings in

flux, as unfinished projects, in search of themselves, engaged together in a same endeavor where they suffer, struggle, and find fulfillment together.

Seeing the piggy bank episode through the lenses of the TO, Mrs. Giroux appears as engaged with Albert, Krista, and Albert. Mrs. Giroux is not doing the same thing as the students. Yet, they appear as working together, trying to bring to the fore a form of co-variational thinking about sequences. What they are doing is the creation of what Hegel (2001) called, in a more general context, a common work. In the TO common work is defined as the sensuous appearance of knowledge (e.g., the sensuous appearance of a co-variational algebraic or statistical way of thinking through collective problem posing and solving, and discussion and debate in the classroom). Common work is the bearer of dialectic tensions because of the emotional and conceptual contradictions of which it is made. Through it, knowledge appears sensuously in the classroom (through action, perception, symbols, artefacts, gestures, language), much in the same way, and with similar aesthetic force, that music appears aurally in a concert hall through the common work of the members of the orchestra.

This view of the teacher and the students is in harmony with the general educational project in which the TO is based. The TO is inscribed within an understanding of mathematics education as a political, societal, historical, and cultural endeavor. Such an endeavor aims at the dialectic creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical practices, and ponder and deliberate on new possibilities of action and thinking. What the TO attends to is hence not only the realm of knowledge but also the realm of becoming. That is, in the TO the focus shifts from how students receive knowledge (traditional teaching) and how students construct their own knowledge (constructivism), to how teachers and students produce knowledge in the classroom against the backdrop of culture and history. But the focus also shifts to how teachers and students co-produce themselves as subjects, in general, and as subjects of education, in particular.

2.2 Artefacts, signs, and language

I have just said that in the account of teaching and learning, the TO focuses on the teachers and the students. What about the artefacts, the mathematical signs, language, and the material culture at large? They are also taken into account. Yet, the theoretical positioning is different from what we find in other sociocultural research. Let me explain the idea by considering the case of language. In Section 1, I mentioned that language was one of the theoretical orientations at the base of the emergence of sociocultural theories in mathematics education. The interest in language has not diminished. On the contrary, in the past two decades language has become one of the most prominent research areas in mathematics education (Radford & Barwell, 2016). And such prominence should not be a surprise.

Indeed, at the heart of sociocultural theories developed in anthropology and sociology we find a constant struggle to understand the problem of the relationship between the individual and the social, the subjective and the objective, inner and outer life, experience and form. It is in this context that, in the work of Wilhelm von Humboldt (1998), Ernst Cassirer (1980), and Valentin Voloshinov (1973), language came to be considered the link between cultures and their

individuals. Voloshinov (1973, also spelled as Vološinov), for example, turned to the sign. He considered the sign to be both subjective (in the sense that it is produced by an individual and expresses the individual's intentions) and objective (in the sense that the expressivity of the sign is embedded in a cultural system of expressions and values). The sign ensures an entanglement between the subjective and the objective so that "the very notion of a fundamental, qualitative difference between the inner and the outer element [of life] is invalid to begin with" (Voloshinov, 1973, p. 85). In Voloshinov's account, the sign par excellence is the word. "Word is present in each and every act of understanding and in each and every act of interpretation" (Voloshinov, 1973, p.15). "The word is implicated in literally each and every act or contact between people" (p. 19). Drawing on this conception of the word, Voloshinov, as well as Mikhail Bakhtin (1986), came to see language and literature not just as one of the fields of aesthetic experience and cognition, but the central field through which the other cultural fields are refracted. For them, it is ultimately through language and literature that reality is produced and interpreted. The problem of the individual and the social, the subjective and the objective, inner and outer life, and experience and form, is resolved, in Voloshinov's account, in the dialectical tension between the relatively stable centripetal forms of culture (epitomized by the novel) and the centrifugal forms of resistance and novelty (epitomized in Bakhtin's idea of carnival). Yet, as Tihanov notes,

Vološinov ended up reducing culture to a single-ideal-mode of existence . . . [one in which] language had a conspicuous and autonomous role in the formation of social life. Thus, . . . Vološinov pursued a language-centered type of theorizing in the field of social thought. (Tihanov, 1998, p. 615).

And so did Lev Vygotsky (1987), although for other reasons.

The TO takes a different route that will become clear in the next section. For the time being, it might be worth noticing that the exact role of artifacts, signs, and language in the social formation of mind has been a disputed question since the insertion of socio-cultural theories—and even in the evolution of the same sociocultural theory. This is the case of Vygotsky's cultural-historical theory. We know, for instance, that in the 1920s, trying to understand the role of society in the individual's cognition, Vygotsky formulated his famous "genetic law of cultural development." Vygotsky's genetic law states that "Every [psychic] function in the child's cultural development appears twice: first, on the social level, and later, on the individual level" (Vygotsky, 1978, p. 57). Internalization is precisely that which ensures the passage from the social to the individual level. The idea of internalization is formulated in terms of the role that signs play therein. Without signs, the concept of internalization does not make sense: "The internalization of cultural forms of behavior," Vygotsky says, "involves the reconstruction of psychological activity on the basis of sign operations" (Vygotsky, 1978, p. 57).

As we can see, the concept of internalization and the concomitant ideas of signs as tools and semiotic mediation were the central theoretical categories in Vygotsky's study of the mind around the period that roughly goes from 1927 to 1931. These concepts belong to what contemporary Vygotskian scholars call Vygotsky's instrumental "phase" or "moment" (for details see Presmeg, Radford, Roth, & Kadnuz, 2016). González Rey (2009) qualifies the 1927-1931 moment as an instrumentalist turn. However, at the end of his life, Vygotsky was moving towards a more encompassing research program featuring consciousness as a systemic semiotic system. While signs and tools remained present, they however lost their technological flavor of

the instrumentalist phase, “where the psyche was still viewed by analogy with a machine, a technical device” (Zavershneva, 2010, p. 72). Signs and tools became considered in terms of what they signify, and yielded room to a programmatic study of consciousness in which affect, emotions, and meaning were brought to the fore. Vygotsky noted: “Consciousness as a whole has a semantic structure. We judge consciousness by its semantic structure . . . Semiotic analysis is the only adequate method for the study of the systemic and semantic structure of consciousness” (Vygotsky, 1997b, p. 137).

In some manuscript notes from 1934, Vygotsky writes: “[T]here are 2 unities of dynam[ic] activity: think[ing] and real activity . . . In fact, the role of think[ing] in activity consists of introducing new dynam[ic] possibilities to activity” (Quoted from Zavershneva, 2010, p. 83). Within the program of research that Vygotsky was envisioning at the end of his life, thinking and activity became intertwined. Such a move does not mean that the importance of signs and artefacts is diminished. It appears now subjected to a different and more encompassing concept: the concept of activity.

I discuss the concept of activity in more detail in the following section.

2.3 Activity

The central theoretical category and the methodological unit of analysis of the TO is activity. From the outset, hence, the TO places itself within a different theoretical perspective from the one in which language, discourse, and literature appear as the ultimate field of aesthetic experience and cognition. This theoretical and methodological choice does not dismiss the role of language in the processes of knowing and becoming, but asserts the fundamental ontological and epistemological role of matter, body, movement, action, rhythm, passion, and sensation.

Behind this theoretical stance rests a specific anthropological conception of the human. Humans, following Marx’s (1998) Spinozist stance (see Spinoza, 1989), are considered to be part of nature: they are natural beings. That humans are natural beings means that they are sensible beings, unavoidably affected by the other parts of nature. In this context, sensations and passions are conceptualized as ontological affirmations of the individual’s nature as natural beings (Fischbach, 2014).

One important consequence of this theoretical stance is that the individual’s existence cannot be conceived of as a substantial entity, produced from within, as articulated by the humanist trend of the Enlightenment. The individual’s existence is relational through and through. It appears to be profoundly linked to an ensemble of relationships with other parts of nature—including social relationships—and is based on culturally and historically constituted conditions of life. In this line of thought, to be a natural being means also that, like other natural living beings, humans are (1) beings of need who (2) find their satisfaction in objects outside of themselves.

To meet their needs (needs of survival and also artistic, spiritual, and other needs created by/in society), humans engage themselves actively in the world. They produce. What they produce to fulfil their needs occurs in a social process that is, at the same time, the process of the individuals’ inscription in the social world and the production of their own existence. In dialectic

materialism, the name of this process is activity. Sensuous, material activity is considered the ultimate field of aesthetic experience and cognition.

This conception of activity is very different from usual conceptions that understand activity as a series of actions performed by an individual in the attainment of his or her goal. In dialectical materialism, activity is something else. It is precisely the specific form in which the individuals express their life. “As individuals express their life, so they are. What they are, therefore, coincides with their production, both with what they produce and with how they produce” (Marx, 1998, p. 37). Activity, in short, is a social form of joint endeavor that comprises self-expression, intellectual and social development, and aesthetic enjoyment. It is a process in a system of social relations that realizes the societal nature of human beings.

In articulating a psychological approach based on the dialectic materialist idea of activity, Leont’ev (1978) suggested a basic structure of activity. An activity for him is characterized by its object and its motive. The object and motive of an activity are the engines that keep activity in motion. In practice, in the pursuit of the activity’s object, individuals break down the object into a sequence of goals to which actions are associated. In the “Supplement” to his important 1978 book—a supplement dedicated to educational matters—Leont’ev discusses the conditions under which a certain theoretical learning content can be meaningfully perceived or attended to by the student. He contends that

in order that the perceived content be recognized, it is necessary that it occupy the structural place of a direct goal of action in the subject’s activity, and thus that it appear in a corresponding relation to the motive of this activity. (Leont’ev, 1978, p. 153)

It is hence through activity and the structural interconnection between motive, object, goals, and actions that the learning content becomes disclosed to the student’s consciousness.

Activity Theory, as this sociocultural approach has come to be known (see, e.g., Engeström, 1987; Kaptelinin, Kuutti, Bannon, 1995), has had an important impact on education in general and mathematics education in particular (see Roth and Radford, 2011). Yet, in focusing on the procedural aspect of activity, activity is reduced to its operational and functional dimension, eradicating from it the aesthetic and political dimensions of making and creation. The account of activity culminates, unfortunately, in a technological dull analysis of what was originally thought of as the sensible experience of human life.

2.4. The concept of joint labor

In dialectic materialism (see, e.g., Ilyenkov, 1977), knowledge (mathematical, scientific, artistic, legal, etc.) is considered to be constituted of forms of human action that have become historically and culturally synthesized. They are synthesized forms of action bearing, in sedimented ways, the political contradictions of human life. They are always in the process of continuous movement, incessantly transformed in practice, constantly born and reborn. Instead of being conceptually neutral, knowledge allows one to always see the world in a certain way. That is to say, unavoidably, knowledge already conveys a specific ideology. The symbolic algebra of the Renaissance, for instance, conveys the theoretical stance of the instrumental reason of the

Western 16th century and the social abstractions brought forward by the emerging mercantilist capitalism. Knowledge belongs to an immaterial sphere of culture that is intertwined with the material world of objects and human actions. This immaterial sphere of culture is part of what Marx (1998) called the non-organic realm of nature and is also part of the conditions out of which human existence is crafted.

However, the students cannot usually perceive, notice, and grasp knowledge directly. The measuring tape that we use in construction settings is a good example. The work of Lacroix (2014) shows how difficult it is for pre-apprentices enrolled in pipe trade training to make sense of the arithmetic of $(1/2)^n$ behind the measuring tape. The human intelligence embodied in the measuring tape does not show up in a direct and immediate fashion; nor can it hardly be reconstructed on the basis of personal experience alone. Indeed, no personal experience, regardless of how rich it might be, can manage to establish on its own a cultural system of ideas, such as the arithmetic of $(1/2)^n$ behind the measuring tape, the Roman legal system, formal logic, or set theory. An undertaking of this magnitude requires not one lifetime, but thousands (Leontiev, 1968), and perhaps even more.

The production/grasping of knowledge is indeed a mediated phenomenon. Knowledge is created and recreated only through sensuous cultural-historical activity and can come into sensuous existence only in and through activity. It is only in the idealist subjectivist accounts of knowing that the identity between the knower and the object of knowledge is presupposed.

From the TO's viewpoint, the classroom production of knowledge and its progressive disclosing to the students' consciousness is entangled with the evolving classroom activity that makes such a production/disclosing possible. As a result, the kind of classroom mathematical activity becomes extremely important in teaching and learning. In traditional learning, the students' participation in the disclosing of knowledge is minimalized, the result being that the students are alienated from meaningful learning. They cannot express themselves in the products of their learning. In the so-called reformed learning and its student-centered Piagetian pedagogy, the students are left to their own cogitations; they do express themselves, but remain imprisoned within the confines of their subjective world, living a one-sided existence, cut off from cultural and historical perspectives at large, and hence alienated from them.

The TO is an attempt to restore the idea of activity in general and classroom activity in particular as non-alienating forms of life. To avoid confusions with other meanings, and to emphasize the idea of activity as an historically produced aesthetic form of life where matter, body, movement, action, rhythm, passion, and sensation come to the fore, activity, in the TO, is termed joint labor.

The concept of joint labor offers a reconceptualization of teaching and learning. In joint labor, the students are not reduced to a role of simple cognitive subjects. They do not appear as passive subjects receiving knowledge or as self-contained subjects constructing their own knowledge. In the same vein, teachers are not reduced to a role of technological and bureaucratic agents—guardians and implementers of the curriculum. They do not appear as possessors of knowledge who deliver or transmit knowledge to the students either directly or through scaffolding strategies. The concept of joint labor makes operational the idea of teachers and students introduced in Section 2.1 as individuals who work together. This concept suggests an educational perspective in which to envision teaching and learning not as two separate activities, but as a

single and same activity: one where teachers and the students, although without doing the same things, engage together, intellectually and emotionally, towards the production of what we termed in Section 2.1 a common work.

Let us pause for a moment and imagine the piggy bank problem that we discussed in Section 2.1, but this time in a different scenario. Let us imagine that the episode unfolds now according to the transmissive educational model of direct teaching. The teacher is in front of the class and tells the students how to solve the problem. The students listen and take some notes. Can we say that there is joint labor here? Let us repeat the same thought experiment, but this time the episode unfolds according to the constructivist student-centered model. The students are trying by themselves to solve the piggy bank problem and the teacher refrains from intervening. Can we say that there is joint labor here? In both cases, what teachers and students are doing does not refer to a dynamic system geared to the satisfaction of collective needs. Teachers and students are involved in classroom activity, but it is an activity in the dialectic sense of the term. More specifically, in the transmissive model, teachers and students are involved in a functional and technical coordination of actions: on the one hand, the teacher tells; on the other hand, the students listen and try to make sense by themselves of what the teacher does and says. In the constructivist model, “the teacher and students each construct individual interpretations that they take as being shared with the others” (Cobb, Yackel, & Wood, 1992, p. 17). What results from such an activity cannot be a “common work” but something that is merely “taken-as-shared.”

In the TO, the students’ encounters with historically constituted mathematical knowledge, materialized in the teachers’ and students’ common work, are termed processes of objectification (Radford, 2008; 2014). The piggy bank episode provides us with an example of a process of objectification. Working together, Ms. Giroux and the students are producing (through gestures, posture, perceptual activity, language, artefacts) a common work, which allows the students to become progressively aware of a different manner to think about the problem (a co-variational way of thinking).

Through these social, material, embodied, and semiotic processes, the students and teachers not only create and re-create knowledge but they also co-produce themselves as subjects in general and as subjects of education, in particular. More precisely, they produce subjectivities; that is to say, singular individuals in the making. This is why, from this perspective, processes of objectification are at the same time processes of subjectification.

The concept of joint labor resorts to: (a) specific collective forms of classroom knowledge production, and (b) definite modes of human collaboration that rest on critical community ethics. The ethical forms of human collaboration are driven by a general attitude towards the world and serve to configure the teachers’ and students’ joint labor in the classroom (for some examples, see Radford and Roth (2011) and Radford (2012)). These critical and community ethical forms blur the borders that separate the teachers from the students. Teachers and students labor in concert as one. This ethical commitment is what we see in the piggy bank example when, for instance, in line 29 the teacher engages with the students and invites them to see the problem solving strategy under a new light. The students respond to the teacher’s call and, laboring with the teacher, try hard to make sense of how this new strategy might look.

The concept of joint labor leads to a new concept of the mathematics classroom. The classroom

appears as a public space of debates in which the students are encouraged to show openness towards others, responsibility, solidarity, care, and critical awareness. The classroom indeed appears as a space of encounters where teachers and students become what Paulo Freire (2004) termed presences in the world. That is to say, the classroom appears as a space of encounters, dissidence, and subversion, where teachers and students become individuals who are more than in the world—they are individuals with a vested interest in one another and in their joint enterprise; individuals who intervene, transform, dream, apprehend, suffer, and hope together.

3. Synthesis

In this article, I have tried to position the TO among other sociocultural approaches. To do so, I started by summarizing the emergence of sociocultural theories in mathematics education, with a particular focus on language-oriented ways of theorizing teaching and learning. The idea was not to compare the TO to each existent sociocultural approach in mathematics education research, but to highlight some of the differences among some important trends — hence my choice of language-oriented theories. I outlined some of the key concepts of the TO and focused in particular on the concept of activity, the central concept of the TO that the theory reconceptualizes as joint labor to bring to the fore the ontological and epistemological importance of activity as a form of life. The concept of joint labor goes hand in hand with a dialect materialist concept of knowledge and the knower. Through these concepts the TO reformulates teaching-and-learning as an ethical cultural-historical phenomenon, providing the TO with a distinct orientation among sociocultural research in mathematics education.

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