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THE ETHIC OF SEMIOSIS AND THE CLASSROOM CONSTITUTION OF MATHEMATICAL SUBJECTS

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In this paper I seek to investigate the production of subjectivities from a non-substantialist viewpoint. I suggest that rather than substantial entities from where intention, knowledge, and meaning rise and proceed, subjectivities are unachievable and always ongoing meaning-making instantiations of cultural coded forms of being. I argue that subjectivities are always in a process of be-com-ing. They are co-produced in activity-bound processes of subjectification dialectically entangled with what children learn and how they learn it. I illustrate these ideas through a classroom episode with pre-school children.

INTRODUCTION: THE QUESTION OF THE SUBJECT

To a very large extent, mathematics education research has drawn—rather implicitly—on the concept of the subject that philosophers of the Enlightenment articulated in the 18th century. As I have argued elsewhere (Radford, 2012), the Enlightened philosophers sought to build in the overcoming of fear, tradition, and feudal hierarchies their idea of the new subject. They found in freedom the subject's most fundamental trait. The idea of subject that they envisioned is someone who is not there to follow what others say or do, but one that has to think and reason by him or herself. Kant illustrates perhaps better than anyone else this idea: the Kantian subject is a subject of reason, the crafter of its own destiny, the architect of its own projects of life, the origin and source of meaning and knowledge. From the Kantian perspective, to be a subject is to be free. And to be free is not to be subjected to anything other than one's reasons. The result is a self-sufficient, humanist, and substantialist idea of the subject. In this paper I explore the question of the subject from a different viewpoint. I suggest that the subject is cultural-historical entity in perpetual transformation—i.e., a subjectivity-in-the-making. My starting point is the rather banal—although rarely considered—fact that mathematics classrooms are not only producers of knowledge but of subjectivities as well. Drawing on the theory of objectification (Radford, 2008a), I am particularly interested in investigating the processes of subjectification out of which subjectivities produce themselves and are at the same time produced by the activities in which they engage.

BEING AND SUBJECTIVITY

To move beyond the Enlightened substantialist concept of the subject, I suggest distinguishing between being and subjectivity. The concept of being that the theory of objectification brings forward highlights what we may call the *being's cultural nature*. What this means is that our idea of what an individual and his/her power of action and will (i.e., agency) are, are relative to their historical moment. If we were born in ancient Greece or another historical period, we would have conceived of ourselves in a manner very different from the way we do today. In the Athens of Plato, for example, in the midst of a society articulated around the distinction between free citizens and slaves, with a negative valence to manual work and a positive valence to intellectual work, our

sense of individuality would have been embedded in a political-geographical criterion of inclusion/exclusion—Athenians vs. foreigners—and defined in terms of the opposition between passion and temperance and the struggle for self-mastery. Very different is the contemporary concept of individual, defined rather as a private owner—an individualist subject, drawn by possessive, consumerist, and instant gratification drives who is continuously urged to express herself "creatively" and "authentically."

The conception of the individual in a specific culture and time is embedded in what in the theory of objectification is termed *Semiotic Systems of Cultural Signification* (SSCS). SSCS are dynamic symbolic superstructures (Radford, 2008a) that include cultural conceptions about the world and the individuals. They comprise ideas about things in the world (e.g., the nature of mathematical objects and their way of existing), ideas about truth (e.g., how truth is and can be established), and ideas about the individuals. SSCS are full of tensions, as are the activities from where they emanate, and have a normative function (which may be explicit, or implicit, or both). They necessarily convey political and ethical views: how we show ourselves to others, how we are expected to behave socially and to be recognized by others. SSCS attempt to explain, among other things, how cultures provide the "raw material" from which subjects draw the ideas of what they are (their meaning, their identity, their power of action, etc.). Clearly, the relationship between the cultural "raw material" and the concrete individuals cannot be seen as a Platonic or logical, or causal, or mechanical relationship. The relationship is in fact *dialectical*. It should, therefore, be important to start by conceptually distinguishing the elements of this dialectical relationship; that is, to start distinguishing between what we mean by "being" and what we call "subjectivities."

To account for the cultural nature of being I suggest the following definition. Being is a general, cultural, dynamic (that is, always changing) ontological category. It is constituted of historically coded forms of conceptions about the individuals and the ways in which individuals are called to present themselves to the world and to interact with other individuals. More precisely, being is constituted of cultural ways of living (i.e., *be-ing*) in the world: ways of conceiving of oneself and of being conceived; ways of positioning oneself and of being positioned; forms of self and otherness (i.e., relationships with oneself and others). In our account, being is *potentiality* (what Aristotle called δύναμις, *dunamis*); that is, something whose mode of existence is not actual but *potential*.

What do we mean by subjectivity? The concept of subjectivity suggested here starts from an idea about the human subject according to which humans are not mere passive recipients of external influences with their own innate structure. Humans are affected by their context but in a reflexive manner. With this idea in mind, we can define subjectivity as the always ongoing instantiation or materialization of being. This unachievable and always ongoing instantiation is a unique, concrete subject (a *subjectivity*), whose specificity results from the fact that it is a reflexive sentient entity always in a process of *be-com-ing*: an unfinished and unending project of life.

Subjectivities co-produce themselves not in contemplation but in the course of a process whose name is *human activity*. Following dialectical materialism, human activity is not a mere set of actions: activity is a *system* (Leont'ev, 2009, p. 84) in constant development, incessantly affected by the entities it mediates. It is, indeed, through human activity that we become subjects-in-the-

making: incomplete individuals in constant flux. It is through human activity that we produce ourselves with, and through, others, within the possibilities and limits offered by our culture (Radford, 2008a). We can ask: How is the human activity that, at school, produces teachers and students? What is its specificity? These questions orient the discussion of the classroom example that I discuss in the next section.

THE CLASSROOM CONSITUTION OF MATHEMATICAL SUBJECTIVITIES

The example that I would like to discuss comes from a pre-school classroom of combined pre-kindergarten and kindergarten children (4-6 years old). The content of my example is mathematics, which occupies an extremely important part of the Ontario pre-school curriculum. Naturally, the emphasized presence of mathematics at the pre-school level is coherent with what the children will find later on: a curriculum axed around mathematics and language. Since the dawn of the 20th century, mathematics came to occupy a privileged position in the school curriculum of those countries that saw in industrialization the path towards modern society. Mathematics became the ally and support of the new capitalist forms of production. Many early 20th century pedagogues understood civilization as synonymous with industry. To a large extent, the main problem of the educational reform was the problem of massive schooling to train the young in the participation and development of a technological society. One century later, things have not changed much. Capitalism has not vanished. It has become trans-national, diversified, and globalized. It is hence not surprising that the preschoolers I see entering the school every morning start the day with activities around counting. If the school has to produce consumers, counting has to be the starting point.

At first sight, counting may seem to be a *natural* activity: the same activity regardless of place and time. On a closer look, however, as anthropological and ethnomathematical research shows, not all cultures have counted in the same way and not all cultures have counted the same things (e.g., Lancy, 1983). Counting can be better conceptualized as a culturally codified numerosity-oriented way of thinking and acting to make sense of the world (Radford, 2008b). Classroom activity is an attempt to provoke the encounter of children with a specific culturally constituted form of thinking about numbers. Rather than natural and conceptually neutral, the form of thinking favoured in the Ontario and in other curricula around the world conveys a specific *worldview* (i.e., what counts as counted), and operates within a particular *rationality* (e.g., how things should be counted) thereby creating its own *regime of truth*. The worldview, its rationality, and concomitant regime of truth are all central elements of the Semiotic Systems of Cultural Significations that organize the school as a social institution and sanction the kind of knowledge to which students are exposed. To avoid believing that cultural mathematical forms of thinking are purely conceptual and to better grasp their economical-political substratum, it is worth recalling the political struggle between merchants and the feudal aristocracy in 13th century Florence about the legitimate way to count—with Arabian numerals or counters—and the prohibition issued by the Guild of the Money Changers to use Indian numerals (Struik, 1968). And all this would have very little bearing on our discussion were it not for the formidable fact that subjectivities are not merely produced as an epiphenomenon of learning: on the contrary, *what* is learned and *how* it is learned are the threads out of which subjectivities are made. From the theoretical perspective that I am sketching here, it would be a mistake to conceive

of the child as already equipped with her proclivities, tastes, and personality—as idealist and rationalist pedagogies do. The child is an individual in an unending process of becoming dialectically entangled with *what* she learns and *how* she learns it.

To sustain the previous ideas, I would like to discuss a classroom activity in the rest of the article. The activity is about an arithmetic game played between two children. The object of the activity, as discussed with the teacher and our research team, was to offer the children an occasion to become acquainted with cultural forms of counting and thinking about numbers as targeted by the curriculum. A plastic sheet contained two rows made up of 10 squares with space enough for the children to place a small plastic bear in each (see Figure 1, pic 2 below). One child received 10 bears of one colour, and the other child received 10 bears of another colour. They received one dice. I focus here on the second part of the game. In the second part of the game, the children started with empty rows. The rules were as follows: taking turns, each child had to place on her/his row the number of bears that corresponded to the number shown by the dice after the child rolled the dice. The winner is the child who fills her/his row first. To fill the row, the child has to roll the dice and obtain the exact number of points on the dice as the number of spaces left on her/his row. To demonstrate the rules, the teacher played a game with a child in front of the class. Then, the class was divided into groups of two. Here is an account of the game played between Carl (C) and Jack (J).

J rolls the dice and gets 6. He says "6!" and places six bears on his row, one bear at a time while counting aloud "1, 2, 3, 4, 5, 6." C follows closely J's actions; he counts and says "6!" He waits for J to finish. Then he says "OK. My turn, my turn!" J responds "OK. I'll just put this [the dice] there for a, for now" and places the dice close to C's row. C takes the dice, rolls it, and says "Oh! 2!" He takes one bear at a time and places them on his row, while counting aloud "1, 2." J follows C's actions, counting closely. J seems to have forgotten how many bears are already on his row. He takes three bears from his row and counts those that remain, pointing to them successively, and says "1, 2, 3." Then, he proceeds to put back those that he just removed. As he puts them back one after the other, he says "4, 5, 6." The page moves a bit; the bears move from their square and now appear not properly placed. A bunch of them are on the same square. In the meantime, C moves the dice close to J's row and says "Ok, it's your go." Up to this point, the children follow the rules of the game. Following the rules, I want to suggest, is an important moment in the children's process of subjectification; that is, in the process through which they co-produce themselves as subjects of mathematics and subjects of education, more generally. Indeed, by following the rules they are not merely practising a way of counting. They are positioning themselves in a social world where their actions are recognized as legitimate. To enter into this world, they have to learn to cooperate. Their cooperation, as it will turn out later, is still very fragile. To enter the social world, they also have to control themselves. They wait, not impatiently, for the other player to finish placing his bears. As Vygotsky noted, "A very young child tends to gratify his desires immediately. Any delay in fulfilling them is hard for him and is acceptable only within certain narrow limits" (1967, p. 7). This is why, Vygotsky contends, it is a mistake to conceive of the child "as a theoretical being" (p. 7) moving from a cognitive stage to another. Yet, the turn-taking process to which I have referred above, although central in the children's subjectification, is not enough. The children have to enact the game's rules, which means that they have to subject themselves to the same *regime of truth*:

they have to count following a *same* culturally and historically constituted way of counting that, despite the presence of the bears, the dice, etc., targets an abstract form of arithmetic thinking that will be required in the abstract commercial exchange network that they will find in society.

Let me continue with my account of the game. J rolls the dice. The upper face shows two points. J is not happy with the result, picks up the dice again, puts it in his hands, shakes his hands vigorously, and lets the dice fall. He utters "5!" Satisfied with the result, he starts adding bears while counting "1, 2, 3, 4." He runs out of bears. C has been looking at what J does, apparently without fully understanding J's actions. At this point a child from another group calls the teacher and C's attention moves to that group. In the meantime, J is busy reordering his bears on his row. Thirteen seconds later, C's attention comes back to J. J is still reordering his bears on his row. C stretches his arm and tries to get the dice, which is in front of J. J prevents C from taking the dice, and says "So, it's … wait! Ok, it's …." C does not pay attention to J and says "Ok my [turn], I …" J interrupts and says "No, wait! Wait! Wait!" After some struggle C succeeds in getting the dice. J continues "So, it's 1, 2, 3, 4, 5, 6," and keeps on placing and counting bears: "1, 2, 3, 4." C is not paying attention to what J does. C rolls the dice twice. J finishes counting and puts his arms in a victory position. He utters "I won! I won! I won! I won! I won! I won! Look!" C turns the dice on his hand, and when he finds the 6-point face, he stops and starts counting the points: "1, 2, 3, 4, 5, 6 … 6!" He tries to start putting six bears on his row. J puts his arms on the page covering all the bears to impede C from placing his bears. J says "I won! … Me, I won!" C moves his body towards the page and in a very frustrated tone says "Ughhhhhh!" (see Fig. 1, pic 1). J insists "Me, I won!" C replies "Me is getting mad at you!" J responds "Me, I won! Won!" J takes the dice and shakes it vigorously as if to start a new game (see Fig.1, pic 2). C exclaims "No! JACK . . . Ughhhhhhh! No! This is enough!" He succeeds in getting the dice. "My was only when [I] have this" (he points to 6 on the dice) "So my turn." J answers: "No, you didn't get that! . . . You did like (he pretends to hold a dice on his hand and to move it around) flip, flip, flip and then you found 6! Um, Carl cheated, he does like flip, flip, flip, flip! . . . (pointing at C) Cheater! Cheater! Cheater! Cheater!" C reacts with his body. He comes very close to J as if he is going to hit him (see Fig. 1, pic 3).

The game turned very bad. At the beginning of this episode, C did not react to J's rolling the dice again after J got the discouraging 2 points. By rolling the dice twice, J transgresses the social dimension of the rule. To some extent, he is aware of it: when he picks up the dice the second time and shakes his hands vigorously, there is a sneaky smile on his face, which may mean something like: "You know, I know that I should not be doing this, but…" Maybe he interprets C's silence as a kind of complicity and continues playing seriously as if nothing had happened. Right after, C got distracted and his attention moved to another group. The result is a rupture in the children's collaboration that was present in the early part of this game. The collaboration includes a *coordination* of actions (e.g., taking turns) but also *paying attention* to what each player does. Part of collaboration is indeed to pay attention to others, even if it is not one's turn. To maintain his attention on the game is a tremendous task for C who is one year younger that J. In turn, although J's attention is on the dice and his bears, he does not realize that C is not paying attention. J is focused on his own actions. When C's attention comes back to the game, his attention is focused on taking his turn, regardless of the position of the game. The *regime of truth* that holds the children together in the first part of the game is no longer there. The social and theoretical common ground

embodied in the rules has disappeared. Without a common ground, the connection between the children is lost. Impulse and desire drive the children's deeds. The other has become an impediment to one's own actions. J disqualifies C by treating him as a cheater. C, who exhibits a lesser mastery of the language than J, responds with unarticulated phrases and with frustrating emotions expressed verbally ("Ughhhhhhh!") and with threatening body language.

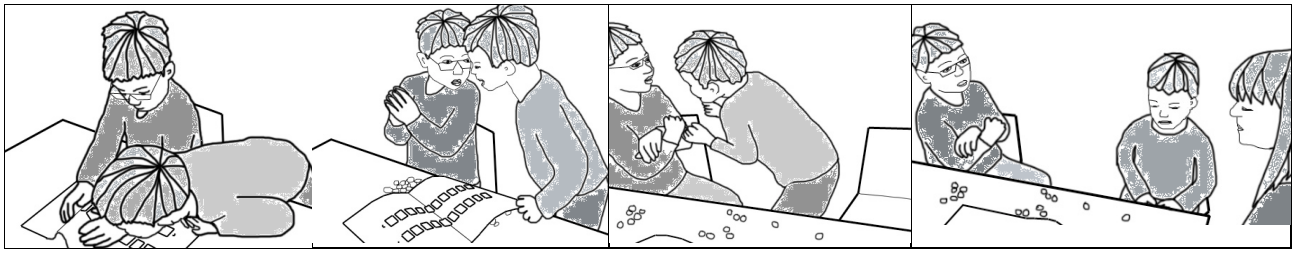

Figure 1. Pic 1: J and C discuss the game. Pic 4: The teacher and the children.

At this point, the teacher (T) comes to see the children:

T: (She positions herself close to C and talks to him in a calm tone.) Sit down.

J: (Furiously, points at C) Cheater!

C: Me no cheater. (Turns to the teacher.) He does not want to listen to me!

T: (Talks to C in a patient, supportive and comforting tone.) He doesn't listen to you? (See Fig. 1, pic 4)

C: No!

T: What are you trying to tell him?

J: (Points at C) He, he cheats!

T: (Talks to J in the same calm tone she talked to C.). OK. Stop saying that.

J: He was doing like (Makes some gestures with his hands.)… and found 6.

T: (Talks to C in a calm tone.) What … what do you want to tell him?

C: Uh…

T: (Talks to both children.) Whose turn is it?

C: Me, me, me rolled like that but he didn't listen.

T: OK. Roll it [the dice] again. We'll restart.

At this point, the children started collaborating again. They started taking turns, paying attention to the other, putting the bears on their row and counting aloud. The teacher remained with them for 12 seconds and left to see another group. The teacher succeeded in calming both children. Through her second and third utterance the teacher shows empathy; that is, as the Greek term *pátheia* intimates, the acknowledgment of the suffering of the other. C responds positively to the teacher's empathic attitude. The teacher also politely asks J to stop calling C a cheater. Not without effort, J acquiesces to the teacher's request, controls himself and calms down. The teacher is now in a position to restore the children's attention. The children can now move beyond accusatorial body, hand, and verbal actions and can focus on the game and its rules. This episode shows the tensions that underpin the processes of subjectification out of which subjectivities are being produced. Through these processes the children encounter forms of being that have been culturally and historically constituted—general coded forms about the ways in which individuals are called to present

themselves to the world and to interact with other individuals. These coded forms are general, and as such cannot be perceived or sensed by the children. To be sensed, to become objects of consciousness and reflection, these coded forms of being need to appear in the concrete material world of action, language, feeling, and thought. And it is the classroom activity that accomplishes that. It is the classroom activity that moves being from its state of potentiality to actuality through semiotic collective processes of meaning-making. In this sense, the activity embeds the children and the teacher. But at the same time, the activity is produced by the deeds of the children and the teacher. This is why, we cannot attend to the participants without attending to the activity in which they are immersed, and reciprocally, we cannot attend to the activity without attending to the participants that produce the activity. This is the dialectic nature of activity and participants through which being is disclosed. In its disclosing, the children and the teacher feel and socially experience anger, frustration, empathy, collaboration, responsibility, solidarity, etc.

These phylogenetically constituted features of being become crucial elements of the always evolving Semiotic Systems of Cultural Signification; they become pointers of action and *be-com-ing*. Such features of being are not encountered, sensed, and experienced equally by the children. In the course of activity, they are understood (not necessarily at the conceptual level) in varied ways. They occur in an emerging interpersonal ethical attitude that, in previous work, we have termed *togethering*; that is to say, a relational attitude based on a non necessarily explicit "ethical commitment participants make to engage in and produce activity" (Radford & Roth, 2011, p. 227). And as we have seen, the encounter with the aforementioned features of being is deeply entangled with the manner in which the teacher interacts with the students. The teacher draws on developed features of being that, in the course of classroom activity, come to interact with the children's emerging conceptual and emotional understanding of the situation. As Vygotsky (1989) once noticed, cultural forms of knowing and being (voluntary attention, arithmetic thinking, forms of human collaboration and ethical dispositions) do not result from mere interaction. Contrary to other living species, humans are not pure biotypes. As a result, cultural forms of knowing and being rather result from the interaction between phylogeny and ontogeny.

CONCLUDING REMARKS

This paper has been an attempt at investigating the production of subjectivities in the classroom. To do so, I resorted to semiotics understood as a theory of signification. Semiotics deals with the production of signs and their meanings, but it also deals with the manner in which individuals are signified and signify themselves. To try to reach my goal I followed a non-substantialist viewpoint. I argued that subjectivities are always in a process of *be-com-ing*. They are co-produced in activity-bound processes of subjectification dialectically entangled with *what* children learn and *how* they learn it. What children learn and how they learn it is deeply entangled with Semiotic Systems of Cultural Significations, which offer a normative symbolic superstructure to the individuals' actions and reflections. These ideas were illustrated through a classroom activity game around counting. The activity allows us to see some of the dynamics and tensions that underpin classroom processes of subjectification. The game was based on a rule that, as all rules, is much more than conceptually following a sequence of instructions. The rule appears indeed as a social and conceptual structuring element through which the children become endowed with generality: they recognize themselves and are socially recognized as players with the ensuing entailment of actions and expectations. In

short, the children become endowed with a generic element that applies to them, as well as others or to any player of the game for that matter. Rules in play (as in life), as well as other more implicit regulative mechanisms of behaviour, extricate the individual's experience from its pure and private significance. They bring experience to a new realm. They position individuals into a cultural, historical, and social world. The rule is enacted in classroom activity, which opens up the space for the children to encounter evolved features of knowing and being (e.g., counting, as well as sophisticated forms of cooperation, action coordination, but also empathy, respect, solidarity, responsibility, etc.). For instance, despite all his frustration, we saw C positively *responding* to the call of the teacher. To answer to the call of the other is part of *responsibility*. In his book *Éthique et infini*, Lévinas notes that responsibility is "the essential, primary and fundamental structure of subjectivity. . . It is in ethics, understood as responsibility, that the very node of the subjective is knotted" (Lévinas, 1982, p. 91). He goes on to say that "Responsibility in fact is not a simple attribute of subjectivity, as if the latter already existed in itself, before the ethical relationship. Subjectivity is not for itself; it is, once again, initially for another" (Lévinas, 1982, pp. 92-93). In short, in the example here discussed, the classroom activity opened up a space for the children in which existential and ethical areas of human life appeared. The activity makes it possible to envision new non-individualist, communitarian based, aesthetic forms of human collaboration (Radford, 2014) out of which we may move towards what Marcuse (2007, p. 227) called "the liberation of both the senses and reason from their present servitude."

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