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ON SEMIOTICS AND EDUCATION

Luis Radford
École des sciences de l'éducation, Université Laurentienne, Ontario, Canada

A "semiotic approach" to education, that is an educational approach that draws from semiotics, cannot consist in the mere amalgamation of a semiotic theory and an educational one. In this article, I argue that the manner in which semiotics can contribute to education is mediated by the epistemological assumptions that underpin educational theories and the extent to which those assumptions can be cast in, and even be transformed by, semiotic concepts and constructs. After discussing the epistemic role of signs in two major epistemological theories (Leibniz's and Piaget's), I present an example of a semiotic approach to education that I illustrate around a classroom teaching-learning episode.

Keywords: semiotics, language, semiotic function, corporeality, objectification.

Sémiotique et éducation

Une perspective didactique qui cherche à inclure des idées sémiotiques ne peut pas se limiter à effectuer une amalgamation d'une théorie sémiotique et d'une théorie didactique. Dans cet article, je suggère que la manière dont la sémiotique peut contribuer à la recherche en didactique est conditionnée par les supposés sur lesquels s'appui l'approche didactique elle-même et par les possibilités d'exprimer (voire transformer) ces présupposés à l'aide de concepts sémiotiques. Dans la première partie de l'article, afin de montrer la complexité de la problématique sous-jacente à l'articulation de sémiotique et didactique, je m'arrête sur le rôle épistémique des signes. Pour ce faire, j'examine deux théories épistémologiques importantes (celle de Leibniz et celle de Piaget). Dans la deuxième partie, je présente un exemple d'une perspective sémiotique éducative à l'aide d'un épisode de salle de classe.

Mots-clés : sémiotique, langage, fonction sémiotique, corporéité, objectivation.

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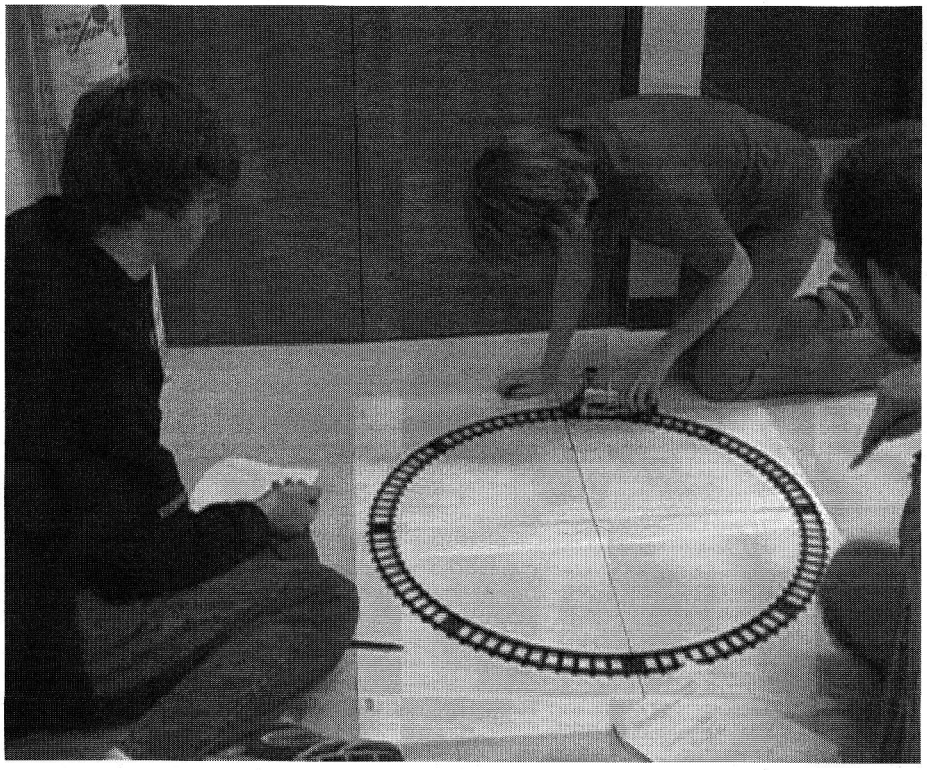
INTRODUCTION

In the investigation of teaching-learning processes substantial attention has been paid to the written and oral registers. Recent studies suggest, however, that gestures, body posture, kinesthetic actions, artifacts and signs in general are a fruitful array of resources to be taken into account in order to investigate how students learn and how teachers teach (Arzarello, 2006; Bautista & Roth, 2012; Forest & Mercier, 2012; Radford, 2009; Radford, Edwards & Arzarello, 2009). Instead of being mere epiphenomena surplus of teaching and learning, these resources, it is argued, mediate the teacher's and the students' classroom activity in substantial manners. Figure 1 shows three Grade 11 students during a trigonometry lesson where they are devising a formula to describe the position P(x(t), y(t)) of a train that moves at a constant speed along a circular route. The student to the left measures time with a chronometer; following the train, the student in the middle indicates with inked signs the train's position along the path of the train at different times; the student to the right coordinates the other two students' action and takes notes on a field sheet.

It is in the investigation of the varied arrays of resources to which we resort in our daily life to think, signify and communicate, that semiotics may offer an interesting contribution to education. Semiotics, indeed, is concerned with signs of any kind and with the various modes in which humans signify. Semiotics, Eco (1988, p. 26) suggests, is devoted "to describe the functioning of communication and signification."

However, the contribution of semiotics to education cannot be taken for granted. There is no such thing as a direct or unproblematic application of semiotics to education. In order to properly account for the educational problems we deal with —e.g., how teachers and students learn specific contents, or how they use and become familiar with digital artifacts— the semiotic concepts to which we may resort have to be suitably *integrated* into theories of teaching and learning. A simple *amalgamation* of semiotics with learning theories would lead if not to failure then at least to limited success. We should not lose sight

Figure 1. Grade 11 students investigating the equation of the point P(x(t), y(t)) of a train that moves around a circle at constant speed.



of the fact that the main semiotics systems were not devised to tackle educational problems. They were oriented to answer questions of a linguistic nature (e.g., Saussure (1916)) or appeared integrated in the development of general theories —e.g., a phenomenological theory of knowing (Peirce, 1958-1966); a phenomenological theory of consciousness (Husserl, 1970); or a phenomenology of perception (Merleau-Ponty, 1960)[1].

If the amalgamation of semiotics and education does not seem to be the best option, how can these two disciplines be brought together? I do not think that, at this point in the development of educational research, a reasonable answer can be provided. What seems reasonable to assert, by contrast, is that because of the semiotic focus on signs and signification, and the educational focus on knowing in particular contexts (e.g., classrooms, workplace, etc.), a semiotic approach to education rests unavoidably on assumptions that are made about the *epistemic role of signs*. In other words, the manner in which semiotics can contribute to education is mediated by the epistemological assumptions that underpin educational theories and the extent to which those assumptions can be cast in, and even be transformed by, semiotic concepts and constructs.

It is this idea that I intend to articulate in this article. In doing so, I hope to contribute to our understanding of the possibilities, limits, and challenges that semiotics offers to educational research. I start with a discussion about the cognitive role of signs in the work of the German philosopher Gottfried Wilhelm Leibniz. Although Leibniz does not offer a semiotic approach to education, his theory of human understanding shows, in an interesting and clear way, that our recourse to signs in our attempts to account for the manner we come to know is, indeed, enmeshed in assumptions about knowledge and the structure of reality. In subsequent sections I shall deal with Piaget's semiotics and contrast it with contemporary sociocultural trends. In the last section I present an example of a semiotic approach to education.

LEIBNIZ AND THE EPISTEMIC ROLE OF SIGNS

Against Medieval theories of signs, Renaissance semioticians argued that signs are not merely bound to the realm of logic. Signs, the Renaissance semioticians contended, have a cognitive and epistemic dimension. As expressed by Portuguese Pedro Margallo, in his 1520 *Logius utriusque scholia in divi Thomae subtilisque Duns doctrina ac nominalium*, "*signum est res faciens cogitare*," that is, "a sign is something that makes think" (quoted in Meier-Oeser, 2011). Charles S. Peirce went even further and claimed, in *Some Consequences of Four Incapacities*, that we do not have the power of thinking without signs (Peirce, 1868). But what is the exact role of signs in what we think?

This question is at the heart of Leibniz' theory of signs. In his 1677 paper *Dialogue on the connections between things and words*, he invites us to consider whether we "can perform any [e.g., two- or three-digit] arithmetical calculation without making use of any number-signs" (Leibniz, 1951, p. 8). What Leibniz had in mind, however, was much more than the idea of signs as mediators or facilitators of thought. Leibniz was fully aware of the fact that a cognitive discussion about signs must not dismiss the problem of the manner in which signs relate to ideas and things. In other words, for Leibniz you cannot turn to semiotics without making explicit your own epistemological and ontological principles.

Leibniz' semiotics rests on a view according to which there is a correspondence between language, thought, and the conceptual structure of reality. Indeed, Leibniz assumed that all existing things are amenable to representations that would reflect their essence and convey their relationships to other things. The world, in other terms, lends itself to be expressed through signs. In *What is an idea?* (a paper written in 1676), he says: "there must be something in me *which not only leads to the thing but also expresses it*" (Leibniz, 1951, p. 281; italics in the original). In Leibniz's view, there is an ontological *commonality* between the thing expressed and its semiotic expression. It is this very commonality that ensures the success of our inquisitive endeavours and that we come to know things:

> the model of a machine expresses the machine itself, perspective drawing in a plane expresses a solid, a speech expresses opinions and truths, letters express numbers, an algebraic equation expresses a circle or some other figure; and it is because these means of expression have something in common with the conditions of the thing expressed and studied, that we can come to know the corresponding properties of the thing expressed. (Leibniz, 1951, p. 282)

However, the harmonious correspondence between *ordo rerum* and *ordo idearum* that, in Leibniz's view, semiotics realizes is haunted by difficulties. If Leibniz is one of the 17th century philosophers who championed the idea that there is a link between the human capacity for language and the human capacity to understand reality, natural languages, according to him, are nonetheless far from reflecting reality. Given the epistemological and ontological principles that he adopted—in particular the assumption that things and ideas have a common deep semiotic structure—he was led to believe that there should be a language that would overcome the deficiencies of natural languages. It should be possible, he thought, to find a language with suitable definitions to capture the essence of things; it should also be possible to find the characters (i.e. signs) to represent things and their essence. These characters would allow the individual to obtain, in a transparent manner, the fundamental knowledge of the thing that they represent.

It is within this context that Leibniz envisioned a language whose signs or characters (and the combinations of them) would result in clear thinking and a clear manipulation of syllogisms and judgements. This language is what he termed the *universal characteristic*. As Rutherford puts it,

> the universal characteristic would enable us to construct linguistic characters which are transparent representations of intelligible thoughts, something the signs of natural languages typically fail to be, and to reduce logical reasoning to a mechanical procedure relying solely on the substitution of formal characters. (Rutherford, 1995, p. 225)

Considering mathematical signs as a kind of model, Leibniz endeavoured to find a method to assign characters or signs to our thoughts so that they would be combined or operated unambiguously as in arithmetic calculations or geometric proofs:

> *De la il est manifeste, que si l'on pouvoit trouver des caracteres ou signes propres à exprimer toutes nos pensées, aussi nettement et exactement que l'arithmetique exprime les nombres, ou que l'analyse geometrique [i.e., l'algèbre] exprime les lignes, on pourroit faire en toutes les matieres autant qu'elles sont sujettes au raisonnement tout ce qu'on peut faire en Arithmetique et en Geometrie. Car toutes les recherches qui dependent du raisonnement se feroient par la transposition de ces caracteres, et par une espece de calcul (Leibniz in Couturat, 1961, p. 155 ; italics in the original)*[2]

It is within this context that Leibniz attempted to improve Euclid's definitions of fundamental geometrical objects (such as points and lines) and that he cast doubts about the potential of algebra to represent geometric objects and calculations, as Descartes did in his geometric investigations. As he put it in a letter to Huygens in 1675,

> *Mr. Descartes ... a donné une méthode de digérer par ordre les courbes et de les accommoder aux équations. Mais il ne s'y est pas pris de la manière la plus simple et la plus naturelle pour ce qui est de les accommoder aux équations. (Leibniz, 1995, p. 11)*[3]

To sum up, Leibniz's account of knowing rests on the assumption of a natural correspondence between knowledge, signs, and thought. It is within these assumptions that Leibniz envisioned the quest for a universal language or characteristic based on the model of a mathematical symbolism and its unending possibilities of substitution and formal transformations. In a certain sense, Leibniz articulated a widespread cultural and intellectual view of the European 17th century. As Wiener (1951) notes in his Introduction to his *Leibniz. Selections*, although societal structures of Leibniz's time remained supported by theological, legal and political traditions, "The development of more exact methods of measuring time, latitude and longitude, interest and insurance rates and new techniques of agriculture" conveyed a view in which mathematical reasoning and symbolism became highly regarded (p. *xx*). It is not surprising to find Leibniz engaged in the search for an automation of human actions, such as the invention of adding machines for banks and ready-reckoners for commerce (Wiener, 1951). Leibniz appears hence as a truly modern thinker that offered a great insight into the complexities that underlie the relationships between signs, thought and reality. Although Leibniz's epistemological and ontological commitments may be found controversial or problematic today, his work help us to see that semiotics cannot be merely juxtaposed against a theory of knowing. Semiotics becomes ineluctably tied to epistemological and ontological assumptions

of the theory by helping us to cast these assumptions through semiotic concepts and constructs.[4]

PIAGET'S SEMIOTICS

With the development of new entrepreneurial forms of mercantile production and the expansion of commercial activity in the 17th and 18th centuries arose a new form of subjectivity that progressively asserted itself as the foundation of meaning and knowledge. While in medieval times to know something was *to see it in God* —considered as creator of everything existing and to come— from the 17th century on, as Arendt (1958) reminds us, to know something became increasingly equated with knowing the process of its formation. Fabrication became the metaphor of the new epistemological paradigm.

Such a shift was not accomplished easily. Leibniz was certainly well positioned to feel the tremendous tension and significance of it. He attempted to reconcile the remnants of the old paradigm with the requirements of the new one, by arguing that the two views amounted to the same thing: to know something is to produce our own ideas and to see them in God:

> As to whether we see all things in God ... or whether all ideas are our own, we must realize that even if we do see all things in God, it is still necessary for the ideas through which we see to be our own at the same time; that is, our ideas are not little replicas, so to speak, but affections or modifications corresponding to what we perceive in God. (Leibniz, 1951, p. 289)

In Kant's time, the secularization of knowledge progressed further and bit-by-bit knowledge became understood as that which the individuals produced on their own. For Kant, knowledge is something that the individual *constructs*. Thus, referring to mathematics, he says:

> mathematical knowledge is the knowledge gained by reason from the construction of concepts. To construct a concept means to exhibit a priori [i.e., prior to experience—LR] the intuition [or representation—LR] which corresponds to the concept. For the construction of a concept we therefore need a non-empirical intuition. The latter must, as intuition, be a single object, and yet none the less, as the construction of a concept (a universal representation), it must in its representation express universal validity for all possible intuitions which fall under the same concept. (Kant, A713/B741; 2003, p. 577)

This short passage of Kant's *Critique of Pure Reason* provides us with a neat window through which to observe the accomplishment of the shift Leibniz saw coming in his own time. By enquiring into the limits of human reason, Kant shook up the assumed belief of a harmonious correspondence between thought and reality. His emphasis on the subject as the doer of his or her own knowledge unveiled at the same time the boundaries of human finitude and the limits of possible experience. The possibility of knowing the *noumena* or the *things-in-themselves* was no longer ascertainable. There is, hence, a change at the ontological level. The ontological change does not diminish our possibility to know something. What Kant says is that, in our human endeavours, we do know, but what we end up knowing is not the *noumena* but rather the products of our own experience. In doing so, Kant adds a new ontological layer, where the individual lodges his/her own constructions. His ontology of things-in-themselves remains, as Rockmore (2011) notes, extremely traditional, Platonic in fact. What is truly innovative in Kant's theory of knowing is his epistemology. Along with Giambattista Vico (2002), he articulates a view that remained in slow incubation for two centuries or so, namely a view of the individual at the center of the epistemological arena. And it is here that Piaget comes in.

Indeed, in Piaget's genetic epistemology, knowledge is theorized as something that emanates from the individual's actions and is primarily produced by the individual itself. Annoyed by the excessive focus on language by the proponents of logical positivism—who, not really far from Leibniz's views, considered the logic of human thinking susceptible to expression through a symbolic processual language—Piaget argued again and again that the origins of intelligence are not to be found, ontogenetically speaking, on the side of language, but on the side of sensory-motor actions. In a paper presented at a symposium in *Neuchâtel*, in 1962, he claimed that

> *L'un des fondateurs de cet « empirisme (ou positivisme) logique » , R. Carnap, a commencé par soutenir que la*

logique entière ne consistait qu'en une syntaxe générale, au sens linguistique du terme. Dans la suite et en parallèle avec Tarski, il a été conduit à y adjoindre une « sémantique » générale, mais ceci ne nous fait pas non plus franchir les frontières du langage. Enfin Morris a montré la nécessité (non d'ailleurs reconnue par l'Ecole entière), pour rendre compte du caractère opératif de la logique, de compléter la syntaxe et la sémantique logistiques par une « pragmatique », mais il s'agit, toujours encore, des règles de l'utilisation d'un langage et nullement d'une logique de l'action. (Piaget, 1972, p. 84)[5]

In the same paper, in articulating the question of the primacy of action over language, he reminds his readers that operations, insofar as they result from the interiorization of actions and their coordination, "remain relatively longtime independent from language" (1972, p. 88). A few years earlier, in his article *Le langage et la pensée du point de vue génétique* [Language and thought from a genetic viewpoint], Piaget had held the same view and asserted that

Les operations + et – [c'est-à-dire d'associement et disassociemente] sont ... des coordinations entre actions avant d'être transposes sous une forme verbale et ce n'est donc pas le langage qui est la cause de leur formation : le langage étand indéfiniment leur pouvoir et leur confère ume mobilité et généralité qu'elles n'auraient pas sans lui, c'est entendu, mais il n'est point à la source de telles coordinations (Piaget, 1954, p. 55)[6]

This is how he answered a famous and profound question that he formulated several times during his career, namely whether or not language can constitute a necessary condition to the achievement of the logical-mathematical operations without being nonetheless a sufficient condition to their formation (e.g., Piaget, 1951, 1954, 1972,).

To understand Piaget's answer we have to bear in mind that language, in his account, can only be understood in its relationship to a semiotic instrument that is ontogenetically contemporaneous to it (Piaget, 1954): the *semiotic function*, which Piaget defined as "the ability to represent something by a sign or a symbol or another object" (Piaget, 1970, p. 45). The semiotic function "commences when signifiers are differentiated from what is thereby signified and when signifiers can correspond to a multiplicity of things signified" (Piaget, 1980, p. 29,). It includes language but does not coincide with it: it also includes gestures, differed imitations, mental imagery, which he equated to interiorized imitation, symbolic play, and sign language (Piaget, 1980 p. 28). For Piaget, language and the semiotic functions have first of all a representational nature that operates at the level of things and the individual's actions. It is the semiotic function that is responsible for the transition from action to representation (1980, p. 28). The sensorimotor signifiers are replaced with symbolic signifiers that signify things that are not necessarily present but evoked. From this representational conception of language and the semiotic function more generally, Piaget does not have any problem showing that thought precedes language. "We can conclude," Piaget says, "that thought precedes language, and that language is limited to transform thought in helping it to attain its forms of equilibrium through a greater schematism and a more mobile abstraction" (Piaget, 1954, p. 54).

Like in Leibniz's case, Piaget presents us with an interesting example of how semiotics appears entangled and underpinned by epistemological and ontological premises. Drawing on Kant's epistemology and giving up Kant's aprioristic assumptions, Piaget elaborated a genetic epistemology where knowledge is constructed by the subject as a result of sensorimotor actions that give rise to a symbolic function. This allows the subject to imitate his/her previous actions, which are now *evocated* through symbolic signifiers[7]. His genetic epistemology is one of the most celebrated achievements in psychology and epistemology. I would like to suggest that Piaget's genetic epistemology is to the 20th century what Leibniz's *Nouveaux essais sur l'entendement humain* is to the 17th century. Yet, as we have previously seen, the Piagetian theoretical edifice rests on a limited conception of language and the semiotic function. For Piaget, semiotics more generally is of the order of representation and remains hence a referential mechanism of knowers and their actions.[8]

In the next section I dwell on some contemporary trends where the human mind is considered to arise from language. Such a contrast will allow us, I hope, to have a better grasp of some of the intricate and complex relations between semiotics and theories of knowing and learning. Then, I shall deal more directly with semiotics and education.

SOCIOCULTURAL TRENDS

In contrast to the Leibnizian conception of semiotics— a conception that, as we saw in Section 2, endows signs with a role of expressible mediators between the alleged harmonious structures of thought and reality— Piaget adopted a conception according to which a semiotic mechanism (the semiotic function) provides the individual with the elements to go beyond actual actions with objects in order to reach the level of conceptual schemes of reflective abstraction. Signs acquire the status of signifiers of objects or actions. But what about the ontological dimension of the objects of knowledge? Rotman (1977) has discussed at length this dimension of Piaget's genetic epistemology in his book *Jean Piaget: psychologist of the real*, and shows how Piaget adopts a form of biological realism. Here, I would like to refer to an interesting passage that comes from Piaget's response to the comments that René Thom offered during the famous Piaget-Chomsky debate. In his comments, which deal with the question of the ontological status of space, Thom asked whether space is a subjective construction or something real, out there. "One of two things: either exterior space exists as such, as the universal framework in which all reality is localized ... or it is constructed from non spatial elements" (Thom, 1980 p. 361). Piaget responded:

> In regard to the concept of space Thom starts by offering an alternative which is precisely one I claim to have made obsolete: either a physical space outside or a construction of the subject. My answer is, on the contrary, that if mathematics is adapted to reality, it is because the subject, in his organic sources, is a physicochemical and spatial object, among others, and because, in the construction of his own cognitive structures, he starts from neurological and biological sources whose laws are those of reality. It is thus through primarily endogenous and not uniquely exogenous pathways that the space constructed by the subject fits in with the outside space; therefore, both exist without conflict and converge without merging. (Piaget, 1980, p. 369)

This ontological assumption is what Glasersfeld calls Piaget's "metaphysical realism" (Glasersfeld, 1988, p. 27).

In contemporary sociocultural trends (see e.g. Arzarello, 2006; Bartolini Bussi & Mariotti, 2008; Cantoral & Farfán, 1998) signs are often considered as mediators of thought without assuming nonetheless a Leibnizian pre-given relationship between the ontological structure of reality and the epistemological mechanisms that lead individuals to know this reality. Without dismissing the specific biological constitution of our species, there is no assumption either of a "natural" fitting between a reality out there and the subjective constructions of it. In a similar vein, in contemporary sociocultural trends, language is not empowered with the possibility of deciphering a world whose nature is supposed to be semiotic from the outset. Nor is language reduced to the semiotic role of producer of signifiers out of which the individual becomes capable of moving from the sensorimotor realm to a more abstract conceptual layer. In some contemporary sociocultural trends, in particular those that identify themselves as discursive, language and discourse build reality in manners that are contingent, and historically and culturally situated.

At the cognitive level, one of the differences between these theoretical positions is the following. In the Leibnizian case, it is considered that, ontogenetically speaking, signs derive from a mind that in its cognitive endeavour tries to find the correct signs to express itself. In the Piagetian case, signs derive from the constructive deeds of the subject and serve the purpose of bringing the individual's actions to a higher cognitive level of hypothesis, propositional operations and structures. Language, as Piaget conceptualized it, is one of the semiotic mechanisms to which individuals resort to overcome the perceptual immediacy of the objects; through language and the semiotic function concrete objects become evoked objects that can now be handled in hypothetical situations. Even if language is also something inter-individual to be used in cooperation with others, for Piaget it remains overall an instrument of abstraction. For sociocultural trends (e.g., Edwards, 1997), by contrast, cognition derives from culture. Although the theoretical articulations of how cognition actually derives from culture is thematized differently (e.g., as participation or as internalization), sociocultural trends agree on the fact that it is misleading to understand the relationship between cognition and culture in causal terms. A causal reading of this complex relationship would lead us back to the Enlightenment mechanist model of scientific explanation that is at odds with the understanding of culture as a continuous, dynamic, emergent movement of conflicting views and perspectives.

Within the large spectrum of sociocultural trends, communication comes to the fore. These trends understand language in a social and cultural way. Some of them argue that cognition arises out of acts of communication and is hence a social formation. For instance, Harré and Gillett (1994, p. 22) say that "[t]he idea that the mind is, in some sense, a social construction is true in that our concepts arise from our discourse and shape the way we think." Sfard suggests that thinking can be defined as "the individualized form of the activity of communicating" (Sfard, 2008, p. 174). In an explicit manner, this definition asserts that, ontogenetically speaking, cognitive processes are built on the basis of the social relations out of which our cognitive endeavours take place.

We see the tremendous difference between the Leibnizian, Piagetian, and the contemporary discursive trends to signs and knowledge. The Leibnizian trend works under the Cartesian assumption that ideas are *in* us. This assumption does not amount to considering the human mind as a sort of container. "What I mean by an idea," Leibniz says, "is not a certain act of thinking, but a power or faculty such that we have an idea of a thing even if we are not thinking about it but know that we can think it when the occasion arises." (Leibniz, 1951, p. 281). Ideas are dormant in us as "the statue of Hercules is dormant in the rough marble" (p. 290) from which it will one day emerge. In Piagetian epistemology, ideas are not *in* us in the Leibnizian sense; they are constructed by the individual in the course of its ontogenetic development. In sociocultural research, ideas are neither in us nor constructed in the Piagetian sense: ontogenetically speaking, they are in culture, embedded in its various practices, and —like the practices in which they are embedded— they are always evolving.

There is still another way to express the previous differences. Leibniz's epistemology rests on a potentially harmonious fitting of thought, reality, and language, whereby to know something is both to produce, or to account for, our own ideas *and* to see them in God. By contrast, Piaget's work is embedded in the foundational project of modern epistemology, a project whose main characteristic is the dismissal of a transcendental dimension (dimension that still survives in Leibniz's work in the form of God's knowledge), and a concomitant emphasis on the individual's actions as producers of knowledge. Different as they are, these epistemologies converge, although for unconnected reasons, in an identification of subject and object: there is no fundamental gap in *what is* (the ontological) and the epistemological (*how we know it*). Mind and things are coterminous. Sociocultural research re-introduces a transcendental dimension. That is, sociocultural research starts from the premise of a cultural (material and ideal) reality that precedes the cognitive activity of the individual, and as such transcends the individual from the outset. This reality is not a Platonic one: it is an always evolving conflicting reality shaped by political, economical, legal, ethical and other dimensions of everyday life. In other words, in contrast with modern epistemologies of which Piaget's is arguably one of the best examples, sociocultural research starts from the non-identity of mind and things. Although the world that we come to inhabit at birth is a historical world made by humans, ontogenetically speaking, we are not automatically, by the event of our birth, *in* the world in a cultural sense. There is an ontological gap that we progressively cross in a lengthy march as we come to inhabit our culture —a march in the course of which we remain in a state of unfinished and always changing selves, that is, in a state of becoming. If, hence, subject and object do not coincide, they are, as Adorno (1973) argues, mediated from the outset by cultural artifacts, as well as discursive and other practices.

We may appreciate better the difference between the epistemological systems sketched above if we return to the place that language and semiotics occupy therein. In sociocultural perspectives, culture and its various artifacts are continuously mediating subjects and objects. Language is the mediating cultural artifact *par excellence*. It is ubiquitously affecting the subject from its birth. For Piaget, from an ontogenetic viewpoint, language intervenes when the child starts using it. Before that crucial moment, the child has to be imagined if not as epistemologically deaf then at least as indifferent to language, as if language would be something like the apathetic and lethargic continuous noise of water running in a river.

AN EXAMPLE OF A SEMIOTICS APPROACH TO EDUCATION

Let me now turn to a semiotic cultural approach to education on which some colleagues and I have been working for the past few years. The semiotic

approach to education that I would like to sketch draws on Hegel's philosophy, chiefly his account of mediation as a central part of the dialectic between the general and the particular. For Hegel (1977), the particular is not a mere instance of something general. For example, a particular triangle drawn on paper is not just a specific case of the general or ideal form "triangle." Between the general and the particular there is a dialectical relationship that has to be understood as *mutually constitutive*. The general is the particular's *mode of existence*, which is what happens when we say that a specific triangle is part of the general category "triangle." Reciprocally, the particular *is* the mode of existence of the general. In this case, the specific triangle is the specific triangle itself (T=T), but is, at the same time, in its role of mediator, the *mode of existence* of the general. The same could be said of a poem, a musical composition, or a physical cultural performance—a dance, for example.

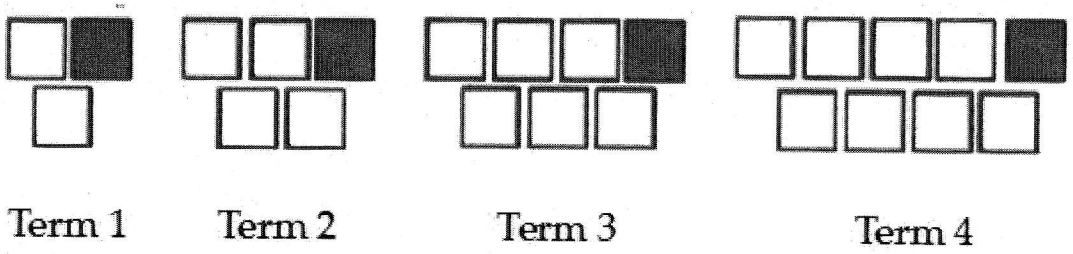
Bearing these remarks in mind, it turns out that knowledge is not only of the order of the general, for knowledge exists, and can only exist, in the form of concrete temporal-spatial processes enacted in activity rather than of impalpable things or products. This is why, within this perspective, knowledge (for instance mathematical knowledge) cannot exist in a kind of Platonic realm, detached from culture. Nor can it be reduced to its material dimension. In the same way as music does not exist in the score or in the violin, mathematics does not exist in rulers and compasses or in the dry written text of a theorem and its proof. A book or a written theorem is an artifact, a trace of activity. For, again, mathematics (as a set of practices) and mathematical knowledge (as defined previously) *exist* in the form of activities, in *performances*.

I can rephrase these ideas in other terms: In the *ontology* that underpins the semiotic approach to education that I am sketching, knowledge appears as an ensemble of culturally and historically constituted embodied processes of reflection and action. In the case of arithmetic, those processes would be processes of reflection and action about quantity mediated by *embodied forms* of representation, as in the case of the Oksapmin investigated by Saxe (1982) or *material forms* of representation, as in the medieval abacus, etc. In the case of music, it would be processes of aesthetic and aural expression mediated by violins, pianos, etc.

In the *epistemology* that underpins the semiotic approach to education that I am describing, to *know* is to enact (through embodied and other types of signs and artifacts) cultural forms of action and reflection. To *know* is to make them present, to expand and to generalize them, and also to criticize and subvert them.

As we can see, semiotics is crucially embedded in the ontological and epistemological concepts of the semiotic approach to education that I have been sketching. Now, how does *learning* enter into this picture? Learning appears as follows. The students cannot necessarily discern the historical-cultural forms of action, reflection, and expression that constitute, for instance, projective geometry or algebra. Let me refer here to an example that comes from a Grade 2 class of 7–8-year-old students. The example is part of a lesson whose goal was to get the students acquainted with a historical-cultural form of reflection and action that each one of us, as competent adults, recognize as algebraic — a cultural form of thinking that would easily lead us to generalize the sequence shown in Figure 2 in order to find, let's say, the number of rectangles in Term 100, as well as a formula for Term n.

Figure 2. The first terms of a sequence that Grade 2 students investigated in an algebra lesson.



When asked, mathematicians—and even adolescents having some familiarity with algebra (Sabena, Radford, & Bardini, 2005)— often report that they "see" the figures as divided into two rows. Then, they generalize this property to all (visible and non-visible) figures of the sequence, and easily come up with both a formula to calculate the rectangles in remote terms, such as Term 100 (see Figure 3a), and, although not without difficulties in the case of adolescents (Radford, 2003, 2010a), another formula, such as 2n+1, to calculate the number of rectangles in Term n (Figure 3b).

Figure 3.

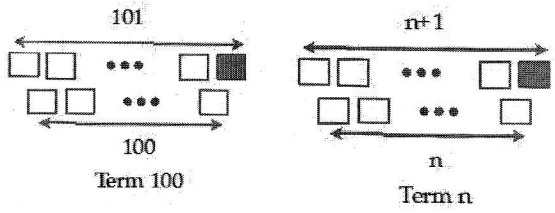


Fig. 3a	Fig. 3b

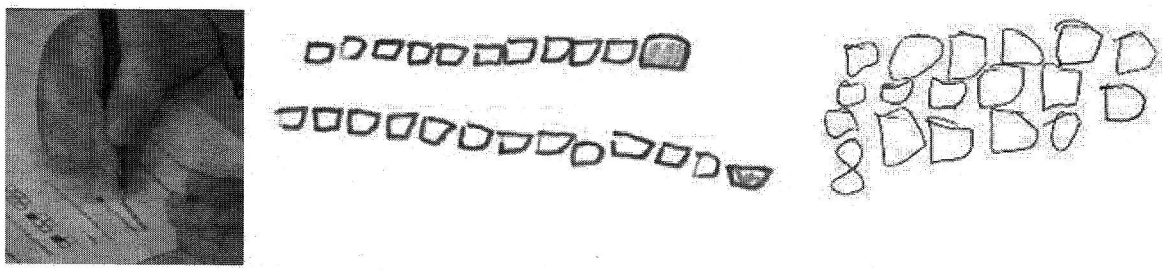
For the trained eye, the terms are often reported as divided into two rows.

For young students, however, discerning what we, as competent adults, could easily discern as algebraic is not necessarily easy. In fact, in our Grade 2 class, when the students extended the sequence and drew Terms 5, 6 and 8, they produced answers like those shown in Figure 4.

It is here where learning comes into the picture. Learning, I suggest, is the social, embodied, and sign-mediated process of creatively and critically discerning and getting acquainted and conversant with historical, cultural forms of expression, action and reflection. Learning hence rests on the following idea, already briefly mentioned before: At birth, we all enter a world that is not only populated by concrete objects but also by systems of thinking (mathematical, scientific, aesthetic, ethical, juridical, etc.). Learning is the creative and critical encounter with those forms of thinking. Those encounters occur in what we call processes of objectification.

It might be apparent now that in order to define learning in this manner, I had first to talk about knowledge and knowing. *What is knowledge?* This is the ontological question. Knowledge, I suggested, is an ensemble of culturally and historically constituted embodied processes of reflection and action. Examples of these processes are varied in and across cultures. They can be divinatory processes, as in the case of oracles in ancient Greece and in the African Azande's methods described by Evans-Pritchard (1968); they can also be the reflective corpus of making sense of the universe through numbers as developed by the Pythagorean brotherhood or the rather self-contained contemporary theories of numbers, etc. *What is knowing?* This is the epistemological question. Knowing is the enactment of these

Figure 4.



To the left, a student finishing drawing Term 6. In the middle, Terms 5 and 6. To the right, Term 8 according to another Grade 2 student.

culturally constituted forms of action and reflection. Knowledge and knowing mediate each other in the Hegelian sense. That is, one is the mode of existence of the other. Now, since this enactment in which knowing consists may not be apparent for the students, they learn it through processes of *objectification*—a term that acquires a specific meaning in our approach and from which the approach borrows its name. Indeed, we call this approach the *theory of objectification* (Radford, 2008, 2011).

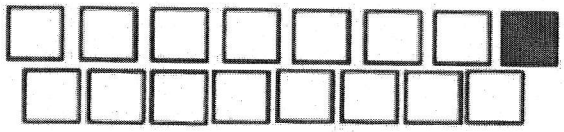
I deemed it necessary to spend some time trying to explain our concept of learning mainly because it differs from traditional concepts where learning is reduced to an assimilation of something already there. Our concept of knowledge as a process precludes this interpretation. If knowledge is a process, to learn and to know something can only be disclosed through our participation in those social practices that make knowledge available. It is only if you consider knowledge as a product that you can imagine that knowledge can be transmitted. For us, knowledge is not a product, but a process. Knowledge is action.

Our concept of learning is also different from the one established by educational approaches inspired by Piaget's epistemology. For the theory of objectification, knowledge is not something that our students construct. Knowledge exists before the students go to school. We think that the metaphor of learning as knowledge construction is in this sense misleading. It may be better, we suggest, to think of learning as something into which we come to participate. As such, it entails something old and something new. It is old in the sense that knowledge is historical and cultural. It precedes each one of us. It is new, for in each learning activity, it always *appears* differently. In other words, knowing —as the mode of existence of knowledge— is always particular. It is an event, and as such it is situated in time and space. It is unique. This is why the same teacher cannot "produce" the same lesson twice, exactly as the same music director cannot "produce" the same symphonic performance twice. The teacher, like the music director, in fact does not produce a performance. They *participate* in it. Knowing as event entails participation with others.

Let me leave these theoretical considerations there and come back to my Grade 2 example. I would like to illustrate the process of objectification in which the students engaged and out of which learning occurred. As will become apparent, in the account of learning semiotics is crucially present.

To become sensitive to cultural-historical algebraic forms of thinking about sequences like the one shown in Figure 2, the students were asked to extend the sequences up to Term 6. A subsequent question consisted in asking the students to decide whether the term shown in Figure 5 is Term 8 of the sequence. They were told that this term was drawn by Monique (an imaginary Grade 2 student); the students were encouraged to discuss in small groups.[9]

Figure 5.



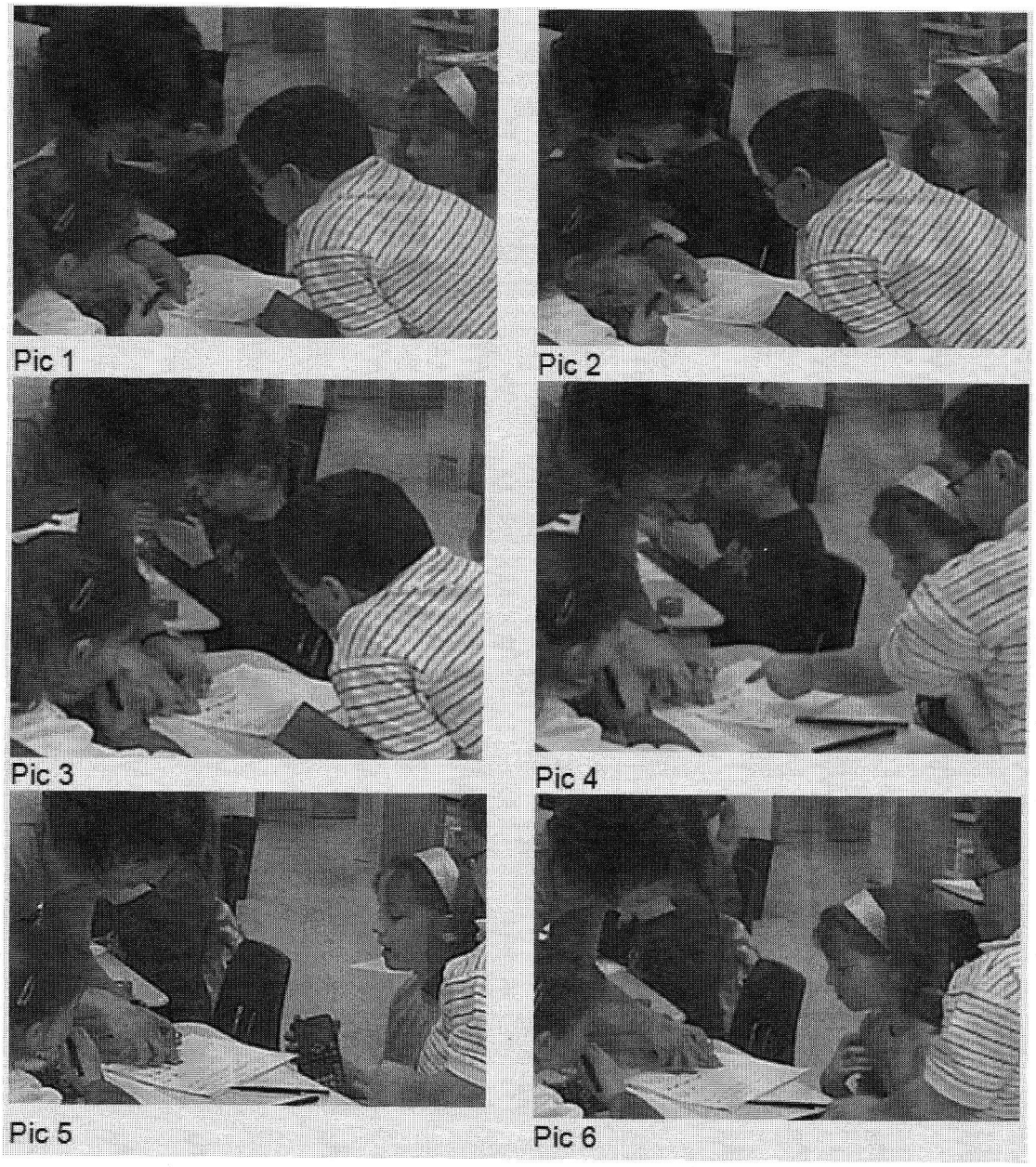
The students were requested to discuss whether Monique's term is Term 8 of the given sequence

Let me focus on the discussion that a group of students had with the teacher—a group formed by James, Sandra and Carla. When the teacher came to see the students' work they had already worked for about 32 minutes together. They had finished drawing Terms 5 and 6, tried (unsuccessfully) to find the number of rectangles in Term 12 and 25, and answered the question about Term 8 (which they considered to be Term 8 of the sequence). The teacher engaged in collaborative actions to create the conditions of possibility for the students to perceive a general structure behind the sequence. In order to do so, the teacher engaged the students in a counting strategy that involved Terms 1 to 4 (drawn on the first page of the activity sheet) and that would be generalized to Term 8. Term 8 of the sequence was not materially present. Monique's drawing was in the next page of the activity sheet. Here is the first excerpt:

- Teacher: We will just look at the rectangles that are on the bottom (while saying this, the teacher makes three consecutive sliding gestures, each one going from bottom row of Term 1 to bottom row of Term 4; Pics 1-2 in Fig. 6 show the beginning and end of the first sliding gesture). Only the ones on the bottom. Not the ones that are on the top. In Term 1 (she points with her two index fingers to the bottom row of Term 1; see Pic. 3), how many [rectangles] are there?
- Students: 1!

- Teacher: (Pointing with her two-finger indexical gesture to the bottom row of Term 2) Term 2?
- Students: 2! (James points to the bottom row of Term 2; see Pic 4).
- Teacher: (Pointing with her two-finger indexical gesture to the bottom row of Term 3) Term 3?
- Students: 3!
- Teacher: (Pointing with her two-finger indexical gesture to the bottom row of Term 4; see Pic 5) Term 4?
- Students: 4!
- Teacher: (Making a short pause and breaking the rhythmic count of the previous terms, as if starting a new theme in the counting process, she moves the hand far away from Term 4 and points with

Figure 6.



The teacher's and students' sensuous (perceptual, gestural, tactile, aural, vocal) engagement in the task.

a two-finger indexical gesture to the place where hypothetically one would expect to find Term 8; see Pic 6.) How many rectangles would Term 8 have on the bottom?

- Sandra: (hesitantly, after a relatively long pause) 4?

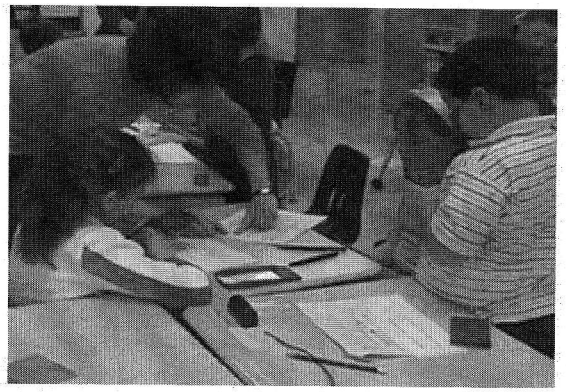
In Line 1, the teacher makes three sliding gestures to emphasize the fact that they will count the bottom row of the four given terms. Then, in the following lines, she starts a joint counting process suggesting a cultural form of perceiving the terms of the sequence—one in which the mathematical ideas of variable and relationship between variables are emphasized. Since Term 8 is not there, the teacher *pretends* that Term 8 is on the empty space of the sheet, somewhere to the right of Term 4. She points to the empty space, as she pointed to the other terms, to help the students imagine the term under consideration. Yet, as Line 10 intimates, the passage from Term 4 to Term 8 was not successful. The objectification of the manner in which sequences can be algebraically perceived has not yet occurred. The teacher hence decided to restart the process, with some important modifications. Indeed, during the second attempt, the teacher does not go from Term 4 to Term 8; this time she counts on the visible Terms 1 to 4 and includes then Terms 5 to 8, which have to be imagined by the students with the help of gestures and words:

- Teacher: We will do it again...
- Teacher: (Pointing to Term 1 with a two-finger indexical gesture) Term 1, has how many?
- Carla: (Pointing with her pen to the bottom row) 1, (without talking to the teacher points to Term 2 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term 2) 2, (again without talking the teacher points to Term 3 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term 3), 3, (same as above) 4, (now moving to the hypothetical place of Term 5 and doing as above) 5.
- Teacher: Now it's Term 8! (*The teacher comes back to Term 1.* She points again *with a two-finger indexical gesture to the bottom row of Term 1*) Term 1, has how many [rectangles] on the bottom?
- Students: 1.
- Teacher: (Pointing with a two-finger indexical gesture to the bottom row of Term 2) Term 2?
- Students: 2!
- Teacher: (Pointing with a two-finger indexical gesture to the bottom row of Term 3) Term 3?
- Students: 3!
- Teacher: (Pointing with a two-finger indexical gesture to the hypothetical place where bottom row of Term 4 would be) Term 4?
- Students: 4!
- Teacher: (*Pointing as above*) Term 6?
- Students: 6!
- Teacher: (*Pointing as above*) Term 7?
- Students: 7!
- Teacher: (*Pointing as above*) Term 8?
- Students: 8!
- Sandra: There would be 8 on the bottom!

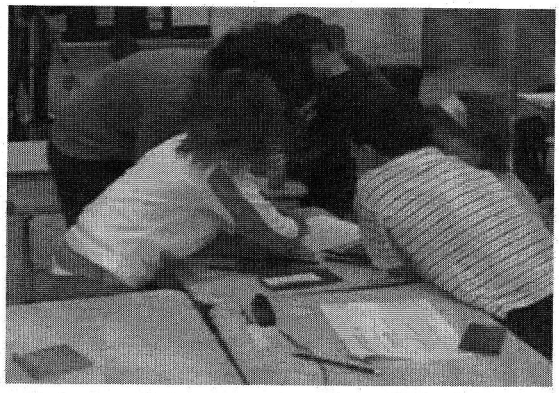
The teacher and the students went to page 2 of the activity sheet and counted together the rectangles on the bottom row of Monique's term and realized that the number was indeed 8. At this point the relationship between variables started becoming apparent for the students. The relationship became objectified. The teacher then moved to a joint process of counting the rectangles on the top row:

- Teacher : Very, very good. Now, we will verify if Monique has the good amount [of rectangles] on top. We will just look at the top... (*like in the previous episode, she makes two sliding gestures, but this time pointing to the top row ; see Figure 7, pic 1*). Term 1 has how many ?
- Students : 2 !
- Teacher : Term 2 ?
- Students : 3 ! . . .
- Teacher : Term 3 ?
- Students : 4 !
- Teacher : Term 4 ?
- Students : 5 ! (*see Pic 2*)
- Teacher : Term 6 ?
- James : 7
- Teacher : (*Repeating*) 7 ... Bravo ! Term 8, will have how many ?
- Students : 9 !
- Teacher : Ok. Oh ! Excellent. Are there 9 [rectangles] here (*pointing to Monique's term*) ?
- Sandra : Yes, there are 9.
- Teacher : We will count it together.
- Students : (*After turning the page where Monique's term is, the teacher points orderly and rhythmically to the rectangles one after the other, while Sandra says*) 1, 2, 3, 4, 5, 6, 7, 8... ! ? (*long pause following a general surprise. See Pics 3 and 4 in Figure 7*).

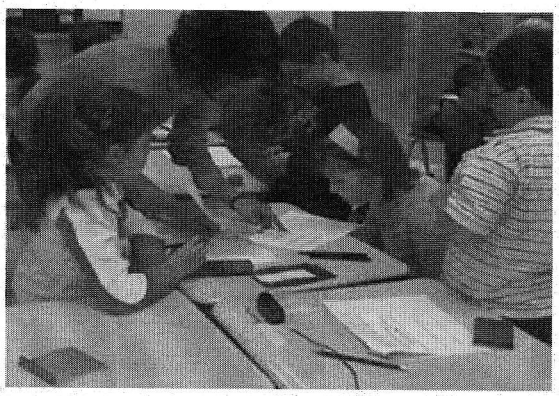
Figure 7.



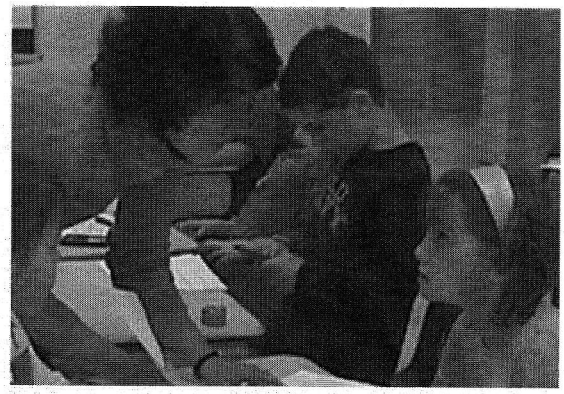
Pic 1



Pic 2



Pic 3



Pic 4

The students were perplexed to see that, contrary to what they believed, Monique's Term 8 did not fit into the sequence. Here the activity reached a tension. Pic 4 in Figure 8 shows Sandra's surprise. The students and the teacher remained silent for 2.5 seconds, that is to say, for a lapse of time that was 21 times longer than the average elapsed time between uttered words that proceeded the moment of surprise (for details of this *poetic* moment in the teacher-students' objectification process, see Radford, 2010b).

Later on in the lesson the students were able to quickly answer questions about remote terms, such as Term 12 and Term 25, which were not perceptually accessible. They refined the manner in which the terms of the sequence could be perceived. The number of rectangles on the bottom row was equated to the number of the term in the sequence, while the number of rectangles on the top row was equated to the number of the term plus one (identified as the dark rectangle in the corner). Here is an excerpt from the dialogue of Sandra's group as they discuss without the teacher:

- Sandra : (*Referring to Term 12*) 12 plus 12, plus 1.
- Carla : (*Using a calculator*) 12 plus 12 ... plus 1 equal to ...
- James : (*Interrupting*) 25.
- Sandra : Yeah !
- Carla : (*looking at the calculator*) 25 !

At this point, the target cultural knowledge has been objectified. The students no longer need to *see* the terms of the sequence to determine the number of rectangles in it. A formula (a counting procedure) is now available. And, as I have argued elsewhere (Radford, 2010b), even if this formula does not contain letters, it is algebraic in nature: it is rather a *formula-in-action* (to rephrase Vergnaud). Through this formula the students can now ascertain the calculations they need to carry out to determine the number of rectangles in any *particular* term of the sequence (e.g., For Term 12: 12 plus 12, plus 1).

As the transcripts suggest, the objectification was accomplished through an intense interplay between various sensorial modalities and different signs. These embodied modalities and signs are epistemologically fundamental in making the general appear through the particular, or to say it otherwise, to make the event (i.e., the teacher-students' joint activity) the mode of existence of the general—its *incarnation*, so to speak. While in Leibniz's semiotics, the senses and the signs as tangible entities cannot be considered as a fundamental part of cognition, for the theory of objectification the embodied and material dimension of activity is unquestionably central. As Leibniz reasoned, "necessary truths such as found in pure mathematics, and particularly in arithmetic and in geometry, must have principles whose proof does not depend upon examples, nor consequently upon the testimony of the senses" (Leibniz, 1949, p. 44). For Leibniz the concrete examples that necessarily appear in material form and are grasped through our senses may be required to awake (*réveiller*) the principles and theories that are already dormant in our soul (Leibniz, 1887, p. 34). The material world that we grasp in a sensuous manner cannot be considered as part of knowledge itself. For knowledge, in the rationalist camp, is of the order of reason and logic alone. For the theory of objectification, by contrast, the bodily materiality of our actions are conditions of the existence of knowledge. Without the former the latter cannot exist.

SYNTHESIS AND CONCLUDING REMARKS

This article seeks to contribute to current discussions about semiotics and education. A "semiotic approach" to education, that is an educational approach that draws from semiotics, I argued, cannot consist of simply the *amalgamation* of a semiotic theory and an educational one. A semiotic approach to education is bound to the manner in which its epistemological and ontological assumptions are cast in, and even transformed by, semiotic concepts and constructs. To support my claim, I discussed, to a certain extent, Leibniz's view of semiotics. His theory of human understanding shows how the manner we come to know is intimately related to our resorting to signs and to assumptions about the ontological and epistemological structure of reality (i.e., what reality *is*, and *how* we come to know it). Then, I briefly discussed the role that Piaget ascribed to semiotics in his genetic epistemology; I made an effort to show that Piaget's semiotics is inscribed in the epistemological tradition of the modern era where transcendentalism is bracketed and the emphasis is put on the subject (Radford, 2012). I suggested that, without discarding the intersubjective nature of language, in his experimental analyses, Piaget ended up conceiving of language as a mere instrument of representation of actions, and objects of actions. This might have been the price to pay in order to keep his genetic epistemology coherent. Be that as it may, the subsequent brief allusion to contemporary sociocultural trends seemed to me appropriate in order to convey an idea of alternative ways in which to theorize the role of signs and artifacts in knowledge and knowing. The allusion to sociocultural trends was an occasion to bring forward one striking difference between modern and cultural epistemologies. Briefly stated, the difference consists in how, ontogenetically speaking, the epistemologies of modernity emphasize the individual as the source of meanings and significations. Sociocultural trends, by contrast, find such a source in culture and its practices.[10] The last part of the paper was devoted to offer an example of a semiotic approach to education—the theory of knowledge objectification. By no means is this approach paradigmatic of the various contemporary trends in mathematics education, let alone of education as a whole. The interested reader is encouraged to refer to some recent work in this area.[11] The theory of objectification draws from the Hegel-Marx-Vygotsky line of thinking and suggests an alternative way in which to conceive of knowledge and knowing. Endorsing a dialectic dynamic between ontology and epistemology, knowledge is considered as an ensemble of culturally and historically constituted embodied processes of reflection and action; knowing is the embodied, and material semiotic enactment of such processes. Now, since for the students the culturally and historically constituted embodied processes of reflection and action may not be apparent, the classroom activity (considered as a teacher-students joint process) moves towards their objectification. If objectification in general is to become aware or conscious of something (Radford, 2002), objectification in classroom teaching-and-learning activity occurs in that (stretched) moment when the object of knowledge appears refracted in the participant's consciousness. Although the teacher

in the Grade 2 example knew very well before the classroom activity started that Monique's term was not Term 8 of the given sequence, in the example here discussed, the objectification required the *event* of the refraction of this fact both *in* the teacher's consciousness and *in* the students'. For objectification is a central part of the event or performance of the Hegelian "particular" in which the students and the teacher are engaged. As part of an event, objectification is always old and new. It is old in that it points to a historical form of thinking about patterns that can be found in the mathematical inquiries about figural numbers in ancient Greece. It is as new as the always particular, contextual, and situated—in short unique— aesthetic experience of each symphonic performance.

Through the example, it is possible to appreciate the various semiotic resources that mediate learning. Thus, at the beginning of the first excerpt, the teacher makes many gestures; through some gestures, she points to terms that are perceptually available (Terms 1-4). We can call these indexical gestures, gestures *ad oculos* to distinguish them from the gestures where what is pointed at is not there, in front of the eyes, but has to be imagined. We can call these indexical gestures *at phantasma*. But there is more. The teacher did not gesture silently. Gestures were coordinated with utterances. More precisely, the teacher coordinated eye, hand, and speech through a series of organized simultaneous actions that oriented the students' perception and emergent understanding of the target mathematical ideas.

The recourse to semiotics, as it appears in the interpretative account of the Grade 2 example, makes sense only within the context of the epistemological and ontological assumptions that I discussed previously. Naturally, other assumptions may lead to a different recourse to semiotics and other accounts of learning. Semiotics cannot be merely juxtaposed with educational theories, but it can help to enrich them, or so I hope.

NOTES

1. Vygotsky's work is a remarkable exception. Vygotsky's educational and psychological work is certainly entangled in semiotic constructs (e.g. Vygotsky, 1981, 1997).
2. "It is obvious that if we could find characters or signs suitable for expressing all our thoughts as clearly and exactly as arithmetic expresses numbers or geometrical analysis [i.e., algebra] expresses lines, we could do in all matters insofar as they are subject to reasoning all that we can do in arithmetic and geometry. For all investigations which depend on reasoning would be carried out by the transposition of these characters and by a species of calculus."
3. A concise translation would be as follows: "Mr. Descartes ...has figured out a method to deal with curves and to put them into equations. But he has not proceeded according to the simplest and most natural manner."
4. A similar outcome would be obtained if, instead of investigating Leibniz's work, I would have focused on Peirce's or Husserl's work (see, e.g. Radford, 2006).
5. "One of the founders of this logical empiricism (or positivism), R. Carnap, started by claiming that the entire logic was nothing more than a general syntax, in the linguistic sense of the term. In this way, and in parallel with Tarski, he was led to add a general semantics; but this does not allow us to go beyond the boundaries of language. Then, in order to give an account of the operative character of logic, Morris showed the need (not entirely acknowledged by the whole school of thought [i.e., logical positivism]), to complete the syntax and the semantics of logic with a 'pragmatics.' However, this is again to determine the rules behind the use of language and not at all those of a logic of action."
6. Associative and dissociative operations are coordination of actions that take place before they are transformed into a verbal form; language is not hence the cause of their formation. Of course, language extends indefinitely their power and endows them with a mobility and generality that they otherwise would not have. But it [language] is not the source of such a coordination.
7. Piaget gives a nice example of the imitation of one's actions and the symbolic play that the actions acquire in his 1954 paper (see Piaget, 1954, p. 52).
8. This is why it is misleading to see the difference between Piaget and Vygotsky as a question of emphasis on language. One of the differences is precisely in the *conception* of language, and signs more generally.
9. The trained eye would not have difficulty noticing the missing white rectangle on the top row. The untrained

eye, by contrast, may be satisfied with the apparent spatial resemblance of these terms with the other terms of the sequence and might consequently fail to note the missing rectangle.

10. Of course, the distinction that I am trying to make does not amount to saying that the sociocultural realm is not taken into account in the epistemologies of modernity, and in Piaget's genetic epistemology, in particular (see, e.g., Nicolopoulou and Weintraub, 2009); nor am I saying that sociocultural theories do not take into account the subject or that the subject is reduced to a simple outcome of cultural production—see, e.g., the discussions about the concept of self in Mikhailov (1980), or more recently in Stetsenko and Arievitch (2004) or Roth and Radford (2011).

11. A partial (chronological) list includes the following items (full bibliographical details can be found in the Reference section):The book *Sémoisis et pensée humaine* of R. Duval, published in 1995.

- The book Educational Perspectives on Mathematics as Semiosis: From Thinking to Interpreting to Knowing, edited by Anderson, Sáenz-Ludlow, Zellweger, and Cifarelli, in 2003.
- The book *Activity and Sign: Grounding Mathematics Education*, edited by Hoffmann, Lenhard and Seeger in 2005.
- The Special Issue of Educational Studies in Mathematics *Semiotic perspectives on epistemology and teaching and learning of mathematics* edited by Sáenz-Ludlow and Presmeg in 2006.
- The Special Issue of Relime *Semiotics, Culture and Mathemtical Thinking*, edited by Radford and D'Amore in 2006 (http://dialnet.unirioja.es/servlet/revista?codigo=7978).
- The book Semiotics in mathematics education: Epistemology, history, classroom, and culture, edited by Radford, Schubring, and Seeger, in 2008.
- The book Mathematical representations at the interface of the body and culture, edited by W. -M. Roth in 2009.
- The book *Semiotics education experience,* edited by I. Semetsky in 2010.
- The Special Issue of Educational Studies in Mathematics *Signifying and Meaning-Making in Mathematics Thinking, Teaching and Learning: Semiotic Perspectives*, edited by Radford, Schubring, and Seeger, in 2011.

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