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La Matematica e la sua didattica
quarant'anni di impegno

Mathematics and its didactics
forty years of commitment

In occasion of the 65 years of Bruno D'Amore

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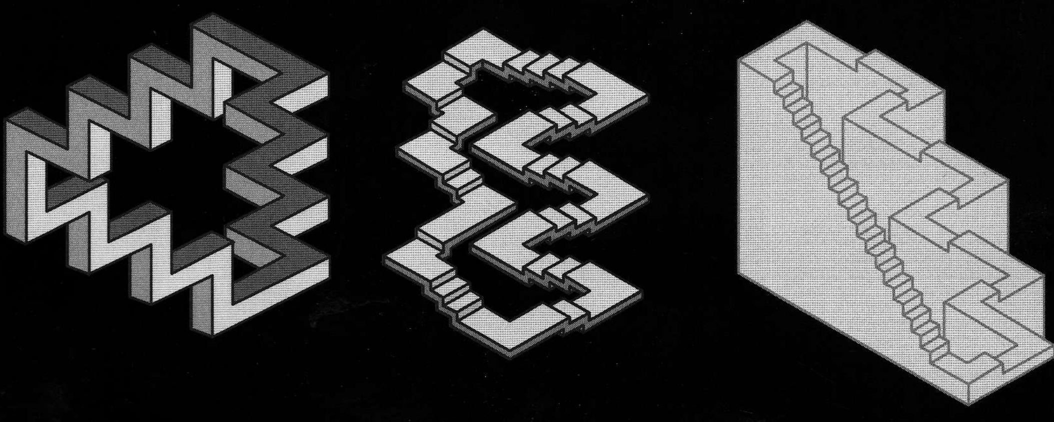
Preface
Bruno D'Amore



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http://www.pitagoragroup.it
e-mail: pited@pitagoragroup.it

Dialogism *in absentia* or the language of mathematics

Luis Radford
Laurentian University, Ontario, Canada

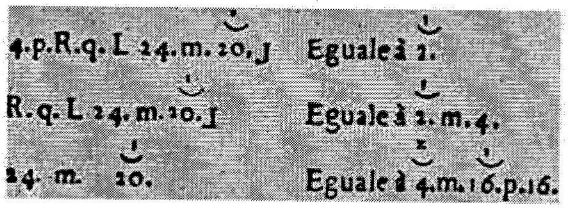
Abstract. *This paper deals with the question of language and mathematics. I suggest that mathematics' language is neither monological nor voiceless: it is Other-oriented. It entails a form of abstract dialogism - a 'dialogism in absentia.'*

Mathematics is often considered a curious form of human activity: not only is it addressed to nobody but, overall, the concrete subject who practices it is excluded from it. Yet, mathematics talks about something. And to talk about something requires a language. Naturally, this must be a strange language, since it has neither a sender nor a receiver. It is a language entirely oriented towards its objects: a language with objects and without subjects at least in the usual sense of communication.

Of course, the conditions of possibility for such a language can only be found in history. Indeed, it would be misleading to think that the language of mathematics has always been the abstract one of our own times. The eradication of the subject from the language of mathematics is a modern phenomenon. It suffices to recall here that Pythagoras and his brotherhood practiced a form of *oral* mathematics. And orality presupposes a subject, in fact a face-to-face contact with someone. We also find in Plato allusions to the mathematical activity of the geometers, who used to discuss their problems by drawing figures on sand. The addressee was physically present. The use of parchments was a practical form to communicate with someone who, because of the distance, could not be addressed directly. It was also a way to systematize knowledge. The original form of communication of the mathematics of antiquity was disrupted in the Renaissance. Two important interrelated elements that underpinned this disruption were the invention of the printing press and the appearance of new forms of subjectivity. The invention of the printing press was not the "ultimate cause". It was rather the symptom of a more general cultural phenomenon. It was the symptom of the systematization of human actions though instruments and artifacts. It replaced the laborious and lengthy medieval process of copying manuscripts. Such a systematization of human actions in the Renaissance radically modified human experience, highlighting factors such as repeatability, homogenization and uniformity proper to mass production. But the social spreading of the book could only be achieved through the invention of a new form of subjectivity: one that is an *abstraction* of the situated and familiar view of the author and the reader. We can get a sense of this abstraction if we compare two mathematical problems that are more or less contemporary to each other. The first one comes from Piero della Francesca's *Trattato d'abaco* (15th century): della Francesca states the problem as follows: «A gentleman hires a servant on salary; he must pay him at 25 ducati and one horse per year. After 2 months the worker says that he does not want to remain with him anymore and wants to be paid for the time he did serve. The gentleman gives him the horse and says: give me 4 ducati and you shall be paid. I ask, what was the horse worth? » (Arrighi, 1970, p. 107). The solution starts as follows: «Do this. You know that he has to give him 25 ducati per year, for 2 months it comes to 4 I/6; and the horse put that it's worth Ī thing, for 2 months it is worth 2/12 of the thing that is I/6 (*sic*). You know that you have to have in 2 months 4 ducati and I/6 and I/6 of the thing. And the gentleman wants 4 ducati that added to 4 I/6 makes 8 I/6. Now, you have 1/6 of the thing, [and] until Ī there are 5/6 of the thing; therefore 5/6 of the thing is equal to 8 I/6 number. Reduce to one nature [i.e. to a whole number], you will have 5 things equal to 49; divide by the things it comes out to 9 4/5: the thing is worth so much and we put that the horse is worth Ī, therefore it is worth 9 ducati 4/5 of a ducato» (Arrighi, 1970, p. 107).

The language of mathematics is the language of the speaker, supplemented with some mathematical signs such as those to designate fractions and a curious sign to designate the unknown - a short line on top of the number I, signifying one "thing". But most important for our discussion is the extensive use of pronouns and verbs – for instance "Do this", you will have", etc. The language of mathematics includes the mathematician and his/her addressee in a vivid manner.

Now, the other problem that I would like to discuss comes from Bombelli's *L'Algebra* (1572, p. 251). The problem is presented below, with a translation into modern symbolism to the right.



$$4+\sqrt{24-20x}=2x$$
$$\sqrt{24-20x}=2x-4$$
$$24-20x=4x^2-16x+16$$

The mathematical language in this text is very different from the one of della Francesca. There is a difference in terms of how now the unknown is represented. Instead of the cumbersome although very creative symbolism of della Francesca to represent the unknown and its powers, Bombelli offers us a sophisticated way in which to express the unknown. However, the most crucial point is not there, I want to argue. The most important point is the eradication of the speaking subject. True, in other passages, Bombelli explains in natural language how computations should be carried out. But we approach here the *ideal* form of a computational language that suffices itself and that displaces

the speaking subject to the margins of the book and beyond. We find here the first steps towards what is going to become a language without a subject.
But is it really a language without an addressee as well?
The thesis that I would like to submit is that the peculiarity of the language of mathematics from the Renaissance onwards lies in how the speaker and the addressee are abstractions. They are abstractions of situated and concrete subjects. More precisely, they are correlated abstractions of those that occurred in the field of commodity exchange that characterized the Renaissance as a historical and cultural period. In the same way as value replaced the concepts of exchange and usage value by measuring them quantitatively without reference to the particularities of the involved commodities, the speaker and the addressee came to be seen as abstractions where one or the other could take a same (abstracted) position in a discourse where particularities and idiosyncrasies were dismissed. Such an abstraction rested of course on the idea that everyone could (in principle) compute, make calculations and establish the mathematical properties of the objects of discourse. This idea is of course indebted to the Renaissance idea of the subject, where the individual starts becoming emancipated from tradition and, as the sociologist Norbert Elias put it, owns himself for the first time. Della Francesca's problem is interesting in this regard too: we see in the problem how a servant can sell his labour, a practice foreign to the Middle Ages. In the Renaissance, for the first time, the world appears to the subject as something to be seen not from the angle of tradition and religion, but from his/her own viewpoint—an idea crystallized in one of the most important concepts of that period: the concept of perspective of the visual arts. What is curious is this: the abstraction that led to a discourse without voices is rooted in its opposite, namely a form of first-person subjectivism. If there is no sensuous voice left, if the speaker's and audience's voices participate now in a *dialogism in absentia*, there is nonetheless an undeniable *aesthetical* experience in our going through an apparently disembodied mathematical proof or symbolic calculation. It is as if the voice of the Other captures us at another level. The aesthetical experience is no longer in the sensuous voice of the speaker that reaches us with its intonations, fears and joys, but in an abstract communicative fusion of our silent voice with that of others.