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Reflecting and shaping the world of mathematics education

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Report of Working Group 5

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Mathematics education: an ICMI perspective

1. *Preamble*

The *Symposium on the Occasion of the 100th Anniversary of ICMI* attracted considerable attention within the mathematics education research community. The working groups, an important component of the program, were appropriately seen as a productive opportunity for participants to discuss and showcase aspects of their own research. The parameters for participation in Working Group 5 were broad and were summarised as follows:

> It can be argued that ICMI's impact on the field of mathematics education is both defined and reflected through the affiliated groups that it has recognized (and in some cases incubated) and through the ICMI studies it has spawned. What has been gained and lost by the mathematics education communities through the choices made? For example, what has been the influence of psychology, philosophy, history, ethnography, anthropology, [...] on mathematics education. Have their approaches been adopted? Have some been privileged over others?

We refined the overall focus further: The broad theme of this working group can clearly be addressed in diverse ways. We considered the following topics particularly relevant:

- The evolution of theoretical frameworks in mathematics education;
- The influence of other disciplines in this evolution;
- ICMI contributions to the dialogue between the mathematics education community and other disciplines – for example anthropology, ethnography, and philosophy;
- Outcomes of interdisciplinary dialogue;
- Further possible ICMI roles in promoting the dialogue (for example, should new disciplines be considered?).

In the remainder of this report we have captured the overall thrust of the material presented, the discussions these generated, and our more general deliberations as best we can within the space allocated.

2. *Report*

In response to the call for contributions, a number of papers were submitted. These are summarised below.

The fuller papers, prescribed to be limited to five pages, can be downloaded from www.unige.ch/math/EnsMath/Rome2008. A number of participants, not listed in the table, also contributed to the discussions in the WG.

As planned, participants in WG5 were drawn from a range of different countries, with Africa the only continent not represented among the presenters. Collectively, presenters touched – in varying degrees - on the sub themes listed above.

Summary of the paper presentations

Authors: *Marcelo Salles Batarce* (London South Bank University & Universidade Estadual do Mato grosso do Sul – Brazil) & *Adriana Cesar de Mattos* (London South Bank University & Methodist University, University of Piracicaba, Brazil, SP)

Title: The ICMI's Grammar.

Brief summary based on the written paper: Issue raised: Is ICMI a chapter in the history of mathematics education or has ICMI set the scene for (shaped) mathematics education? Reform of the school mathematics curriculum needs a language to express its aims and new directions. ICMI and its affiliates often mirror (or anticipate?) these needs. What is needed to achieve the aims of mathematics to be both a universal and international (i.e., non political) science? Are new concepts and terminology needed to achieve this?

Key points raised during the presentation & discussion: The *Commission Internationale de l'Enseignement Mathématique* was established in 1908; the *International Commission in Mathematics Instruction* in 1952. How closely are these two linked? The concepts 'internationalisation of the educational system' and 'mathematics universality' are not synonymous. The former is broader than the latter.

Authors: *Christer Bergsten* (Linköping University, Sweden).

Title: On home grown and borrowed theories in mathematics education research – the example of embodied cognition.

Brief summary based on the written paper: Using embodied cognition [EC] as an exemplar, three broad issues are explored in this paper: the dimensions of relevance; the influence on scientific discourse, the furthering of scientific knowledge, and on educational practice; and issues of compatibility with other theoretical perspectives already applied in the field.

Key points raised during the presentation & discussion: What happens

when a 'new' theory from outside feeds into an existing field of study? And in particular, what has been the impact on mathematics education of embodied cognition? Two separate waves can be identified, with the first in particular leading to research presentations at PME, one of ICMI's affiliated groups. The influence of embodied cognition on the field can usefully be considered under three headings: dimensions of relevance; implications; and compatibility. Those who have drawn on the theory include those interested in the link between the acquisition of mathematical knowledge, gestures, and more general semiotic perspectives, and those exploring the epistemological foundations of mathematics and its relevance for education.

Authors: *Paolo Boero* (Università di Genova, Italy).

Title: Processes and products, structures and meaning in mathematics classroom: some snapshots from the last century.

Brief summary based on the written paper: Various 'movements' in mathematics education are discussed. Evidence is presented to illustrate how positions in other disciplines have influenced changes and priorities in the teaching of school mathematics. It is argued that mathematics education should not merely rely on tools and theories from other disciplines but should also develop as a relatively autonomous scientific discipline.

Key points raised during the presentation & discussion: To show the importance of developing mathematics education as an autonomous scientific discipline, using tools from other disciplines when helpful, was a core theme. Changing priorities in the teaching and learning of mathematics in school were driven by dominating positions in epistemology and psychology, and in particular by: Formalism and logicism (from Hilbert, Frege, Russel); by Modern (or New) Mathematics (Piaget + Bourbaki); and by Constructivism(s). However, the inertia of the school system (teachers, parents, etc.) proved a force against change.

Authors: *Rolf Biehler* (University of Kassel (Germany) & *Andrea Peter-Koop* (University of Oldenburg, Germany). The authors were unable to attend the Symposium.

Title: The development of mathematics education as a scientific discipline – some reflections from a German perspective.

Brief summary based on the written paper: The authors provide, and build on, three historical snapshots – involving Felix Klein, ICME3 in Karlsruhe, and the work of the *Institute for Didactics of Mathematics*, highlighting the link between ICMI and developments in mathematics education in Germany.

Authors: *Gerd Brandell* (Lund University, Sweden).

Title: Using multiple theoretical perspectives to connect, clarify and convey research results.

Brief summary based on the written paper: Do different theoretical perspectives facilitate or impede insights into complex teaching situations? Similarities/overlap between apparently different theoretical approaches to understanding how learning takes place can be highlighted by using the overlay of the characteristics of a 'deep' and 'surface' approach to learning. It is argued that diversity can be handled constructively by exploring unifying, integrating, competing, and networking notions and that the characteristics of deep and surface approaches to learning are particularly useful for considering similarities and overlap between different theories.

Key points raised during the presentation & discussion: Several examples were used to illustrate that the same empirical data can be analyzed using different theories or approaches. A more global theory can help to communicate research results to an audience outside the mathematics education community (e.g. mathematicians). There are various ways to build on different theoretical perspectives: unifying; integrating; competing and comparing; and networking the different and possibly competing interpretations.

Authors: *Tony Brown* (Manchester Metropolitan University, UK).

Title: Subjectivity: An alternative to the psychology of mathematics education.

Brief summary based on the written paper: How should mathematics education research position itself? «Mathematics education research needs to move away from earlier instrumentalist tendencies concerned with understanding and improving mathematical performance against unproblematised social registers».

Key points raised during the presentation & discussion: Mathematics education research is widely premised on psychologically oriented research perspectives centred on control and perspectives which often rely on questionable assumptions. But using a 'Lacanian conception of subjectivity' may be more profitable. Using this lens, the student is understood through participation in the linguistic structure of learning; teachers are understood through the filters of curriculum structure; and researchers are considered according to how they conceptualise psychology/ subjectivity.

Authors: *Bruno D'Amore & Martha Isabel Fandiño Pinilla* (University of Bologna, Italy).

Title: Change of the meaning of mathematical objects due to the passage between their different representations.

Brief summary based on the written paper: Transforming 'everyday language' into algebraic expressions involves «a constant change of meaning ... within various semiotic systems». Examples are given to illustrate how disciplines outside mathematics education are already

influencing our explanations of mathematical learning and activity.

Key points raised during the presentation & discussion: What are the causes of the changes of meaning, what origin do they have? Some theories 'external' to mathematics education – and in particular philosophy, sociology, anthropology, and psychology - have a strong influence on the ways in which different interpretations are analyzed.

Authors: *Nadia Douek* (IUFM de Nice, France).

Title: The determination of mathematical objects of didactical activities.

Brief summary based on the written paper: Theoretical constraints influence (limit or expand) the theoretical and practical dimensions of mathematics education. Particular emphasis is placed on the work of Vergnaud and Vygotsky.

Key points raised during the presentation & discussion: The idea of a mathematical object is problematic, the author proposed, because the boundaries of a mathematical object (its determination) seem to depend on the philosophical choices made, for instance on the epistemology and/or the cognitive psychology perspective(s) chosen to guide our analysis. Thus, it is argued, it is preferable to consider mathematical objects as emergent from activity which is dynamic and rich with its socio-cultural components.

Authors: *Helen Forgasz* (Monash University, Australia).

Title: Positioning gender and mathematics education research.

Brief summary based on the written paper: Evidence is presented of diverse ways in which ICMI has broadened the research horizons of gender and mathematics.

Key points raised during the presentation & discussion: ICMI's decision to host a Study on 'mathematics and gender' stimulated international academic debate on this topic, strengthened the area as a credible research domain, highlighted different theoretical perspectives, and also drew attention to gender differences within ICMI's organizational structures. Further work on gender has been promoted and supported by two affiliated groups, IOWME and PME. Has ICMI's influence on the field waned in recent years? And what can we realistically anticipate as the future contributions of ICMI?

Authors: *Michael N. Fried* (Ben Gurion University of the Negev, Israel).

Title: History of mathematics and the future of mathematics education.

Brief summary based on the written paper: The founders of ICMI (including F. Klein and D.E. Smith) considered that history of mathematics was important to mathematics education. Yet can the history of mathe-

matics be readily incorporated into the school curriculum?

Key points raised during the presentation & discussion: The personalities and activities connected with ICMI have, in the past, shown a deeply rooted interest in history of mathematics. But what practical problems are posed when the history of mathematics is incorporated into mathematics education, and are these practical problems, in fact, entwined with more deeply seated theoretical matters? How can curriculum requirements be accommodated? To what extent does history become a tool rather than a subject that is studied? How can mathematics education be reconceptualized to allow the history of mathematics to become an integral part of mathematics studies? And what is lost if this is not done?

Authors: *Fulvia Furinghetti* (Università di Genova, Italy).

Title: Mathematics education in the ICMI perspective.

Brief summary based on the written paper: The development of mathematics education as a discipline is traced. Lively descriptions are given of cooperation, hostilities, and frictions between the communities of mathematicians and mathematics educators.

Key points raised during the presentation & discussion: The aims of 'communication', 'solidarity', and 'internationalization' underpinned the establishment in 1899 of the Franco-Swiss journal *L'Enseignement Mathématique* which in its turn contributed to the eventual creation of ICMI. The current more equal relationship between ICM and ICMI is in sharp contrast with past traditions. Though part of the International Congresses of Mathematicians, ICMI was for many years 'the poor relative': the 'pedagogy of mathematics' was not considered worthy of a plenary session, ICMI presentations were invariably relegated to one of the final sessions of the congress, and – for the first 100 years – the ICMI president was a male mathematician and not a mathematics educator. Only through a partial rupture with the community of mathematicians could the mathematics education community construct its own identity as an academic discipline.

Authors: *Solomon Garfunkel* (COMAP, Inc., U.S.A.).

Title: The Faffufni-Chaim Yankel effect: A cautionary tale.

Brief summary based on the written paper: Reflections on «the patterns of how projects and programs are evaluated». Implicitly: how can we move beyond 'accepted' recipes for the conduct of research? Or why good ideas are lost ...

Key points raised during the presentation & discussion: What have we (in the USA) gained and lost by moving from earlier funding approaches to the current mechanisms for funding projects? How can we ensure that an appropriate balance is retained between quality based and popularity based approaches to project filtering? Are we now so weighed down by

bureaucratic processes that innovative, formula-challenging projects are doomed not to be supported? Should ICMI become involved in championing certain projects?

Authors: *Gabriele Kaiser* (University of Hamburg, Germany).

Title: Meaning in mathematics education. Reflections from various perspectives.

Brief summary based on the written paper: Meaning in mathematics education is influenced by diverse factors. Experimental evidence is provided that modelling examples can make mathematical learning more meaningful and that comparisons with students from different cultural backgrounds might enhance our understanding of the meaning students develop in mathematics education.

Key points raised during the presentation & discussion: What is meant by 'meaning' in mathematics education? Will modelling examples increase the likelihood that students will find mathematics learning more meaningful? Should we distinguish between personal and objective meaning? Two specific teaching 'experiments' set in two different cultural contexts are used to explore these and related issues.

Authors: *Stephen Lerman* (London South Bank University, UK).

Title: Theories as intellectual resources in mathematics education research.

Brief summary based on the written paper: A rich overview is provided of different theoretical perspectives relevant to mathematics education research, their evolution, potential, and limitations.

Key points raised during the presentation & discussion: Mathematics educators often draw on a range of disciplines – frequently psychology, anthropology, and sociology - as they grapple to understand better issues of teaching and learning mathematics. How best to make sense of multiple theories was explored and the danger of mindless application of theory to educational research stressed. The concluding section of the written paper captures some of the thrust of the presentation: «I am not surprised by the multiplicity of theories in our field and the debates about their relative merits, nor do I see it as a hindrance. ...I am particularly troubled by the attacks on educational research as an inadequate shadow of a fetishised image of scientific, psychological or medical research».

Authors: *Po-Hung Liu* (National Chin-Yi University of Technology, Taiwan).

Title: A 4-dimensional analysis of the practice in mathematics education.

Brief summary based on the written paper: Practice in mathematics education can conveniently be considered in terms of four components: global vision, local focus, mathematical knowledge, and time span. Such

an analysis highlights not only tensions and contradictions but also opportunities and strategies for further developments.

Key points raised during the presentation & discussion: Mathematics education issues, it was forcefully argued, are not only pedagogical, but also epistemological, social, and even political. ICMI should respond to diverse calls and needs among all countries. Rather than merely look for solutions to problems, ICMI should provide a structure for facilitating interactions and exchange of views. A four-dimensional view, described in some detail in the written paper, for achieving this was discussed. Meanwhile: whose voices are given prominence in research journals and at conferences and whose voices are ignored? Whose problems are considered worthy of research and whose situations are seen as unproblematic? Might research in mathematics education *damage* rather than facilitate mathematics teaching/learning under certain circumstances and in certain settings?

Authors: *Immaculate Namukasa* (The University of Western Ontario, Canada). The author was unable to attend the Symposium.

Title: The contribution of multiple disciplines of influence to mathematics education: A complexity science interpretation.

Brief summary based on the written paper: Different paradigms for conceptualizing mathematical thinking are described: Individual and content psychology, cognitive and information processing, co-emergent and context paradigm; coherence and post-structuralism, and ecological and systems paradigm. Drawing from complexity theory the author demonstrates how considering varied perspectives in light of each other might bring forth novel perspectives on mathematical thinking.

Authors: *Marie-Jeanne Perrin-Glorian* (DIDIREM, Université Paris Diderot et IUFM Nord-Pas-de-Calais, France).

Title: From producing optimal teaching to analysing usual classroom situations... The notion of milieu.

Brief summary based on the written paper: The theory of didactic situations, its evolution, and the integral component of milieu are described in some detail. Strong reference is made to Brousseau's work.

Key points raised during the presentation & discussion: Various aspects of the theory of didactic situations were discussed: the theory's historical development, the notion of *milieu* and its constituent elements; the contribution of the theory to analysing regular lessons. The theory's assumptions, viability, and broad applicability generated considerable audience debate.

Authors: *Norma Presmeg* (Illinois State University, USA).

Title: Semiotic theoretical frameworks: creativity and imaginative rationality in mathematics education.

Brief summary based on the written paper: Theories from many different disciplines are germane to mathematics education research. Considerable emphasis was placed on the work of Charles Sanders Peirce. Various rich examples are provided.

Key points raised during the presentation & discussion: The strong focus on logical thinking in mathematics education should not exclude recognition of the creative elements that can enrich mathematics learning. Similarities and differences between deduction, induction, and abduction, and their relevance to problem solving in mathematics, were considered at some length. The different representations of mathematical objects and processes embodied in the Peircean triad of iconic, indexical, and symbolic signs were also highlighted.

Authors: *Wolfgang Schlöglmann* (Johannes Kepler Universität, Austria).

Title: Is cognitive neuroscience relevant to mathematics education research?

Brief summary based on the written paper: Learning is now regarded as an increasingly complex process, involving cognitive and social processes, affect, emotions and motivation. Concepts from many fields are invoked in our attempts to explore and understand how it occurs. An argument is made to focus as well on neuroscience and areas that may benefit from neuroscience based research are listed.

Key points raised during the presentation & discussion: Recent developments and findings in cognitive neuroscience appear to be opening interesting new research pathways for increasing our understanding of factors facilitating or inhibiting mathematics learning. Various examples, discussed in the written paper, were highlighted.

Authors: *Nathalie Sinclair* (Simon Fraser University, Canada).

Title: Notes on the aesthetic dimension of mathematics education.

Brief summary based on the written paper: Aesthetics, it is argued, contributes to 'understanding the rationality of mathematics itself, and to enriching existing theories in mathematics education'. Various examples are given and issues worthy of further investigation are put forward.

Key points raised during the presentation & discussion: Different disciplines raise different dimensions and questions with respect to aesthetic values in mathematics and mathematics education. For example, should aesthetic considerations in mathematics and school mathematics be the same or different? What can mathematics educators learn from mathematicians? Is aesthetic sensibility linked to mathematical motivation and curiosity? How do theories of embodied cognition relate to aesthetic perception?

Authors: *Heinz Steinbring* (Universität Duisburg-Essen, Germany).

Title: Mathematical knowledge as a social construct of teaching / learning processes – the epistemology oriented mathematical interaction research.

Brief summary based on the written paper: Three broad areas are covered in this paper with its focus on constructing meaning for mathematical signs: mathematics teaching as an autonomous culture; epistemological constraints of mathematical signs in the culture of teaching; and the interactive constructions of mathematical knowledge – social and epistemological conditions.

Key points raised during the presentation & discussion: The theoretical bases and practical implications of Epistemology Oriented Mathematical Interaction Research were outlined in more detail and illustrative examples were provided. The specific rules of the social behavior of the participants is investigated through an interaction analysis of the classroom culture. How the mathematical signs of concepts and of operations are used and interpreted is examined via an epistemological analysis of the classroom culture.

3. *Synthesis and discussion*

The contributions, it can be seen, were diverse and often provocative. A number of broad themes emerged.

3.1 *The evolution of theoretical frameworks in mathematics education*

Historical identification of changes:

Formalism and logicism; Modern mathematics; Constructivism(s); Sociocultural approaches.

Two contemporary trends were highlighted:

Mathematics Education modeled after the natural sciences; Mathematics Education modeled after the social sciences.

3.2 *Interdisciplinary perspectives*

Semiotics, neuroscience, critical theory in art and philosophy, embodied cognition, epistemology, history, psychology were all areas on which mathematics education research has drawn.

3.3 *ICMI Future contributions*

ICMI, it was also argued, could profitably shape the research landscape of the future in a number of ways, and in particular:

By fostering a dialogue between the mathematics education commu-

nity and other groups (policy makers, government...) to ensure that sensible and practical criteria be applied to prioritizing and funding research;

To maintain, and if necessary reinforce, a challenge to the role of dominant (male) Western paradigms as drivers for our research and interpretation of international results.

3.4 *Some issues*

A number of issues attracted particularly lively debates. These included:

The role of creativity and aesthetics in mathematics and the different theoretical perspectives within which this might be explored;

Whether educational research might harm students, e.g., by frequent advocacy of new, and not necessarily well tried, teaching approaches rather than offering stability of learning environments.

Perhaps the most vigorous discussions involved trying to reconcile different theoretical perspectives. Attempts to do so were not always successful!.

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