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Rethinking Representations¹

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The concept of representation is one of the more powerful psychological concepts used in the field of Mathematics Education in order to explain some important phenomena about children's way of thinking (see e.g. Janvier 1987). This concept has a long-standing tradition in philosophy where it has been used with different connotations. For instance, as a faculty of the mind to *reproduce* the external world or as a kind of view that the individual *produces* from his/her own perspective. While the former has its roots in Cartesian philosophy and the idea of *re*-presentation as a mental mirror, the second one is related to the neo-Kantian idea of an individual who, in his/her acting in the world, produces an idea of it and its objects. Cognitivism proposed a distinction between external and internal representations whose link was theorized as a double-sense mapping from one kind of representation to the other and in which the external representations and mathematical notations were seen as aiding devices for the accomplishment of the cognitive mental processes (Meira 1995). Since it is acknowledged that internal representations cannot be directly observed (De Corte et al. 1996, p. 502), experimentally, in the mapping perspective, external representations (e.g. students' use of mathematical notations, drawings) are seen as key elements to conjecture what is occurring in the head.

We argued elsewhere (Radford 1998a) that the aforementioned classification of representations is clearly related to the traditional philosophical opposition between the individual and his/her milieu and the view according to which the

milieu is the scene or space where individual thinking finds expression. However, with the progressive abandonment of the Cartesian cogitator and the solitary mind in contemporary approaches in psychology, the discourse about representations tends to include more and more the role played by others in the mental representations that an individual comes to form. It is beyond the scope of this paper to offer an overview of the many possibilities to theorize about these views. Therefore I shall limit my discussion to what a mental representation may be within the theoretical perspective I am advocating (Radford 1998b).

1. Representations as a social construct

First of all, a mental representation, I want to suggest, is not the direct product of the visual or sensual organs of the individual. The reason for this is that the individual is never in direct contact with his/her surrounding: this contact is mediated by the culture and the arsenal of concepts that the culture makes available to the individual. How Babylonian scribes conceived squares as different objects from us is an example. For us a side of the square is a segment. For them, it was *also* a side provided with a canonical projection, so that in fact the square had four rectangles added to it. If the side of the square has a length equal to "s", the width of the added rectangle will have "s" units and its length (the canonical projection) will have 1 unit (see Radford 1996, in print). Their mental representation was different from ours. Of course, they did not necessarily use all four projections. It depended on the problem and the situation they were studying.

2. The context of mental representations

The previous point brings us to another feature we want to stress about mental representations, namely, their *contextual* nature. A mental representation is not independent of the context in which it is used. The "reality" represented in a mental representation is not the reflection of reality but the reflection of the reality as constructed by the individual in his/her interaction with others and participation in social practices. In this sense, in a child's drawing, it is not the object itself which is stressed but his/her *experience* with it. The experience, however, is not an individual act. To follow through with our example, a Babylonian scribe experienced a square differently from us. There is always a cultural aspect in experiencing objects. As it results from what we said in Section 1, any object is

culturally embedded in a web of meanings which penetrate the ways we experience our world.

3. A paradox: The elusive concepts

The point stressed in Section 2 leads to some paradoxes. Indeed, the contextual and situated nature of mental representations can be seen as the impossibility for us to think of concepts in abstract terms. For it may be argued that if any mental representation is situated, and mental representations are our instruments of thought in order to think about mathematical concepts, then our thoughts will always remain situated and hence "concrete". When we produce a mental representation of a triangle, for instance, we do it for a specific triangle (it will be right or acute or equilateral or something else but it will be a *specific* triangle). A solution to this paradox is to consider mental representations as pointers (like signs) to abstract objects. But, in doing so, the problem is not solved. We merely succeeded in hiding it, for the question of the link between the concept and the mental representation still remains open. The concept appears as a sly fox always eluding the hunter's trap.

4. The concrete texture of mental representations

I suggested elsewhere (Radford 1998a) that mental representations are genetically constructed on the basis of signs (e.g. words, mathematical notations, gestures). This position, of course, has been previously pleaded by Kaput (1991), Duval (1995) and others. What I have in mind is, however, something different. Since signs are not used at random but exist only within systems, within semiotic systems which provide the rules for their use and understanding (Radford 1998b), in my view a mental (private) representation does not appear as a pointer to an abstract idea but as the contextual instantiation of social modes of knowing as expressed and contained in signs. As a result, the mental representation does not go beyond the signs but remains caught *in* them. 'Mental images' can be considered as materialized pieces of thought in the external sphere of the individual activity, colliding with the tasks at hand, and hence externally situated (Radford 1998a, p. 291).

A practical pedagogical implication of this point of view may be illustrated through the case of functions. Functions play a central role in the new Ontario Curriculum of Mathematics. The learning of functions is seen as the capability of

a student to move from one representation to another (tables, graphics, algebraic formulas, etc.). These “external” representations are seen as the expression of the *same* concept—that of function. In my view, each semiotic system (tables, graphics, etc.) leads to a particular concept (that, here, would be the one genetically constructed on the basis of the semiotic system in use). The mental image produced through sign use is not the same. It does not mean that all those conceptualizations are incommensurable among them. What this means is that a contextualization among representations will be needed to link them, and this requires a different pedagogical action.

In a longitudinal research program that I am conducting, a considerable effort is being made in order to understand the way students use representations and signs in the learning of algebra from the theoretical perspective of which I noted some aspects above. Instead of seeing external representations as merely the concrete manifestations and reflections of internal life, we are taking the former as the concrete aspect of the latter (see e.g. Radford in print-b).

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