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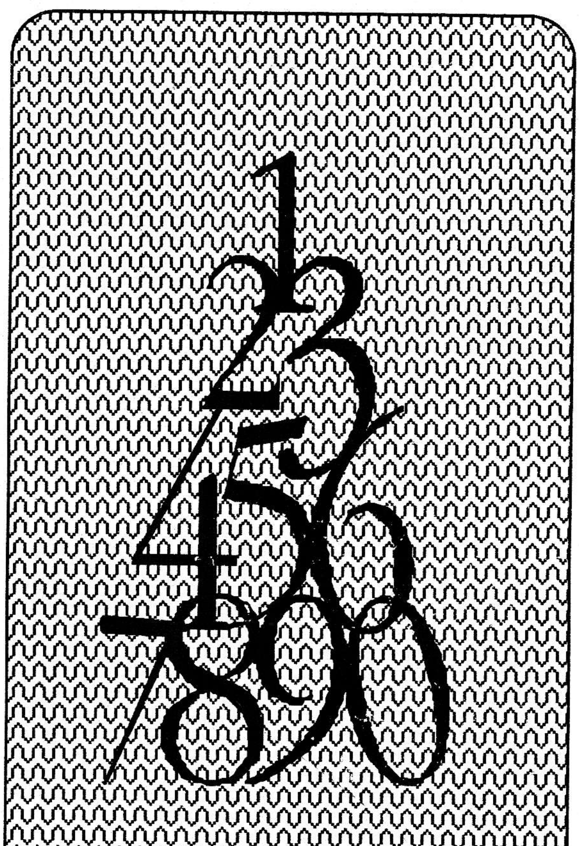

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An Historical Incursion into the Hidden Side of the Early Development of Equations[3]

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Abstract

Often, the development of mathematical concepts is merely seen as reification processes (i.e. processes of abstraction and/or generalization) with little or any relationship to sociocultural factors. Through a case study taken from history –that of the rise of the algebraic concept of equation– we shall attempt to show that mathematical reification processes do not happen in abstract spheres reserved for the mind only but are encompassed by sociocultural processes. Our work makes it possible to see that the rise of the algebraic concept of equation was historically related to (i) the development of writing and (ii) to socially elaborated forms of mathematical explanation. Two different stages of the development of equations can be detected: (i) equations as heuristic tools and (ii) equations as genuine mathematical algebraic objects. The transition from one stage to the other is discussed in terms of the factors that, in the 15th and 16th centuries, led to the elaboration of arbitrary signs designating the unknown and its powers. Some experimental data about the children's acquisition of the concept of equation will be discussed through our historico-epistemological results.

1. Introduction

Epistemology, as it is usually understood, deals with the study of knowledge. From an educational point of view, some of the most important

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epistemological problems are those focused on the comprehension of the genesis and development of knowledge. The ways in which these questions may be tackled depend upon some basic assumptions about knowledge itself. In the case of mathematics, it may be correct to say that most of the current research programs follow a constructivist point of view. Even though it is often taken for granted that constructivism provides some room to consider the social aspects of knowledge, these aspects remain, in most cases, a complementary axis to the supposedly main axis –that in which the individual ultimately engages itself in his or her most private intellectual intimacy in order to construct *his* or *her* knowledge. The resulting scenario gives us a view of mathematical knowledge as being an essentially dominant socioculturally-free human activity (Radford, 1996a, 1996c). Consequently, the rather few educational studies that scrutinize the history of mathematics for epistemological purposes pay little –if any– attention to the sociocultural factors[4]. However, a closer look at the sociocultural contexts in which mathematics develop unveils interesting facts that can shed some light on the comprehension of mathematical knowledge –facts which cannot be understood within the individual sphere only.

In this paper we want to deal briefly with the algebraic concept of equation. The classical historical account consists in seeing how clumsy past efforts developed throughout time until they reached our modern concept of equation. One assumes the hypothesis that, for *logical* necessities, ancient mathematical concepts had to reify in order to attain the perfection of our modern concepts. Studying the concept of equation, we shall attempt to show, through three historical episodes, that reification processes are intimately linked to sociocultural reasons.

2. The Ancient Near-East

It is interesting to note that, while equations play a central role in our modern mathematics curricula, equations (considered as genuine mathematical objects) appeared very late in history. In fact, algebra emerged as an intellectual activity centered around solving puzzles –more specifically, non-practical problems. In the case of Mesopotamia, this activity was carried out

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2 The primacy of the mind over its milieu has been recently challenged by several scholars (e.g. Harré and Gillet, 1994; Wertsch, 1991). Lerman (1996), in an enlightening work, argues that the insertion of sociocultural factors to the radical constructivism cannot be done without introducing contradictions in the account of the development of knowledge.

in schools or teaching centers called "tablet-houses" where the *dub-sar* or "tablet-writers", that is, the scribes, were trained to read and write in order to primarily keep track of commercial transactions and to draw up official notarial documents (e.g. contracts, letters, sales, rentals, adoption; see Høyrup, 1991; Radford, 1996b). As Høyrup pointed out (*op. cit.*), the reason underpining the rise and spreading of such riddles from which algebra emerged was a social one linked directly to the need to prove intellectual virtuosity.

The way in which mathematics (and particularly algebra) was practiced was deeply and determinately shaped by the social structure of the time –an autocratic society centered around the king (for a more detailed account, see Radford, 1996b). Many tablets show different contracts and oaths that people at the service of the Palace had to sign in order to ensure loyalty as well as honesty and correctness in the handling of the royal goods. On the eve of a possible invasion, all the kingdom could be requested to sign a loyalty oath. The contract-tablets reveal the complete alienation of the servant and his or her submission to the king, who, in the case of the Palace's employees, controlled all the contact that they could have with the exterior (see Durand, 1991). The autocratic king-servant relationship was a general scheme that we find recurrently in the different social levels. Some historical evidence suggests that this scheme underlaid the teaching model of the scribes' houses (Radford, 1996b). Within this context, it is not surprising then that the mathematical tablets contain little information about the problem-solving procedures followed by the ancient scribes. Although there were clear technical writing limitations imposed by the small clay tablet (a constraint that is evident in the official reports that the diplomats had to address to their king), the written language used for educational purposes –a language that operated as a complementary tool to the spoken language– focused on the *procedural* aspect of the problem-solving tasks. Generally, in the tablets you are told *what* to do without being told *why*. This does not mean that the scribes' procedures were found at random, as it has been often suggested, nor that the scribes had at their disposal a hidden algebraic language. Høyrup's reconstruction of Babylonian mathematics suggests the scribes' utilization of geometrical drawings as auxiliary tools to guide the actions needed to solve many problems (Høyrup, 1990). In all likelihood, the whole solution of mathematical problems was mainly exposed orally, relying heavily on the scribe's memory. Very often, on the tablet containing the problem-solving procedure of a mathematical problem, after some calculations have been made and giving as a result a certain number, for instance 30, we find the explicit instruction: "30, that your head retains".

Contrary to the social role played by literacy in the Greek culture where literacy was accessible to anybody wanting to learn to read and write (see Pfeiffer, 1968) –even though such an access was evidently more difficult for slaves–, literacy in Mesopotamia was reserved for a small *élite*.

The "explanation" that a written mathematical text may convey was hence restricted to the core of the problem-solving procedure –a choice very coherent with the general autocratic scheme mentioned above. The search for scientific texts containing a more complete "explanation" was a later sociocultural phenomenon that ran parallel with the social opening of literacy: in contrast to the post-Pythagoreans who were influenced, without a doubt, by the intellectual activity surrounding the work of the emerging sophists in the fifth century and were engaged in the production of writting works, Mesopotamian mathematical scribal texts did not reach an autonomous life as did Greek "books".

In this context, it seems possible to understand that *equations* did not appear in Mesopotamia as an explicit symbolic written equality between quantities. The unknowns were referred to by their own names: for instance, the length, the width, the area and so on of a rectangle or the weight of a stone. Let us consider here problem 1 of a tablet conserved at the British Museum, identified as BM 13901. The statement of the problem, which seeks to find the length of the side of a square, is the following:

> The surface and the square-line I have accumulated: 3/4.

The solution, as it appears in the tablet, is the following:

> 1 the projection you put down. The half of 1 you break, 1/2 and 1/2 you make span [a rectangle, here a square], 1/4 to 3/4 you append: 1, makes 1 equilateral. 1/2 which you made span you tear out inside 1: 1/2 the square-line. (Høyrup, 1986, p. 450)

According to Høyrup, the solution is underlying by a geometrical configuration upon which the oral explanation was based.

The scribe thinks of an actual square (fig. 1) which together with its side makes up 3/4. The side is seen as provided with a canonical projection that forms, along with the side, a rectangle (fig. 1); the quantity 3/4 refers then to the total area of fig. 1. Then the scribe cuts the width 1 into two parts and transfers the right side to the bottom of the original square.

Now the scribe completes a big square by adding a small square whose side is 1/2. The total area is the 3⁄4 (that is, the area of the first figure) plus 1/4 (that is, the area of the added small square). It gives 1. The side of the big square can now be calculated: that gives 1; now the scribe subtracts 1/2 from 1, he gets 1/2: this is the side of the original square.

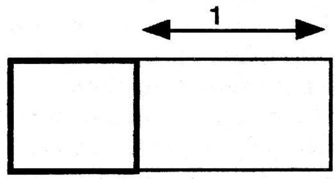

fig. 1

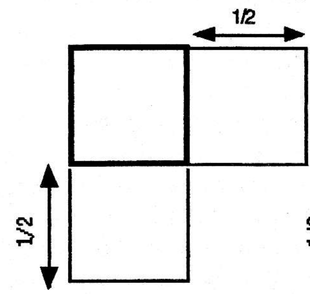

fig. 2

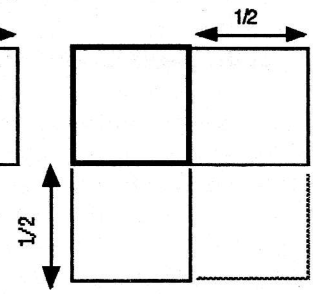

fig. 3

Our question is: where is the equation? In fact, there is not an explicit equation. In contrast, the problem-solving procedure is based upon: (i) an implicit principle of conservation of areas that we may express by saying that "parts of a figure may be cut and translated without altering the area" and (ii) a *double* expression of the area: (ii_a) first the area is given by a known quantity –in this problem the total area, A, of the final square is 1, and (ii_b) a relational formula linking the area of the square to its side –something that we express in modern symbols as $A=x^2$. Combining (ii_a) and (ii_b) the scribe obtains the side, x, and then the side of the initial square. It is important to note that while the reasoning allowing one to find that A=1 is essentially iconic and, hence, based on perceptual properties of the geometrical figures, the transitive step that leads to the deduction $x^2 = 1$ is achieved at an abstract level. Following a semoitic distinction introduced by Frege, we may say that the success of the solutions was based on the possibility of handling two different *senses* of the area and merging them into a single *meaning*. This, I believe, is an important step in the development of symbolic thinking and a necessary one for the rise of algebraic thinking.

3. Equations in Diophantus' *Arithmetica.*

There are some particular conceptual aspects that surface in the way in which equations were historically handled. Nevertheless, this aspect is often ignored or overlooked in traditional historical mathematical accounts, mainly because the ancient equations are unfortunately seen through modern lenses.

Let us discuss the case of Diophantus' *Arithmetica* –a monumental work very likely intended to be used in some schools of Alexandria at the end of Antiquity. As we mentioned earlier, the explicit rise of equations seems to have beeng related to the development of writing and to the sociocultural modes of expressing and transmitting mathematical contents. Although in the *Arithmetica,* equations still constitute a heuristic tool without being considered as mathematical objects *per se,* their explicitness is couched in the peculiar role played by explanations in the Greek scientific style of thinking.

Inheriting from Egyptian and Babylonian traditions of mathematics, Diophantus, as well as some of his Greek predecessors and contemporaries, was interested in riddles about numbers. Thus, in his introduction to the Arithmetica, he says:

> "Knowing my most esteemed friend Dionysius, that you are anxious to learn how to investigate problems in numbers, I have tried, beginning from the foundations on which the science is built up, to set forth to you the nature and power subsisting in numbers." (Heath, 1910, p. 129)

In all likelihood, Diophantus' main contribution was to create a theoretical foundation for the ancient riddles about numbers in order to convert them into a scientific domain. To do so, he classified numbers into categories according to the *eidos* of numbers, that is, according to the *form* that they may share: one such category was that of squares, another was that of cubes, another was that of square-squares, etc., that is, the numbers that our modern symbolism may awkwardly translate by x^2, x^3, x^4, and so on (actually Diophantus considered categories up to x^6).

Most of the problems of the *Arithmetica* seek to find numbers verifying a certain combination between the aforementioned categories of numbers. One such example (problem 8, Book 2) is the following problem whose roots date back to the Pythagoreans:

> "To divide a given square number into two squares."

Note that this problem is translated very poorly by the modern expression $X^2 + Y^2 = a^2$, for Diophantus did not use two unknowns. In fact, while the problem seeks two sought-after quantities, the problem-solving procedure uses just one unknown –Diophantus calls this unknown the *arithmos*, i.e the number, which refers to the number that the problem-solving procedure will uncover and from which the sought-after quantities will be inferred (for our distinction between sought-after quantities and unknowns, see Radford, 1994). The emergence of several unknowns was a later invention (see Bednarz *et al*. 1995).

Diophantus' solution is as follows: he takes the case where the given square number is 16.

> "Then", he says, "let the first number be the square of a number. Thus, the other number will be 16 units minus the square of the number. Therefore 16 minus one square of the number must be equal to a square. I form the square from any number of the number minus as many units as there are in the side of 16. Let us take the square of two of the numbers minus 4 units. This square will be 4 squares of the number plus 16 units minus 16 of the numbers which is equal to 16 minus the square of the number. Add to each of them the missing parts and subtract the similar terms. We will have that 5 squares of the number is equal to 16 of the numbers, hence the number is equal to $^{16}/_{5}$. Thus, one of the numbers is $^{256}/_{25}$ and the other will be $^{144}/_{25}$. The sum of both numers is $^{400}/_{25}$, that is, 16." (See Ver Eeck, 1926, p. 54; Heath, 1910, p. 144-145)

There are two points to be stressed. First, the fact that there is not a specific number for the unknown requires a special attention in order to understand what number we are talking about in each step of the problem-solving procedure. Secondly, the equation results from the fact that a *same* number is expressed in two different ways. Whatever the expressions may look like, they refer to the same object: the second sought-after quantity. We find here the same phenomenon as in the case of Babylonian algebra, except that in Diophantus' case, the objects have gained a very important level of abstraction. Indeed, although Diophantus' categories of numbers have an evident geometrical taste, they cannot be directly linked to any sensible object beyond the cube (whereas the Babylonian scribes formulated most of their problems in terms of rectangles, their area and sides[5]). The fact of being able to consider and to name objects that do not have any perceptual relationship to the sensible world is in complete hamony with Greek philosophical thinking and reflects clearly the distinction between the mind and hand: while the every-day arithmetical calculations were left to the slaves, the

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[5]Even though there are some evidences that in some cases the geometrical objects were thought of metaphorically, thereby showing an important step in the abstraction process (see Høyrup, 1994), most of the time the geometrical objects remained concrete objects.

intellectual class from the post-Ionian period confined itself to the elaboration of abstract scientific and philosophical systems (see Restivo, 1992, pp. 10-17).

The previous remarks show, I believe, that the emergence of equations as heuristico-algebraic objects was underlaid by the social meaning of writing as well as a reification process through which quantities were thought of as abstract numbers. Nevertheless, this reification process did not occur independently of the social meaning of the mathematical speculations. Indeed, the reification occurred intimately linked to the categorical distinction between the sensible world and the world of ideas advocated by Parmenides and followed by Plato (see Radford, 1996a) –a movement that led to a reconstruction of the Greek counting-stones mathematicians' results that were later reformulated into a "scientific format" as we find them in the arithmetic books of Euclid's *Elements* (see Lefèvre, 1981).

4. From the Middle Ages to the Renaissance

The spread of commercial activity in the late Western Middle-Ages led merchants and bankers to face new mathematical necessities –e.g. how to quickly and accurately calculate the exchange of different currency as well as the equivalence between domestic and foreign commercial products measured in different metrological systems.

To fulfil these necessities, it was necessary to train people in the mathematical calculations needed in business. This led to the rise of new educational institutions called *bothegas d'abaco* that followed the same functioning format as the *bothegas d'arte* (i.e. workshops of art in which painters and sculptors were trained to satisfy the demands of the princes, the church and the class of rich merchants).

Algebra was taught in those new commercial schools as an advanced topic probably reserved for an élite group of students –among them, those who wanted to become maestri d'abaco, that is, teachers of mathematics (Franci, 1988). Thus, once again, algebra served a social purpose. The specific shape that algebra took at that time, as in the case of the arts, was intimately related to its social role –that of allowing its practitioners to shine in their milieu in order to capture students as well as public contracts and royal patronages. Unlike the case of Diophantus, whose algebra was focused on solving problems that reflect the supposedly deepest nature of numbers, the algebra developed in the late Western Middle-Ages was centered on solving numerical and geometrical riddles whose mastering reflected an intellectual virtuosity welcomed within the dominant social and economical classes. In many aspects, the social sense of medieval algebra was much like the Babylonian one.

The study of the particularities resulting of the scientific contact between different cultures in the history and the mechanisms that allowed the cultures in contact to draw and to adapt the foreign ideas is beyond the scope of this short article. For our discussion, we shall limit ourselves to note that the Italian algebra took over the main organization of Arabian algebra –an organization in terms of *equations*[6]. This does not mean that there was a shift in the main focus of algebra from word-problems to equations seen as mathematical objects *per se*. Indeed, equations appeared because of a need to organize and simplify the resolution of problems. As we mentioned previously, the main focus was still that of solving word-problems and the role of algebra was that of providing a very powerful tool with which to solve them (see Radford, 1995).

An interesting point to recall now is the lack of algebraic symbols. During the Middle-Ages, the resolution was done by writing, in *extenso*, all the steps of the solution. In some cases abbreviations for main recurrent words were used, resulting in a language that has been called *syncopated* –something merely imposed by the mode of the production of mathematical texts before the invention of printing. However, there was an historically important shift when the abbreviations stopped to be functioning as confortable writing devices and started functioning as a means to simplify the calculations.

The passage from word abbreviations to autonomous signs to represent the unknown and its powers as well as the operation between known and unknown numbers was not easily accomplished (see Radford and Grenier, 1996) and, once again, cannot be seen just as a *logical* reification process. In fact, the emergence of algebraic signs –something that happened within the walls of a *bothega d'abaco*, was undoubtedly motivated by teaching reasons. As Vygotsky (1981, p. 157) pointed out, "A sign is always originally a means used for social purposes".

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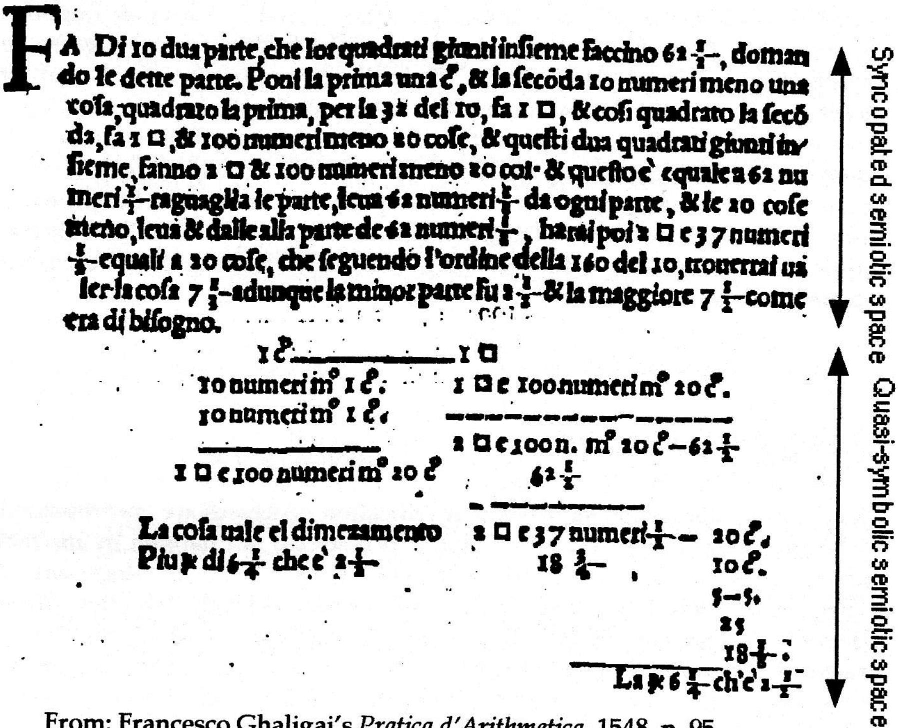
[6] In *The Condensed Book on the Calculation of al-gabr and al-Muqabala*, written in Bagdad *circa* 830 B.C., the author, Al-Khwarizmi, presents a classification of equations in 6 cases. In modern notations the cases are: (1) $ax^2 = bx$; (2) $ax^2 = c$; (3) $bx = c$; (4) $ax^2 + bx = c$; (5) $ax^2 + c = bx$; (6) $bx + c = ax^2$ (see Hughes, ed., 1986, pp. 233-34).



From: Francesco Ghaligai's *Pratica d'Arithmetica,* 1548. p. 95.
"Fa di 10 dua parte, che lor quadrati giunti insieme faccino 62 1/2"

(fig. 4)

Luca Pacioli, in his *Summa de Arithmetica, Geometrica, Proportioni et Proportionalita* (1494) used a syncopated language to solve word-problems witout being interested in equations. This was also the case of his master Piero della Francesca –one of the mathematicians who proposed a symbolic system to solve problems by algebra. For them, equations were still powerful heuristic tools with which to solve word-problems. In his *Practica d'Arithmetica* (1548), Francesco Ghaligai went a step further by delimiting two semiotic spaces in the problem-solving procedure: (i) a *rhetorical space* containing the statement of the problem and the syncopated problem-solving procedure and (ii) a *symbolic space* in which the calculations are shown (fig. 4). However, the content of his problems was not symbolic expressions but riddles about numbers (e.g. To divide a number into three parts such that ...).

Equations as genuine mathematical objects can already be found in Rafael Bombelli's *L'Algebra* (1572, p. 183 ff.) and Michael Stifel's *Arithmetica Integra,* published in 1544 –a delightful didactic work inspired by Christoff Rudolff's *Behend vnnd Hubsch Rechnung durch die kunstreichen regeln Algebre so gemeincklich die Coss genent werden* (1525). In the *Arithmetica Integra,* the author tackles problems like the following: 6x+6=12x-30. "First", he says, "subtract from each part 6x, thus it remains 6 equal to 6x-30. I Transfer 30 and get 36 equal to 6x." (Stiefel, 1544, p. 232B) There are also problems dealing with second degree equations. The important step that we can note in Stifel's work does not mean, however, that algebra was no longer related to commercial applications and solving riddles about numbers. There are, in the *Arithmetica Integra,* many such problems (see p. 256 ff.).

5. A Concluding Remark

The previous discussion suggests that reification processes are encompassed by sociocultural processes. Reification processes do not happen in abstract spheres reserved for the mind only. The study of the development of equations (and algebra in general) provides a neat example: equations have always had a *meaning* shaped by the social structures in which they were practiced. Of course, if we see the past from our modern mathematical point of view and we confine ourselves to the narrow realm of mathematics, what we see is but a distorted landscape. Mathematics, like any human activity, needs to be relocated in its different sociocultural contexts. Doing so, we start seeing past (and modern) events in a richer way, capable of providing us with a deeper understanding of mathematical knowledge. In turn, this can enable us to better understand the teaching and the learning of mathematics.

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