

Ethics in the Mathematics Classroom

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Abstract. *Despite some recent progress, ethics remains a peripheral topic in mathematics education research. My intention in the first part of the article is to show that ethics is omnipresent in the mathematics classroom. However, ethics does not always operate in the same way. To better grasp the nature of ethics, in the second part of the article, I discuss two ethical systems that have been influential in Western thought (Hobbes's and Kant's). These systems have largely informed the understanding of the mathematics classroom, even if, more often than not, they remain implicit. Then, I move to a short discussion of ethics in postmodern times and try to pinpoint what is at stake in ethics. The previous theoretical considerations pave the way to approach, in the last part of the article, ethics from an educational viewpoint. I argue that, in educational contexts, such as the school, ethics appear framed by the way in which we understand teaching and learning. I end the article with an outline of the communitarian oriented relational ethics articulated in the theory of objectification—a communitarian ethics whose practice features responsibility, commitment, and care.*

Keywords: ethics, theory of objectification, Vygotsky, Spinoza, Kant, Hobbes

Introduction

The first question that might arise is the following: What does ethics have to do with mathematics education? Let me present a short twofold answer.

First answer

Teaching-learning mathematics cannot avoid facing the question of the *legitimation* of particular forms of knowledge and knowing that arise in the classroom. Classroom discussions usually lead to *conflicting views* about what counts as mathematically valid and authentic.

Here is an example. In a Grade 5 class (10-11-year-old children) of a French-speaking school in Sudbury, Canada, the students were invited to write a text for a student from another class explaining how to solve linear equations. To write and solve simple equations, the students had been using an iconic semiotic system (ISS) that comprises four signs:

{ , , =, ↑ }.

In the ISS, the small rectangles represent cards; the envelope represents the unknown (as each envelope contained the *same* unknown number of cards). The problems with which the students had been dealing involved two individuals (e.g., Claudine and Sylvain) who each had a known number of cards and one or

- relations of *solidarity* and inclusiveness.

Here is an example. In a Grade 4 classroom (9-10-year-old children) of a French-speaking school in Ottawa, Canada, the students were working in small groups of three or four trying to solve geometric problems. The first problem revolved around the classical question of whether squares are rectangles. The pictures in Figure 2 provide a sample of *body positions* of the interaction of a group of four students. In Picture 1, Laura is talking to Sandra, the girl in front of her: “Yes, but they all have four sides.” In Picture 2, Mirna tries to contribute to the group and says: “The squares have same umm ... the same edges ...” Laura turns to look at Mirna for a short period of time; then, turns back to look at Sandra to continue their discussion. In Picture 3, Híria (front left) tries unsuccessfully to get Sandra’s and Laura’s attention and says: “The squares have parallel faces ... because there is ... Look!” In Picture 4, after recurring attempts to be heard, Mirna expresses her frustration and utters an anguished “Ahhhhh!!!”



Figure 2. A group of Grade 4 students dealing with a geometry problem.

We are here in the presence of an ethics of exclusion. Híria and Mirna try hard to enter the conversation, to contribute to the discussion, but they are not heard. The practice of an ethics of exclusion impedes a genuine collective engagement in mathematics and raises an invisible, yet important, wall between *us* and *them*.

The previous examples illustrate one of the main ideas of this article: teaching-learning is unavoidably an *ethical* event—and this is so regardless of the pedagogical model that underpins it.

I would like to go a step further and contend that ethics is not only omnipresent in mathematics teaching and learning but is also a crucial component of it for at least two reasons.

First, ethics shapes the manners in which teachers and students *engage* and *assume* (or not) certain *responsibilities* in the mathematics classroom. Consequently, ethics shapes how teachers and students come to understand mathematics and conceive of themselves as practitioners of mathematics.

Second, ethics shapes the *students' and teachers' relationships with others*—for instance, in the various manners by which the students voice (or not) their values and understandings, and how their voice is heard (or not). In this sense, ethics affects how teachers and students assert themselves as mathematical *subjectivities* (Radford, 2020).

Perhaps what is surprising in the claim that I am making, namely that mathematics teaching and learning is unavoidably an ethical event, is not that ethics is omnipresent in teaching and learning but the fact that, some exceptions put aside (e.g., Atweh, 2014; Atweh & Brady, 2009; Boylan, 2016; Dubbs, 2020; Ernest, 2009; Maheux & Proulx, 2017; Neyland, 2004; Maheux & Roth, 2014; Roth, 2013; Silva D'Ambrosio & Espasandin Lopes, 2015; Walshaw, 2013), we have not been able to notice this presence before—or at least not with the importance it deserves. I would like to suggest that the root of that omission is entrenched in the long-standing conception of mathematics education as a technical process, a conception that reduces mathematics education to a matter of acquiring knowledge, making the question of being and becoming peripheral aspects of teaching and learning. With this same technicist stroke, we erase from view the fact that the mathematics classroom is not only a site of knowledge production but also a site where subjectivities are produced every day.

The importance of ethics in an encompassing account of learning leads us to ask the question of the kind of ethics that we, implicitly or explicitly, nurture in classrooms and research. This question seems to be important in discussions on the diversity of theories in mathematics education; considering theories from the ethical standpoint they convey (even if they do so only obliquely) may shed new light on the differences, similarities, and complementarities between theories.

In an attempt to better grasp the nature of ethics, in Section 2, I discuss two ethical systems that have been influential in Western thought (Hobbes's and Kant's ethical systems). These systems have largely informed the understanding of the mathematics classroom, even if, more often than not, they have remained implied. Then, in Sections 3 and 4, I discuss what is at stake in ethics. Intended always as a means to better grasp the nature of ethics, Section 5 revolves around some ideas of ethics in postmodern times. The previous theoretical considerations pave the way to approach, in Section 6, ethics from an educational viewpoint. I argue that, in educational contexts, such as the school, ethics appear framed by the way in which we understand teaching and learning. I end the article with an outline of the communitarian oriented relational ethics articulated in the theory of objectification (Radford, 2019, 2021a)—a *communitarian ethics* whose practice features responsibility, commitment, and care.

2. Hobbes and Kant

All ethical perspectives are directly tied to conceptions of self and the social

world. Try to think of what self is and you will quickly realize that it is entangled with how we perceive ourselves and perceive others. This perceiving of the other involves social relations, some to keep, some to discard. Our concepts of self are indeed inseparable from the landscape of ethical issues, “from how one ought to be” (Taylor, 1989, p. 112). These ethical issues change from culture to culture and within a same culture through spans of time. Thus, from the end of the Middle Ages and the dawn of Renaissance, when European feudal structures were being shaken by the rise of mercantile capitalism and the ascending power of the concomitant new economic and commercial relations, the social world appeared more and more as an aggregate of monadic individuals (Elias, 1991). On the social plane, life became increasingly regulated by agreements or contracts between people (Le Goff, 1956; Jeannin, 1957). In the 17th century, in his *Philosophical Rudiments Concerning Government and Society*, Thomas Hobbes conceives of these contracts as means to regulate human conduct and to avoid aggression against each other. In his account, these contracts bring individuals out of the state of nature, which “is nothing else but a mere war of all against all” (Hobbes, 1841, p. xvii). These contracts elevate the individuals to civil society; they embody personal aspirations and interests while trying to safeguard each other’s security. Ethics appears here as a contractual mechanism to preserve the social order while affirming the monads’ interests in a way that reflects the underpinnings of the emergent bourgeois ideology.

One century later, when Kant articulated his ethical system, capitalism had expanded and become more systematic. Kant’s society, of course, is still perceived as an ensemble of monadic individuals, but with an increased complexity out of which the modern concept of self finds a more precise shape.

Kant assumed that individuals are by nature provided with a *same* reason—a legislative Reason. He also assumed that the individuals’ reason moves in a world of universal laws that can define the social and moral orders. From those assumptions Kant formulated his famous *categorical imperative*: “act only in accordance with that maxim through which you can at the same time will that it become a universal law” (Kant, 2006, p. 31, original emphasis).

As we can see, the categorical imperative answers the question: *What should I do?* Through it, moral content becomes specified not in terms of social ends that the rule would help to attain or achieve. Moral content can only be derived from the form of the rule as expressed and appraised by the individual.

MacIntyre (1966, p. 209) notes that “The eighteenth-century individualist sees the good as the expression of his feelings or the mandate of his individual reason.” The categorical imperative is an attempt “by the individual to supply his own morality, and at one and the same time, to claim for it a genuine universality” (p. 208). As I argued in (Radford, 2021a), the categorical imperative makes the Other appear as a *reflection* of the self in a mirror through which the self can see how the actions of the Other might affect itself. It assumes a universal human

subject and a universal reason that needs to be understood as a response to “the emergence of a formal law in Europe at that time, the state homogenization that was being imposed [on society], [and] the development of an instrumental rationality” (Bohy-Bunel, 2022, pp. 23-24). This universal human reason is nonetheless an *exclusive* ethnocentric reason in its most intimate nature. As Bohy-Bunel remarks, the Enlightened modern bourgeois individual “arrogates to himself all possible humanity” (p. 24). This individual is

the Westerner whose social, classist, patriarchal and *colonial* project of domination becomes clearer in the modern age . . . Its "science", which ends up fitting perfectly into this project, postulates that it relies on some universal "human reason," precisely in order to better assign to non-humanity those whose activity and being must be submitted to a total managerial control. (Bohy-Bunel, 2022, p. 25; emphasis in the original)

It is hence under the assumption of one and only one universal reason, the Enlightened managerial reason, that Kant can think that he solves the problem of the tension between the individual and the social world and the tension of human action in that world.

3. What is at stake

What is hence at stake in conceptions about ethics? The previous discussion provides us with a hint: what is at stake is the social meaning of choices about human conduct. This social meaning refracts, on our dealings with others, the forms of life found in the social world. In the case of Hobbes and Kant, the forms of life were determined to a large extent by the capitalism of their time, which became the “dominant reality in the historical life of European societies” (Foufais, 2020, p. 24), that is capitalism in its commodity form. Confronted with a social understood as an assemblage of monads, Hobbes found in the contractual mechanisms of capitalists the prototype of a juridical form where individuals could negotiate and combine their own interests. Through contractual mechanisms individuals became subject to a new entity, an *assembly*, through which the multitude becomes a state or *civitas*. Pressured by the needs of a more sophisticated social and economic organization and the emergence of a formal law, Kant did not need to have recourse to something extraneous to the individuals, as Hobbes’s did through the idea of *assembly*, to which the individuals relinquished the control of their own actions. Kant did not need to do something like that, as he assumed, as we just saw in the previous section, that there was a universal reason legislating choices about human behaviour.

It might not come as a surprise that the growth and implementation of ethics during the 20th century unfolded torn by two opposite forces that we find already in play in Kant’s *categorical imperative*: on the one hand, there was an emphasis on the *individual* (we saw that the categorical imperative comes from and is enunciated by the individual). This is the view of the existentialist ethics (Sartre’s ethics, for example) where, standing in the debris of WWII, authentic existence

was “to be found only in a self-conscious awareness of an absolute freedom of choice” (MacIntyre, 1966, p. 269). On the other hand, there was an emphasis *on the universal validity of the law*, without assuming, however, that the law emanates from the individual itself. The ethical self should be educated through a series of prescriptions. This is what has been called “prescriptionist” ethics; that is, the ethics practiced in particular by the legislators facing the problem of governing the masses. We find modern prescriptionist ethics at work in post-revolutionary France. For example, in 1833, French Minister of Education François Guizot explains that

The great problem of modern societies is the government of minds... It is not only for the commune and in a purely local interest that the law wants all the French to acquire, if it is possible, the knowledge indispensable to the social life, and without which the intelligence languishes and sometimes becomes stultified... This is because freedom is not assured and regular only among a people enlightened enough to listen in any circumstance to the voice of reason. (Guizot, cited in Mayeur, 2004, p. 335)

Thus “modern legislators and modern thinkers alike felt that morality, rather than being a 'natural trait' of human life, is something that needs to be designed and injected into human conduct” (Bauman, 1993, p. 6). The result was that, in the footsteps of Kant, modern ethics was mainly understood as deontology; that is, as duty and obligation driven by general rational laws to be applied regardless of the context. At the practical level, and with the push of modernity, this deontology was supplemented with a utilitarian outlook of the world. In the contemporary educational context, Neyland pictures the problem as follows:

Recent modernist reforms . . . have resulted in teachers increasingly being represented as objects rather than as subjects in policy discourse; professionalism has been replaced by accountability, and collegiality by competition and surveillance; initiative, creativity and teacher-led innovation have been constrained; teaching has become technicized, and learning experiences impoverished. (Neyland, 2004, p. 62)

Here, as Neyland suggests, ethics amounts to proceduralism; that is, the implementation of the dictates of the educational apparatus as how teachers and students should behave and what they have to accomplish.

Postmodernist thinkers have quarrelled with this proceduralist conception of ethics. What bothers many of them is not the subjective aspect involved in ethical contexts. They retain the Kantian idea that morality does not come from outside the subject but from within. Bauman, for example, argues that “The moral call is thoroughly personal; it appeals to my responsibility” (1993, p. 60). What bothers these postmodernists is the deontological dimension. They have reacted suggesting a non-deontological ethics, an ethics that is context-sensitive and hence impossible to frame through a priori laws or principles. As Bauman puts it, “Moral phenomena are inherently ‘non-rational’... They are not regular,

repetitive, monotonous and predictable in a way that would allow them to be represented as *rule-guided* (1993, p. 11; emphasis in the original) . . . morality is endemically and irredeemably *non-rational*” (1993, p. 60; emphasis in the original).

Now, how can such an ethics possibly work if postmodernist thinkers themselves have been very successful in showing that the individuals are subtly shaped by a multitude of social mechanisms of governance that make them think and act in specified ways? How can we make sure that the ideas with which we ponder a delicate ethical situation are not those prescribed by the system even if they do not appear in a law form? In other words, how can we make sure that it is *me* and not the system through its webs of power and knowledge that is talking through me?

Foucault, the last modern Kantian and one of the first postmodern thinkers, raises the question as follows: How can the subject be the locus of meaning, feeling, intentionality, and ethical decisions if this subject must talk, feel, and intend through thoughts and words that are not its own?

Can I, in fact, say that I am this language I speak, into which my thought insinuates itself to the point of finding in it the system of all its own possibilities, yet which exists only in the weight of sedimentations [that] my thought will never be capable of actualizing altogether? (Foucault, 1966, p. 335)

Rothenberg articulates this problem asking how the individuals produced within the social modes of power, “subsisting as nothing other than the intersection of various discursive determinants” (2010, p. 26), could be able to act independently of ideological operations. These individuals, Rothenberg argues, “have to transcend their ideological determinations somehow and lift the veil of misrecognition, even though they themselves are nothing other than the expression of ideology and thoroughly blinded by it” (Rothenberg, 2010, p. 25).

What is at stake, I argued before, is the social meaning of choices about human conduct. We now see that, going a step further, what is at stake at a deeper level, is our conceptions of human nature. Are humans capable of transcending their ideological determinations or are they trapped in them forever?

4. Self as lack and self as excess

Historically speaking, feminist, multiculturalist, psychoanalytic, and dialectical materialist thinkers, and scholars in related fields have contended that human nature is such that ideological determinations can be overcome. We are not trapped. The “trapped forever” position, they argue, is based on a conception of the self as *lack*. It features a self that lacks wholeness or plenitude, subjected forever to the limits of the given structures and confining discourses. They counter this conception of self with a conception of self as *excess*.

In feminist theory the subject is conceived of as having the power “to reflect

on the social discourse and challenge its determinations” (Alcoff, 1988, p. 417). Following a Hegelian thread, for Butler (1999), the subversion of the subject is possible because all acts of signification not only restrict the subject’s actions but are, at the same time, in their enactment, always located within the possibility of a variation in the “alternative domains of cultural intelligibility” (p. 185).

Rothenberg (2010) approaches the problem from a psychoanalytic perspective and claims that meanings have the peculiar property of being impossible to be totally controlled. Although individuals are socially produced, the producing causes leave “a remainder or [meaning] indeterminacy, so that every subject bears some unspecifiable excess within the social field. Every subject is an ‘excessive’ subject [and] the excess is ineradicable” (2010, p. 10).

Drawing on Lacan, Žižek refers to a fundamental characteristic of the Symbolic: its “openness.”²

The cause of this irreducible "openness" of the Symbolic is not its excessive complexity (we never know in what decentered context our statement will be inscribed), but the much more refined, properly dialectical impossibility of taking into account the way our own intervention will transform the field. (Žižek, 2010a, pp. ix-x)

Through our interventions in the world, Žižek contends, we affect our reality, which means that we change and affect our social determinations. These are not outside an undisturbed reality, nor is our free will.

When we feel thwarted in our freedom by the constraining pressure of external reality, there must be something in us, some desire or striving, which is thus thwarted, but where should this striving come if not from this same reality? Our "free will" does not then in some mysterious way "disturb the natural course of things," it is part and parcel of this course. (Žižek, 2010a, p. xii)

Dialectical materialist thinkers (e.g., Fischbach, 2014; Macherey, 2008) have drawn on Marx’s (1998) work, particularly on the idea of *transformative praxis* quickly sketched in Thesis 3 of *Theses on Feuerbach*. Following Marx’s ideas, in dialectical materialism the subject is featured as one that, while being produced by its circumstances, has, inversely, the power to reflect on these circumstances and to transform them (for an enlightening analysis see Macherey (2008) and Fischbach (2015a)). Following Marx, what contemporary dialectical materialists add to the agentic conceptions of self mentioned above (i.e., the self as excess) is that subversion is accomplished in *praxis, with others* (Freire, 1998). Subversive praxis is the arena of the emergence of a new form of social consciousness. For consciousness is not only a refraction of reality. Consciousness, through its

² The Symbolic is the sphere of the Law. Without submitting ourselves to the Symbolic, we would not be able to function in society. The Symbolic includes grammatical rules, social norms, and unconscious prohibitions (see Žižek, 2007).

varying layers of depth, is a concrete relation that, given our biography and cultural background, *propels us* towards the world and leads us to act on/in it and *transform* it (Clot, 2015). In short, what dialectical materialists add to the agentic conception of self mentioned before is that the subversion of self is not the result of the deeds of a solitary being: it is a political and social project strictly tied to the transformation of the individuals' consciousness. There is still hope, then, that in our Grade 4 example, Mirna and her twin sister, Híria, will be heard. However, for this to occur, there must be a transformation of circumstances. The classroom culture must be transformed. This transformation requires a new *praxis*, a classroom praxis, out of which a new form of social consciousness can emerge.

To recap, in each one of these accounts, although for different reasons, there are always possibilities to *interrupt* the quotidian train of our actions and thinking. It is at this point that Scott's (1990) *The Question of Ethics* as a question of *interruption* of habits and values acquires its whole sense. For Scott, *The Question of Ethics* indicates our capacity to bring forward "an interruption in which the definitive values that govern thought and everyday action lose their power and authority" (p. 4). This interruption makes sense precisely because of our cultural-historical *agentic* nature (i.e., our power to sublimate and surpass the cultural and historical possibilities on which we draw when we re-act to, and re-enact, the world).

5. Ethics in postmodern times

5.1 Ambiguity

As stated before, one of the main features of postmodern ethics is to consider ethics as having an intrinsically *ambiguous* nature. To be ambiguous means that ethics is neither something contractual (like teachers do this, students do that), nor is it something that works on the basis of rules and abstract principles (like *do your homework!*). Valero and Jørgensen (2021) give an example of the ambiguous nature of ethics in contemporary ethical thought. A student submits an essay that, according to a plagiarism software, shows 21.5% of compatibility between a student's text and a referred text. However, the institutional maximum of compatibility is 20%. For the institution, plagiarism is

unacceptable and grounds for expulsion. The teacher knows the student and knows that there is a case of terminal cancer in the immediate family that has affected the student's academic ability ... For the teacher, ensuring access to education, particularly for poor people, is a fundamental part of his understanding of fairness in education. It is part of his social responsibility and commitment as a teacher. What does the teacher decide to do: follow institutional norms or his personal norms? (Valero and Jørgensen, 2021, p. 271)

As the authors remark, "The issue is to enter into a deep reflection to generate an ethical rule that is applicable in *this* situation" (Valero and Jørgensen, 2021,

p. 272; my emphasis).

So, to say that ethics is intrinsically ambiguous means that our acts and relations to others do not have *one* obvious meaning. They are context sensitive. Ethics is seen here as a context-sensitive dynamic and open-ended relational stance that is continuously materialized and assessed as teachers and students engage with each other.

5.2 Ethics as thoroughly personal

Another main feature of postmodern ethics, or at least the one we find articulated in Bauman's work, is, as mentioned before, that ethics is something "thoroughly personal," something that "appeals to my responsibility" (1993, p. 60).

Bauman continues here the Western modern tradition that posits the individual as the origin of intentionality (Husserl, 1982) and ethical concerns (Kant, 2006). This feature of postmodernism is anything but new. It is easily understood once one realizes that postmodernity "one may say, is modernity without illusions (the obverse of which is that modernity is postmodernity refusing to accept its own truth)" (Bauman, 1993, p. 32). In other words, postmodernity is modernity disenchanted; modernity removed of the illusions of the universal abstract reason of the Enlightenment and its overarching grand narratives. Of course, postmodernism, even if it continues the modern tradition of the "new," the "novel," the "creative," is more complex than this (Macherey, 2006). Capital (understood as the general system of production of the individuals' lives, the system of production that characterized modernity) has not disappeared. It continues to be *the* mechanism through which human life is shaped and quotidianly produced. But capital has been refined and expanded: it has become more complex, more nuanced, and much more oppressive than in modern times. Bauman, hence, is right in seeing in postmodernism the story of an unending end; it is still a modernism, but one that seeks with relentless vigour to get rid of the encumbrances of the Symbolic (the "law" in Žizek's sense) that has continuously threatened the autonomy of the modern subject. In this sense, postmodernist ethics is still (or again) a subjective ethics, this time pushed to its radical limits—with all the contradictions that such a push entails (Lipovetsky, 1989; Žizek, 2010b). If postmodern ethics recognizes the role of the Other in some qualified Levinas's sense (as Bauman does), it does so within a monadic understanding of the social featured by modernity, that is, as an arena of "competitive struggle" whose pinnacle is "individual choice" (Bauman, 1993, p. 46).

6. Ethics in the theory of objectification

6.1 Reconceptualizing the social

Ethics in the theory of objectification is neither modern nor postmodern. Although ethics is understood in a non-deontological, context-sensitive manner, ethics is not considered as coming from within. It is not something "thoroughly personal." The reason is not because the individual is exempt of all moral

responsibility. The reason is that it understands the social not as an agglomeration of monads that, once constituted as humans, enter the social realm. This monadic conception of the individual, where the social is seen as a derivative of the individual's life, has been one of the chief marks of modernity and postmodernity, from where we have inherited the "acids of individualism" (MacIntyre, 1966, p. 266).

The conception of the social that underpins the theory of objectification is such that there is no separation between the individual and the social. The social is always appearing within the individual and between individuals. One cannot exist without the other. The social is neither a container, nor a flat surface where we live our life. The social is made up of *relations (rapports)* between individuals. These relations are not merely connectors; they are susceptible of modifying the individuals they put into relations.

To the extent that social relations express and manifest in their enactment political, economic, cultural and social conceptions about the world, ourselves and others, they affirm certain *forms* of dealing with others. This is what we see in our discussion of Kant's and Hobbes's ethical systems: each one of them suggests a social form of self-other dealings (Kant's ethics turned to a universal reason legislating the transactions between individuals; Hobbes's one turned to transactions of a contractual nature). Ethics is found precisely in these forms, which are nothing else but *forms of alterity*. Ethics, then, appears as the form of Self-and-Other.

If the social is understood as made up of social relations in movement, what would then be the kind of relations that are targeted in the theory of objectification?

6.2 Ethics and learning

To answer this question, we need first to make explicit the educational project in which this theory is subsumed. The theory of objectification inscribes itself in a Freirean and Vygotskian educational project that posits the goal of Mathematics Education as a political, societal, historical, and cultural endeavour aimed at the dialectical creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical discourses and practices, and who ponder new possibilities of action and thinking.

As in other educational theories and pedagogical models, ethics in the theory of objectification appears in the relations between students and teachers. Ethics, understood as the form of Self-and-Other that manifests itself in our dealings with other individuals, is directly linked to the understanding of how learning occurs.

Let us consider two examples.

Think of the theory of transmissive instruction. Learning is conceived of as the assimilation, through practice and repetition, of knowledge that the teacher possesses. This theory positions teachers as knowers and the students as lacking

knowledge. The ethics of the transmissive instruction, manifesting itself in the form of alterity that it promotes, reflects, but also operationalizes. It manifests itself concretely in the alienating relations of power and subjection it fosters and maintains, relations that are thematized along the obedient lines of superior/inferior, potent/impotent, knower/ignorant, authority/vassal (Radford & Lasprilla Herrera, 2020).

Now think of constructivism. In contradistinction to the theory of transmissive instruction, constructivism is based on an ethics that stresses the freedom of the student: since knowledge is conceived of as what results from the autonomous deeds of the student, and learning is the very process of the student's subjective construction of knowledge, teachers and students are positioned otherwise: the student's freedom and autonomy configure the constructivist ethical space (Radford, 2012). It alienates teachers and students, but in a way that differs from how transmissive instruction does (Radford, 2021a).

Learning in the theory of objectification is neither about transmitting knowledge (as in direct or transmissive teaching), nor is it about the students' constructing their own knowledge (as in constructivist approaches). In the theory of objectification, learning is conceived of as a *collective* and *truly social* embodied and material process through which students critically encounter culturally and historically constituted ways of thinking mathematically. This encounter happens in what the theory terms *joint labour* (Radford, 2021a, 2021b).

6.3 Joint labour

Joint labour is a sensuous, practical, material *activity*—activity understood as driven by *collective* concerns. The German and Russian languages have a specific term for this type of activity: *Tätigkeit* and *deyatel'nost'*, respectively. Activity in this sense is opposed to activity as being merely busy with something (as in watching TV). Again, the German and Russian languages have a specific term for this other type of activity: *Aktivität* or *aktivnost'*. Unfortunately, in the translation into English (and several other languages), the distinction is lost and both types of activity are rendered as *activity*.

In the case of the theory of transmissive instruction, classroom activity is not oriented towards the satisfaction of collective needs. This activity corresponds hence to *Aktivität* or *aktivnost'*. In joint labour, by contrast, students and teachers work hand in hand to *produce* something *together*, what Hegel termed “a common work,” in our case, *mathematics*. It is this sense of labouring together (as opposed to simply interacting or exchanging with others) that makes joint labour a truly social activity and learning a collective process.

6.4 Transforming ethics

Classroom research has shown us, however, that for learning to become a truly collective process, radical changes in the classroom culture might need to occur.

Often, drawing on experiences shaped by transmissive instruction, the students conceptualize the teacher as the possessor of knowledge and power, and conceptualize themselves as submissive to the teacher and her knowledge, even when the teacher tries to conceptualize herself differently and encourages the students to learn collectively and organizes the classroom into small groups. Often, the students configure small, enclosed groups and erect aggressive or exclusive antagonistic barriers between their group and other groups—they resort to what we may term a *clique ethics* or *gang ethics* (Radford & Lasprilla Herrera, 2020), which is also what we see in the example from the Grade 4 classroom briefly mentioned in the Introduction.

Of course, moving towards a more encompassing, democratic, and inclusive ethics cannot be achieved by imposing new social forms of conduct. “It is not obedience to someone or obedience to something, but the free adoption of those patterns of behavior which will vouchsafe the consonance of all of behavior” (Vygotsky, 1997, p. 233). The educational problem around ethics becomes the problem of the creation of classroom conditions for new ethical relations (new *forms* of alterity) to emerge and to be collectively pondered and discussed against the always contested background of culture and history. The process of emergence of such an ethics has to overcome the dynamic power of capital, which constantly tries to appropriate the social “in order to put it at the service of its own development.” It is hence “a question of not letting ourselves be dispossessed: it is necessary to maintain and to develop [the spring of the production of social life] against capital” (Fischbach, 2015b, p. 71). This can only be done by developing a form of democracy that would “increase our power to generate ourselves our own social life” (p. 71).

6.5 A communitarian ethics

So, what are the new ethical relations that we strive to nurture in the theory of objectification? We focus on a classroom mathematics practice featuring a *communitarian ethics* which is consonant with the conceptual historical-cultural bases of the theory and its conception of learning as a collective process.

Communitarian ethics is based on what we call three ethical vectors. They define a dynamic ethical space. Rather than be seen as fixed terms, we see them as *repères*; that is, reference points against which to gauge our actions. These vectors work as “primary orientation of the ethical self” and should not be seen as “a modern uniform code”; they are “shared ethical ideas, priorities and principles that are subject to ongoing and ‘antagonistic’ negotiation” (Neyland, 2004, p. 57). These vectors (or “virtues” to use MacIntyre’s term) are responsibility, commitment, and care. It is here where we resort to the construct of “voice”—not voice in a linguistic sense exactly; rather we resort to voice as something that brings in the notion of *difference* and the *primacy of the political* (Giroux, 2005). Finding one’s voice or having a voice is “moving from silence into speech,” it is “a gesture of defiance that heals, that makes new life and new

growth possible” (bell hooks, 2015, p. 29), something that “assumes a primacy in talk, discourse, writing, and action” (p. 33).

Coming back to our Figure 1, when Mirna utters an anguished “Ahhhhh!!!” and moves her right hand towards the two other girls who are not listening, she is moving into speech to express her frustration about not being counted and heard. Her voice (which is much more than what she discursively *says*, as it also says things in her body posture, facial expression, pitch, gesture) opens up new possibilities for action (for herself and the other teammates). Mirna’s embodied utterance is, indeed, a *call* to the Other.

6.5.1 Responsibility

The call now must be responded to, and it is responded to within a certain node of social relations that tie the students together. Whatever path the teammates’ response takes, it is cast in a general ethical attribute that Lévinas calls *responsibility*. For Lévinas, responsibility is “the essential, primary, and fundamental structure of subjectivity ... [where] the very node of the subjective is knotted” (1982, p. 101). Since all educational theories put into motion a certain ethics—for, as mentioned before, ethics is the substrate and the form that is continuously materialized in our dealings with the Other—responsibility is a common denominator of all of them. Yet, the *meaning* of responsibility is not the same. In the theory of direct instruction, the students’ responsibility consists in assuming a submissive role vis-à-vis the teacher. The teacher’s responsibility consists in subjecting the students through the power of knowledge. In the case of the theory of didactical situations (Brousseau, 2005), responsibility follows Hobbes’s contractual outlook and appears as a didactical contract; that is, a reciprocal distribution of duties. In the case of the theory of objectification, responsibility means living and acting *with* and *for* others; it means to respond to the call of others as they are on their own terms: in their “existence, in [their] being-for-other[s] . . . as free being[s]” (Hegel, 1978, p. 57).

6.5.2 Commitment

Commitment is both the promise and its realization of doing everything possible to work side by side with others in the course of our joint labour (e.g., trying to understand the process being followed to solve a problem, trying to contribute to the classroom common work).

The classical utterance “I do not understand” of classrooms operating within the transmissive instruction model often conveys the stance of a de-responsabilization; it intimates that it is the teacher who has not done his work properly. Through this utterance, the student invests the teacher with the role of patriarch of knowledge and requests the teacher to deliver knowledge. However, if teachers and students understand that knowledge is produced collectively in the classroom, the same utterance would mean something like: “I do not understand, but let’s work together so perhaps things will become clearer.” Commitment is this resolution to engage in joint labour, to participate in the creation of the

classroom common work (*oeuvre commune*), which can be establishing *collectively* how equations can be solved or theorems proved, for example.

6.5.3 Care for others

Far from being an act of condescending or simply caring for someone, the care for others is a relational involvement entailing the attention to, and recognition of, others and their material and spiritual needs. Although caring for the Other opens up the possibility of seeing ourselves in the Other, of recognizing our vulnerability in the vulnerability of the Other, the importance of caring for the Other is to go beyond ourselves and to be dragged powerfully into the world, to position ourselves there, with-the-Other.

To understand Mirna's "Ahhhhh!!!" within an ethical practice of responsibility, commitment, and care, we need to broaden our conception of language and *recognize* (in its Hegelian sense) this painful expression as *voice*; that is, as something where, as Lévinas suggests, the *saying* (that which I want so much to utter but I will never be able to do) moves beyond the totalizing enclosure of the *said* (that which I manage to say) and becomes rather the possibility of openness to the other (Radford, 2021a). In this conception of voice, power does not disappear since power is not a *thing*, but something imbricated in our relations to others. What we can expect in the transformative movement towards a communitarian ethics is that, through conscious, reflective, and critical stance, power in the classroom goes beyond its own subjecting mechanisms of social order and becomes rather something fluid, dynamic, to be exercised with responsibility *for* the Other.

7. Concluding remarks

In the first part of this article, I argued that mathematics teaching and learning is unavoidably an ethical event. There are at least two reasons for this. The first reason has to do with the fact that teaching and learning always involve interaction between teachers and students and this interaction is based on relations (relations of power and subjection, relations of exclusion, or relations of solidarity, inclusivity, etc.). The second reason has to do with what is to be learned, with knowledge. It has to do with the *legitimation* of particular forms of knowledge and knowing. Teachers are confronted in their everyday practice with a variety of ways that students bring about how to solve mathematical problems. Legitimation appears here as what counts as true, as mathematically right, sound, worth pursuing, etc. I presented an example in which students produced two different mathematical texts explaining how to solve linear equations. The question revolved around the epistemological role of examples: should the text include an example? Should it be rather general? What text is better? As soon as we move beyond Eurocentric views of mathematics (D'Ambrosio, 2006; Powell & Frankenstein, 1997), the question of legitimation acquires its full sense.

In the first part of the article, I complained that despite its omnipresent nature in the mathematics classroom, ethics still remains largely unexplored (although

it is gaining some traction, as we can see in the No. 38 (December 2021) issue of the *Philosophy of Mathematics Education Journal*, edited by Paul Ernest).

To better understand conceptions of ethics, in the second part of the article, I dealt with two ethical systems—Hobbes’s and Kant’s—that have been influential in Western thought and have informed the understanding of the mathematics classroom, even if, more often than not, those ethical systems have remained implicit (for instance, Kant’s subjectivist ethical stance has influenced constructivism; Hobbes’s contractualism has influenced transmissive learning, and both Kant and Hobbes have influenced the theory of didactical situations).

In the third part, I moved to a discussion of what is at stake in ethics. We saw that, on a first level, the answer lies in the social meaning of choices about human conduct. On a second and deeper level, the answer lies in our conception of human nature.

The discussion of ethics in postmodern times, intended always as a means to better grasp the nature of ethics, allowed us to see that, on the one hand, the postmodern project breaks with the modern project, while, on the other hand, continues it. The postmodern project breaks with the modern one in refusing to resort to a universal reason as the basis of the ethical self. At the same time, it continues the modern project in formulating a subjective ethics; that is, an ethics that is at the end of the day “thoroughly personal” (Bauman, 1993, p.60). The postmodern understanding of ethics still rests on the modern conception of the social as an aggregate of individuals, monadic self-constituted agents.

The previous theoretical considerations paved the way to approach, in the last part of the article, ethics from an educational viewpoint. I outlined the ethics in a Vygotskian theory of teaching and learning, the theory of objectification (Radford, 2021a). To do so, drawing on the work of Fischbach (2015a, 2015b), I started offering a reconceptualization of the social as made up of *social relations*, which are much more than simple links between individuals. So, instead of being a mere background, the social appears to be a dynamic totality in perpetual movement and transformation. The relations of which the social is made up express conceptions about ourselves and others. They are always tinged with political and cultural valences. These considerations led me to suggest that ethics can be considered as the *form of alterity* (the form of Self-and-Other) that manifests in our dealings with other individuals. I argued that, in educational contexts, such as the school, ethics appear framed by the way in which we understand teaching and learning.

Against this background, I outlined the communitarian oriented relational ethics that is articulated in the theory of objectification—a *communitarian ethics*. This ethics follows Spinoza in stressing the body’s *power* of acting in the world (*agendi potentia*; Spinoza, 1989, p. 210). The communitarian ethics also follows Lévinas’s (1982) work which radicalizes previous ethical systems in acknowledging that our actions and deeds are always modulated by the presence

of the *Other*—a presence that comes to us in a sentient and fleshy manner: through the *proximity of our bodies*. In this proximity our conceptual epistemological categories and mechanisms are put on hold, and we encounter the Other as *is*. “The Other is appreciated precisely *as Other*, in her radical alterity and irreducible singularity, only when thought renounces its totalitarian hubris and learns to think of the Other on her own terms . . . ‘beyond essence’” (Min, 1998, pp. 573–574; emphasis in the original). The self appears here conceived not as the prerequisite of existence of ethical relations. The self is rather seen as the *result* of those relations.

In conceiving of ethics as the form of alterity, the focus turns not to moral precepts but rather to the fluid and content-dependent relationships between subjects as they appear in the immediacy and banality of everyday life. In this view, ethics is continuously *materialized in praxis* out of a myriad of possibilities, for, in this view, the individual appears as “full of unrealized possibilities every minute” (Vygotski, 2003, p. 76). If we come back to Figure 2, we see that Mirna’s teammates could have opted for other actions. The materialization of the students’ *actions* reflects their understanding of the context (consciousness) and the manner in which the context is lived through their unfolding collective affective experience (emotions), as well as the relational stances (ethics) they (consciously or not) adopt towards each other. Through the previous classroom examples, we see that embodied, emotional, contextual, cultural-historical action is related to ethical postures that students assume and show in practice.

What is specific to the communitarian ethics of the theory of objectification is that, braking with the subjectivist stances of modern and postmodern ethics, its practice features responsibility, commitment, and care. This communitarian ethics provides reference points for pedagogical actions in the classroom, where teachers and students explore together new critical spaces that promote engagement, inclusiveness, debate, and respect (for some classroom examples, see Radford, 2021, 2021b). Underneath the communitarian ethics lies the recognition that our historical, cultural, and material origin embeds and refracts dynamic and antagonistic visions and conceptions of the world and of what a good life can mean. Although impossible to posit and describe a priori, the good life and the common good are understood here as ideas (*idéalités*), more specifically, as “*visées éthiques*” (aims), generated in a joint unending project created itself within the dynamic that produces the social relations, a project that will always be paved with tensions, contradictions, and difficulties. “Let us call ‘ethical aim’ (*visée éthique*) the aim of the ‘good life’ with and for others in just institutions” (Ricœur, 1990, p. 202). It is the vitality of contradictions that gives substance to social human life, always changing, always challenged, and that makes ethics to always unfold in a context of struggle.

Acknowledgments

This article is a result of a research program funded by the Social Sciences and Humanities Research Council of Canada and Laurentian University Research Fund. The article was presented at the TSG 57 (Diversity of theories in mathematics education) of ICME 14. A previous version of it appeared in Radford (2021c).

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Appendix

Translation of the texts shown in Figure 1.

First text:

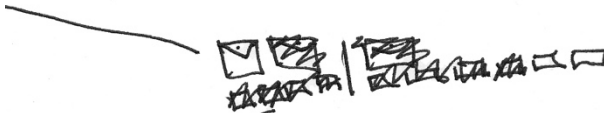
Step 1: Count all cards and envelopes in the equation.

Step 2: Take the number with fewer envelopes and remove all the envelopes. Ex.



Step 3: Same thing for the cards. Ex.

Step 4: Give an answer ex.



$$\overset{0}{x} + \overset{1}{2} = \overset{0}{x} + \overset{2}{2} \quad \square = 2\square$$

Second text:

To write an equation you have to:

1. Read the problem.
2. Take the smaller amount of card[s] on one side and cross it out.
3. Cross out the same amount of cards on the other side.
4. Take the smaller amount of envelopes on one board and cross it out.
5. Cross out the same amount of envelopes on the other side.
6. Circle the remaining cards and envelopes.
7. Count the remaining cards and envelopes.
8. Divide the cards into each envelope if you have more than one envelope remaining.