Of Love, Frustration, and Mathematics: A Cultural-Historical Approach to Emotions in Mathematics Teaching and Learning

Luis Radford

Abstract: Emotions have traditionally been characterized as inner, subjective, and physiological experiences, usually of an irrational nature. Against this subjectivist and physiological position, drawing on cultural psychology and anthropological research, in this article I advocate for a cultural conception of emotions and their role in thinking in general and mathematical thinking in particular. I argue that, rather than momentarily subjective phenomena, emotions (for instance, anger, frustration, love) are historically constituted. Emotions, I contend, are not opposed to thinking, but are an integral part of it. Emotions are as ubiquitous as breathing. I illustrate these ideas through the analysis of Grade 4 students working on a mathematical problem.

Key Words: Thinking and Emotions • Feelings • Cultural Historical Activity Theory• Subjectification • Motives

> Like all other mental functions, emotions do not remain in the connection in which they are given initially by virtue of the biological organization of the mind. In the process of social life, feelings develop and former connections disintegrate; emotions appear in new relations with other elements of mental life. (Vygotsky 1999, p. 244)

1. Introduction

In his Plenary Lecture at the CERME 7 Conference Hannula (2011) offers a detailed review of the problem of affect in mathematical thinking and learning. In particular, he points out the difficulties that mathematics educators encounter when trying to define the key concepts through which the affective domain can be scrutinized and understood (see also Goldin (2002), Furinghetti and Pehkonen (2002)). The result is obvious: as far as the affective domain remains difficult to understand, its link to mathematics teaching and learning will remain difficult to recognize.

What, indeed, do we mean by affect? And how is it different from emotion and feeling? How do affect, feeling, and emotion relate to motives and motivation? Motivation, Hannula remarks, "is perhaps the most difficult [concept] to define" (2011, p. 44). This is so, I would like to suggest, because motive and motivation require that the manner in which individuals' intentions, needs, and interests relate to the social and cultural context be made unambiguous. Motives are the affective component of projects of life that link the individuals and their contexts, present and future. How to explain this link is not an easy matter. Here resides the central problem of the classical distinction between intrinsic and extrinsic motives, a distinction that remains decidedly dualistic. In dealing with motives, such an account assumes that the individual, while screening his/her sociocultural environment for clues and insights, finds in an allegedly insulated

interiority the foundations of what moves him/her towards action. Unavoidably the intrinsic-extrinsic motive account ends up portraying individuals as entities living in solipsistic envelopes. The most profound deficiency of this account is that it assumes a kind of auto-sustained self. Within this model, motives are personal constructs and emotions truly private bodily phenomena.

The point that is missed here is that the affective domain in general and motives and motivation in particular are not only subjective but also sociocultural phenomena. They are subjective and sociocultural in the sense that on the one hand motives are the *motives of a concrete and unique person* but, on the other hand, they relate to a sociocultural and historical world that *transcends the individual*. In its transcendence, the sociocultural historical world indirectly—albeit in a decisive manner—shapes and organizes the individual's motives and emotions. This point, however, is often missed as a result of conceiving the relationship between society and its individuals as a relationship of opposition—society *versus* individuals. Commenting on this oppositional view, A. N. Leont'ev wrote: "the main thing is ignored, that in society man [sic] finds not only his external conditions to which he must adapt his activity, but also that these very social conditions carry in themselves the motives and aims of his activity" (Leont'ev 2009, p. 3).

In the past few years, sociocultural research has made an effort to go beyond the oppositional conception of the individual and the social. Evans and Zan distinguish three trends: (1) a socio-constructivist approach, where "Emotions are seen as social in nature and situated in a specific socio-historical context, because of the social nature of an individual's knowledge and beliefs" (Evans and Zan 2006, p. 44); (2) a discursive approach that considers "emotions as socially organized within a structure of social relations where power is exerted" (p. 43); and (3) an approach based on cultural-historical activity theory where emotions "come from the body. . . [and are] seen as integral to practical action" (p. 45). Evans and Zan (2006) show clearly how these approaches with their different conceptions of emotions address specific problems through different methodologies (see also Evans 2006).

The conception of emotions that I am about to sketch here draws on cultural psychology and anthropological research. It stresses the role of emotions in thinking. My goal is to offer evidence of the manner in which thinking and emotions are intertwined in mathematical cognition and to stress some implications for teaching-and-learning. The cultural conception of emotions that I put forward here is located in an important shift that Evans (2006) and Evans and Zan (2006) note. According to these authors, there has been a shift in mathematics education research that goes from the investigation of more or less durable individuals' features (e.g., attitudes and beliefs, in general scrutinized through questionnaires and interviews) to research on emotions considered as volatile and contextual dynamic processes. I argue, however, that the contextual and dynamic nature of emotions cannot be limited to the analysis of their contextual occurrences. My contention is that emotions are dynamic processes, but rather than being singular and momentarily subjective, emotions (for instance, anger, frustration, love), while being socially organized, are historically constituted. The historicity of emotions (despite their formal acknowledgment in the sociocultural and activity theory trends identified by Evans and Zan) has not been a main theme in mathematics education research. The

inclusion of the historical dimension into the investigation of emotions in mathematical thinking and learning, I contend, may help us to understand emotions not only as socially organized, or as bodily based, but also as historically structured and produced. The point is not, hence, to assert that we are emotional beings through and through. We are emotional beings, for sure. But the *kind* of emotional beings that we are can only be understood within the scope of cultural forms of subjectification that are available to us. Before going into the subject matter, I start with a brief overview of conceptions of emotions, hoping that the overview may provide a background against which we might better understand the affective domain in mathematics teaching and learning.

2. The naturalistic approach to emotions

In his 1932 series of lectures on psychology delivered at the Leningrad Pedagogical Institute, Vygotsky complained that emotions had been conceptualized in biological and naturalist terms only. He lamented that the investigation of emotions was "completely dominated by a pure naturalism of a kind profoundly foreign to other domains of psychological investigation" (Vygotsky 1987, p.325). Darwin's (1886) famous book *The expression of the emotions in man and animals*, preceded by physiological investigations in France, England, Germany and other countries, paved the way to a conception of human emotions as remnants of our animal nature—vestiges of our irrational forces. Meticulous observations were made to ascertain the bodily modifications that animals and human undergo during emotional experiences. Changes of activity in the autonomic nervous system (e.g., perspiration, pupillary dilation, heart rate) were taken as "expressions" of our emotional life. At the end of the 19th century one of the central questions revolved around whether primacy was to be given to the 'bodily disturbances' or to the 'mental states' that occur in an emotional experience. In other words, the question was to determine whether emotion as a psychic state preceded its bodily expression or whether it was the other way around. For the idealist camp, a mental perception of a fact (a dangerous situation, for example) excites a mental affection (considered to be *the* emotion, in this case, fear), which leads to a bodily disturbance (e.g., an increase of the heart rate). For the physio-pragmatist camp, the feeling of the bodily disturbances resulting from an exciting fact (in our case, the feeling of heart rate) *is* the emotion. The latter was William James' (1884) famous position. According to James, "the bodily changes follow directly the *perception* of the exciting fact, and … our feeling of the same changes as they occur *is* the emotion" (James 1984, pp. 189-190; emphasis in the original).

However, several years later some psychologists and physiologists argued that bodily disturbances could not be equated with the sensed emotion. Thus, the works of Sherrington (1900) with dogs and Cannon, Lewis, and Britton (1927) with cats showed that the removal of the body parts where bodily changes reside in situations of anger, fear and rage—e.g., the sympathetic channels for nervous discharge in situations of profound excitement— does not affect the expected emotional states. Thus, the chirurgical-altered cats in Cannon et al.'s (1927) experiments behaved emotionally in the same way as intact cats when a dog approached their kittens or when food was taken away. These experiments suggested that emotional states might continue to be present even when the corresponding physiological support is missing.

One of the most significant contributions of Cannon's (1922, 1927), Cannon et al.'s (1927) and Sherrington's (1900) investigations was the distinction between *emotional feeling* (i.e., the uncontrolled and uncontrollably bodily changes, such as adrenaline production occurring during an emotional experience) and *emotion as such*. The psychic aspect of emotion is certainly intertwined with the physiological aspect, but one cannot be reduced to the other. Both together prepare us for action: physio-psychic emotion is not the end of the emotional phenomenon but the beginning of an action—*fight or flight, as* Cannon formulated it. "According to the argument here presented," Cannon wrote, "the strong emotions, as fear and anger, are rightly interpreted as the concomitants of bodily changes which may be of utmost service *in subsequent action*" (1922, p. 212; emphasis added).

Psychologists such as Lewin (1935) moved the conceptualization of emotions to new grounds by showing that human emotional phenomena is not of an instinctual nature, as in the case of animals, but is linked to the meaning of life: "one must not forget that in dealing with psychical processes one is dealing with life processes" (Lewin 1935, p. 63). On a commentary concerning Lewin's view Vygotsky wrote: "the structure of the individual's character is reflected in his emotional life and his character is defined by these emotional experiences" (Vygotsky 1987, p. 333). Emotional phenomena came hence to be seen not as merely transient experiences rooted in our biological apparatus (although without it no emotional life would be possible), but as something entrenched in the manner in which we understand ourselves in the world. This is the view conveyed by Charles Solomon (1978), who suggests that emotions are a **"**complex system of judgments, about the world, about other people, and about ourselves and our place in our world" (1978, p. 186). Judgments, however, do not refer here to assertive or declarative instances backed up by a logical-deductive apparatus. On the contrary, they mean rather appraising and gauging events involving self and context. As a result, emotions, as systems of judgments or appraisals are not merely declarative or assertive. Through them we do not merely say or state something about the world in a cold, logical way. Through emotions we speak out and relate to events, people, behavior, things, and actions. Emotions do not only drive our affective life; they also shape the manner in which we understand the world and ourselves (Roth 2007). Thus, rather than a crisis or worldly lived incident, emotions are focal points of a whole way of life. They rest on physiological processes, but cannot be reduced to them. They entail a range of cultural conceptual categories that are instantiated differently by different people (e.g. moral and ethical categories; notions of privacy, responsibility, autonomy, etc.).

The picture that emerges from the previous account can be summarized as follows. Emotions are not irrational forces; neither are they momentary incidents or disruptions in our everyday life. Emotions are part of a worldview that, through our participation in cultural and social activities, we come to share. Our emotional life is, in this sense, profoundly shaped by history and culture, although this does not mean that the parameters of what is to come in our emotional life is somehow injected into our being by a kind of mysterious syringe. Like cognition, emotions can only be understood through the interplay of history and the manner emotions develop in ontogeny. That is, emotions can only be understood through the incessant dialectical relationship of past and present and their projection into the future.

3. Emotions as cultural constructs

To better describe the sense in which I take emotions as cultural constructs, in this section I would like to make an excursus into one chief category of emotional life: love. Such a move should allow me to make my point clear when it will be time to see emotions in mathematical cognition. I should clarify, however, that this is so not because my forthcoming classroom analysis is going to be about love in mathematics; nor is it because there is a straightforward transfer from love to the joys and frustrations that students experience in learning mathematics. The complexity of human life makes it impossible to express its affective domain in terms of homomorphisms and transpositions. The reason of my excursus is to show that in the same way as love is a historically and culturally constituted emotion, so are the alienating or fulfilling emotion students experience in dealing with mathematics. Yet, it is not randomly that I have chosen love as the terrain of my excursus. Love is usually conceptualized as the most intimate conceivably repository of individuality. And although this might be true in Western cultures, in particular since the Romantic movement, it is so to the extent that the manners in which we consider love and practice it are embedded in a concept of subjectivity and individuality that is cultural through and through. Once this point is realized I hope that there will be room to envision emotions in mathematics teaching-and-learning not as mere idiosyncratic features of individuality but as culturally and historically constituted dimensions of the self. There is no doubt that, in walking along this path, I am diving in controversial waters. When the late anthropologist Clifford Geertz argued that passions of the Bali in Indonesia are culturally shaped, he was received with skepticism. Thus, in his review of Geertz' (1980) book *Negara*, Edmund Leach asserted that

> I can make no sense of a line of thought which claims that "passions" are culturally defined. From my prejudiced position as a social anthropologist this passage reveals with startling clarity the ultimately radical weakness of the basic assumption of cultural anthropology, namely, that . . . human individuals are products of their culture rather than of their genetic predisposition. (Leach 1981, p. 32)

Let us see, then, in what sense love appears as a cultural construct.

Although it has been argued that love is a part of our instinctual kit and that its function is to ensure the species survival, the manner in which love occurs between two adults and how it is felt is not an invariable concept. This point was already recognized by the Andalusian philosopher Ibn Hazm, author of a treatise on love written around 1022 (*Ring of Dove*) where he admits that love for the Bedouins and for the ancients meant two different things (Hazm 1022; Preface). The same can be said of love in the Western Middle Ages and today.

The Medieval concept of love and the manner in which it was felt was mediated by the social-economic structures of the time. These structures, along with cultural aesthetic concepts (such as "beautiful figure"), social ideas of good human personality (like "excellence of character"), and a praised role of language (referred to as "extreme readiness of speech") structured the space within which love was sought, practiced, and felt. In a famous book written ca. 1184 Andreas Capellanus explains how the aesthetic

elements, the worthiness of character, and mastering of speech should be advantageously put in motion in obtaining love. These components were articulated differently depending on the social range of the individuals. Thus, Capellanus gives a series of examples: one deals with the case where the man and the woman are both plebeians (or commoners); another example deals with the case where the man is a plebeian and the woman is part of the nobility, etc. Love in each case was conveyed as an intense feeling (an "inborn suffering," as Capellanus put it 1960, p. 28) modulated by aesthetic and ethical concepts such as modesty, loyalty, commitment, and generosity. Intimacy and fulfillment as we know them now were not part of love in the Middle Ages. As Ratner puts it (2000, p. 12), "Personal idiosyncrasies were not cultivated during feudalism and they played no part in evoking romantic love." Instead of personal idiosyncrasies, lovers attended to questions of family social position, along with humility, beauty, and dedication. Love consisted in the contemplation of the soul, and the sentiments of the heart. And if it involved a kiss on the mouth and physical contact, it was in a very chaste and modest manner (de la Croix 2013). This is why "Love," Ratner says, "was a spiritual, almost religious, sentiment that sublimated the base instincts and elevated the soul through dedication to one's loved one. One was a better person through caring for (serving) another. Love was thus a moral act" (Ratner 2000, p. 12).

At the dawn of the 20th century, the manufacturing forms of production that emerged progressively since the Renaissance reached an unprecedented level of industrial sophistication. This evolution of the forms of production came hand in hand with a range of new divisions of labour out of which new understandings of the self and concepts of others as well as how individuals relate to each other came to be envisioned. Within these societal transformations in the forms of production and modes of interaction, love, as a specific form of human relationship, found itself transformed.

The modern concept of love required indeed a specific concept of self—one that was defined in individualistic and private terms and which came in tandem with a new ethics of consumerism. Sociologist Eva Illouz notes that "The rise of consumerism coincided with the period between the two world wars when the self became both locus and focus of culture… In the new ethos, individuals were encouraged to express themselves 'creatively' and 'authentically'" (Illouz 1997, p. 35). In his studies about love, William Leach connects the emergence of modern romantic love to "the emergence of economic individualism" and goes on to say that the "romantic lover resembled his economic counterpart the risk-taking entrepreneur" (1980, p. 106).

According to Illouz, the transformation of the concept of love was characterized, among other things, by

> the increasing prominence of the theme of love in mass culture, especially in film and advertising; the glorification of the theme of love as a supreme value and the equation of love with happiness; the association of love and consumption, more specifically, the romanticization of commodities; the inclusion of "intensity" and "fun" in the new definitions of romance, marriage, and domesticity. (Illouz 1997, p. 28)

As in the Middle Ages, loved was shaped by the economical dimension of society. But rather than being refracted through a social hierarchy of church officials,

nobles, bourgeois, and commoners, love was refracted along the lines of advanced capitalism and its ethos. Thus, instead of being mediated by "excellence of character" and an "extreme readiness of speech" and the ethical categories that made it a moral act in the Middle Ages, love came to be mediated by the expanding industry of commodities. This is what Illouz calls the "commodification of romance" (1997, p. 11). Some signs of consumerist love are: dancing, eating dinner and drinking cocktails at expensive and luxurious places, travelling, vacations, and movie-going. The movie theater, the dance, and the candle-lit dinner became signs of a new intimacy that was made possible by the circulation of capital and the expansion of the working class. While the Middle Ages' love themes revolved around the value and practice of humility, commitment, and praise, the seductive themes of love during the first quarter of the 20th century evolved from the Victorian morality of domesticity to a plethora of consumerist notions such as exoticism, expenditure, speed, adventure, intensity and the physical care of the self. Instead of the intense ecstatic longing feeling of the Middle Ages courtly love, modern love, in short, appears stimulated by spontaneous and hedonistic desire for commodities (Ratner 2000) and based on an "experience of intense feeling, uninhibited sensuality, instant gratification, spontaneous pleasure, [and] fun" (Illouz 1997, p. 88). And as in the case of all emotions, love is learned through socialization. In the case of contemporary love, much of its socialization is done through mass culture, which provides adolescents with cognitive responses of romantic mannerism, behavior and skills. In a study conducted in the early 1930s by Herbert Blumer on what adolescents learn from movies, one of the respondents—a 21 year-old male— answered:

> The technique of making love to a girl received a considerable amount of my attention, and it was directly through the movies that I learned to kiss a girl on her ears, and cheeks, as well as on the mouth (Blumer 1933, p. 47).

Love as cultural phenomena means hence that the biological arousal that is at emotion's origin evolves into a psychic emotion that goes beyond the biological realm. Elicited by a concrete element in the world (the direct sight of beauty or its mental evocation), the physiological phenomenon —i.e., the *emotional feeling* or uncontrollably bodily changes that Capellanus (1960, p. 28) candidly referred to as the "inborn suffering derived from the sight of and excessive meditation upon the beauty of the opposite sex"—occurs in a world of cultural significations where it comes to be appraised, labeled, and sensed variously ("longing" and "devotion" as in the Middle Ages, "rational friendship" in the Victorian era, the passionate and sensual in the capitalistic culture).

By unavoidably occurring in the world of sociocultural significations in the form of judgments and appraisals (Solomon 1978), emotions entail a moral and ethical dimension. Anger, for instance, involves more than the production of adrenaline, or a neuronal circuitry in the lateral orbitofrontal cortex; it involves moral categories (e.g., offense and transgression), and concepts of the self[1].

[1] The same can be said about guilt. Murphy's studies suggest that spread of guilt in Africa during the first half of 20th century was often associated with a new concept of self as promoted by Protestantism and proto-capitalist forms of production. Within this societal transformation led by new entrepreneurial activities, individuals came to conceive of themselves as planners and masters of their own actions. Unfolding under the presence of an "omniscient God who can read one's thoughts" (Murphy, 1978, p. 237),

4. Emotions and thinking

In the previous sections I have advocated for a cultural-historical concept of emotions according to which emotions are historically constituted. They are part of the forms of subjectivity that cultures foster. This is why emotions cannot be understood without taking into account the processes of subjectification through which we enter cultural life and come to instantiate the raw forms of being that are culturally available to us at a certain point of culture's development (Radford 2013a). It is indeed within the scope of the various Medieval, Victorian, and capitalistic cultural forms of subjectivity that, in my examples, love is practiced and felt.

Since the self is emotional through and through, it is not surprising that thinking is rooted in emotions too. Yet, a precise functional description of the relationship between thinking and emotions has proved difficult to articulate. Ratner (2000, p. 6) suggests that "Emotions are feelings that accompany thinking. They are the feeling side of thoughts; thought-filled feelings; thoughtful feelings." In a commentary on Ratner's position, however, Menon complains that thoughtful feelings may fail to recognize the embodied dimension of affective life:

> While Ratner succeeds in emphasizing the irrelevance of biological processes to emotional experiences, he goes a little too far, perhaps, when he ignores the body and somatic experiences in his discussion. There is passing mention about 'bodily concomitants' (p. 19) but little more. In my view, it is very necessary to explicitly recognize the body in emotional experiencing, because such experiences are grounded in the reality of the bodily self—although I would not go so far as to claim that emotions can be identified with particular feelings. (Menon 2000, p. 43)

Menon finds missing a clear reference to the somatic correlates of emotions and turns to the work of Rosaldo, who has suggested that emotions are "embodied thoughts" (Rosaldo 1983, p. 143) an idea that conveys unequivocally the fact that "emotions are grounded and experienced in our bodily selves" (Menon 2000, p. 44).

Now, if thought is inherently embodied (Radford 2013b; Shusterman 2012; Varela et al. 1991) and emotions are more than physiological processes, what is then the difference between emotion and thought? Perhaps what we should bear in mind is the fact that there is no dividing line between thought, body, and emotion. To refer to emotions as embodied thought is redundant. Our thinking is *necessarily* embodied *and* emotional. During a match, chess players may seem to be exclusively cogitating before a move. Yet, the cogitation is highly emotional. The tensed and sustained gaze at the chessboard and the muscular tension in the otherwise immobile sitting body are two expressions of the ongoing intense emotional and bodily phenomena. Intermingled with rational calculations and logical thinking are the emotions that underpin chess players' activity. Only computers can "think" without feeling anything. They do not even feel the heat of their chips. They feel nothing. They display pure mechanical calculations of which humans are

---

individuals conceptualized themselves as responsible for their actions, as opposed to a former worldview where actions were understood more in collective terms and events attributed to the collective, chance, bad luck, or witchcraft.

definitely incapable. We can make some calculations, and we can do it while feeling boredom, thrill, excitement, challenge or something else; what we cannot do is simply feel nothing.

5. Emotions and motives

The brief overview of emotions carried out in the first part of this chapter shows that emotions were initially investigated through their *expressive form*. The problem was to understand what happens when we feel something—e.g., anger, fear, or rage. Leont'ev suggested that emotions should rather be investigated in terms of the psychological organizing role that they play in activity, a role that he conceived in terms of "inner signals," and their relationship with the individuals' motives:

> Emotions have the function of inner signals; that is, they do not directly represent the psychological reflection of object-oriented activity. The special feature of emotions is that they reflect relationships between motives (needs) and success, or the possibility of success, of realizing the action of the subject that responds to these motives. (Leont'ev, cited in Holodynski 2013, p. 8)

Emotions are hence related to motives in a time-projection manner: they relate to the *possibility* to succeed (or to fail) to reach the object of the activity. We have to bear in mind here that, for Leont'ev, what characterizes an activity is its object- and motive-orientation (Roth and Radford 2011). Working within the more general 'production paradigm' (Markus 1982) of his time, he conceived of activity as something that is driven towards a result—an outcome. This outcome has to produce something objective, tangible: the product of activity (which can be material or ideal). But activity is not merely a mechanical or technical production of things. Activity has to include the human dimension that Leont'ev captures through the concept of *motive*. This is why "The concept of activity is necessarily bound up with the concept of motive. There is no such thing as activity without a motive" (Leont'ev 2009, p. 6).

However, the concept of motive as theorized by Leont'ev is not easy to formulate. The concept appears at two different levels: the level of activity (where it appears as the activity's motive) and the level of the individual (where it appears as the individual's various motives). To make the distinction, Leont'ev presents the example of hunters, who labour together in order to satisfy their common needs. In this example, there is a perfect match between the motive of the activity and the individuals' motives. However, this coincidence of subjective motives (the individuals' motives) and the motives of activity is rather the exception:

> At the early stages, when people participating in collective labour still have common motives, meanings as phenomena of social consciousness and as phenomena of individual consciousness directly correspond to one another. But this relationship does not endure in further development. (Leont'ev 2009, p. 20)

This non-coincidence between the individuals' motives and the activity's motive is, however, often the rule in classroom activity. What we have there is indeed often a plethora of different motives that may seem to threaten even the possibility of joint activity to occur.

As mentioned previously, Leont'ev resorted to motive and object as the two main vectors of activity. What I want to propose is to see them not as fixed entities but as dynamic and evolving ones. Thus, instead of considering activity as something that has to end up with the materialization of the object in the activity's outcome (which is what Engeström (1987) emphasizes, ending up in a functionalist conception of activity), I suggest that we see activity as an *open system*, driven by an *evolving object* and a *developing web* of interconnected and sometimes *contradictory motives*.

The couple object-motive thus becomes the drive that moves activity and its sentient individuals not towards something to be attained, but rather towards a participation in a cultural way of life and the fulfillment of material and spiritual needs.

Leont'ev did not theorize activity in exactly the way I am suggesting. However, my proposal is not alien to Leont'ev's perspective, as it can be seen in the following passage, where Leont'ev talks about activity's general structure. He says:

> Historically, man's activity does not change its general structure, its "macrostructure". At every stage of historical development it is realised by conscious actions in which goals become objective products, and obeys the motives by which it was stimulated. What does change radically is the character of the relationships that connect the goals and motives of activity. These relationships are psychologically decisive. The point is that for the subject himself the comprehension and achievement of concrete goals, his mastering of certain modes and operations of action is a way of asserting, fulfilling his life, satisfying and developing his material and spiritual needs, which are reified and transformed in the motives of his activity. It makes no difference whether the subject is conscious or un-conscious of his motives, whether they declare their existence in the form of interest, desire or passion. (Leont'ev 2009, pp. 21-22)

To recap, from a cultural-historical perspective, emotions are both subjective and cultural phenomena simultaneously; they are entrenched in physiological processes and conceptual and ethical categories through which individuals perceive, understand, reflect, and act in the world. Their subjective-social link is to be found in the double-faced nature of motives, which are always personal and cultural.

Let me turn now to my classroom example to see how emotions unfold in activity.

6. I hate to give my answers: Frustration, exasperation and disappointment

In the rest of the article, I would like to discuss some passages from a lesson in a Grade 4 class (9–10-year-old students). The class is part of a 3-year longitudinal study. The lesson reported here takes place during the third year. I focus on the work of a group of three students: Jay, Thom, and Laura. Jay and Thom have been involved since year 1 in the study. Laura, by contrast, joined the class the third year and was hence new.

The lesson was about sequence generalization and started with a general discussion of how to continue a sequence of numbers. Before working on the problem, the teacher discussed with the students the meaning of group work. To understand the importance of the teacher's emphasis on group work the reader needs to bear in mind that, within the theory of objectification, learning is not conceptualized as a mere acquaintance with cultural forms of thinking (in this case, algebraic thinking). Learning is

not only about knowing but also about becoming (Radford 2008a). As a result, the design of the classroom activities (which is made by the teachers and our research team) involves both a thorough design of problems of increasing difficulty whose organization requires the mobilization of the target mathematical concepts in depth, as well as the constitution of meaningful spaces of social interactions where students are encouraged to attend to other voices and ideas, to collaborate with others, and to show support and solidarity (Radford 2012, 2013b).

These mathematical and ethical dimensions provide distinctive basic elements for particular forms of subjectification to occur (e.g., forms based on responsible understanding and solidarity). Although these processes of subjectification cannot be anticipated or predicted beforehand, the elements highlight conceptual and ethical features out of which kinds of plausible intersubjective theoretical-emotional experiences may occur.

The aforementioned mathematical and ethical dimensions have been the driving vectors of our 3-year program. The third year, the teacher summarized with the students what they have been practicing the years before. Collectively, the students and the teacher discussed the introductory problem on the white board; they talked about the meaning of mathematical concepts required in the task (e.g., the regularity in a sequence), and the meaning of group work. The teacher wrote on the white board the students' responses, which included: "ask for help," "listen to the others' ideas," "encourage others," "do not get frustrated."

After collectively solving the introductory problem, the students worked on other generalization problems. The third problem revolved around the sequence of numbers indicated in dark in the following table:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The students were invited to find out the next three terms, and then the following three terms.

Jay and Thom engaged in an exploration of the sequence, counting on Jay's page the spaces between the dark cells, and exchanging ideas. Laura worked on her own:

1. Jay: (*Starting from cell 1, he points rhythmically to the following cells with his pen*) 1, 2, 3, 4.
2. Thom : (*Who is following Jay's utterances and gestures says at the same time as Jay*) 4 (*short pause*).
3. Jay: (*Pointing to cell 5*) 1.

4. Thom: (*At the same time as Jay and with the same intonation as Jay's says*) 1.
5. Jay: Wait, 1, 2, (*Thom starts counting with him*) 3, 4, 5 (arriving at cell 9).
6. Laura: (*Who has made no eye or other contact with her teammates says lowly without leaving the eyes from her page and if talking to herself*) yeah, 1, 2, 3.
7. Jay: (*Pointing at cell 10 and continuing uttering in a synchronized manner with Thom*) 1, 2, 3, 4, 5, 6, 7 (*arriving at cell 16*).

During their work Thom and Jay show an emotional tension that results from the search for a regularity that they cannot yet grasp. This tension is reflected in the sensuous counting of squares and the tremendous attention that they have to pay to carry out their actions. Jay's utterance "wait" (line 5) interrupts the flow of the counting process: it marks a moment of hesitation that is overcome and reassumed right after with some assurance. The synchronic work of Thom and Jay creates a feeling of closeness and unity that may colour the tension positively. This feeling of closeness in which utterance and gesture are coordinated is highlighted even further by the aural proximity of voice tonality. Laura remains outside of the synergy that is created between her teammates (see Figure 1). She looks determined and focused.

Figure 1. Thom (to the left) moves towards Jay (middle) and accompanies Jay's counting visually and verbally in a synchrony that relieves the tension of solving a problem with an uncertain outcome. Laura (to the right) works on her own, without making contact with her teammates.

The teacher came to see the students work. Thom and Jay engaged with the teacher in a process of objectification out of which they started noticing that they had to move 3 cells to the next dark cell, then they had to move 5 cells, 7 cells, etc. Laura continued to work alone:

8. Teacher : Can you start seeing the sequence?
9. Jay : They add 2? (*With some uncertainty*).
10. Teacher : Ah! Two are added each time!
11. Thom : (*Thrilled*) Oh!
12. Teacher : Can you explain it to your teammates?

13. Laura : (*As if referring to something trivial*) No, I know already.

The teacher went to see another group. The students continued their work. Adding successively 15, 17 and 19, Jay and Thomas found that the next terms of the sequence were 64, 81 and 100. Adding successively 15, 18 and 19, Laura found that the next terms were 64, 82 and 101. Jay and Thomas were surprised by the difference:

14. Thom: Laura, can you explain to us what is your idea?
15. Jay: Why is it 101?
16. Laura: (*Referring to the difference between terms, she says*) Well it's because right now I calculated 11 and when I continued I found … um… 13 and then 15 and then 18.
17. Jay: Yes, but we don't understand…
18. Laura: Look, (*pointing to the cells with the pen, confidently she starts counting from cell 82; she makes a mistake and counts 82; Thom counts with her*) (*pointing to cell 82*) 1, 2 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 (*She doesn't arrive to 101, but to 100; she seems hesitant*) (pause) I mean 18 (pointing to cell 99).
19. Thom: 19!
20. Laura: (*Makes a gesture in the air*) Oh! (*She rotates the sheet to put it in front of her and tries to understand*) Wait, I think I put…
21. Jay: It's 19
22. Laura: (*Looking attentively to her page*) Wait, wait.
23. Thom: It's 19, because it went…
24. Laura: (*She is scrutinizing the first cells and doesn't want to listen; the tension increases; she makes a "waiting" gesture and says*) Wait, wait (*see Figure 2, Pic 1*).
25. Jay: You put 18.
26. Laura: (*In an apologizing tone and passing her pen over cell 100*) I think I forgot by accident to write 100. (*She starts counting 15 from cell 49; then, she counts 17 from cell 64; Thom follows the counting without talking; he replaces utterances with a sequence of short noddings; Jay follows the counting from his post*) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 (*arriving at cell 81 and not to the expected 82 that she had marked on her sheet. She tries to make sense of the unexpected outcome*).
27. Thom: (*Wanting to help, he says*) Plus, 17 plus 2 equals
28. Laura: (*Pointing to cell 82*) 18 (*pause*). (*Talking rather to herself*) I think I made a mistake in my own work maybe (*she crosses out cell 81, still thinking that the right answer is 82. Disappointed, she hits the table with her pen*).

29. Thom: (*Noticing that she has crossed out 81*) No, it's 81!
30. Jay: Yes, its 81.
31. Thom: Yeah, it's 81.
32. Laura: (*With emphasis and dismay*) Oh greeeeat! (*long exhalation; she holds her head with one hand for a while; then with the arms extended in front of her, she says*) This is why I hate to say what I do . . . Ugh!! (*During 4 seconds she crosses out with intense circular motion square 82; see Figure 2, Pics 2 and 3*) Mmmgh! Mmmgh! (pause) (*Talking to her teammates)* That's why I HATE (she pronounces the word slowly and louder) to give my answers (s*he corrects her mistake on her page*) (pause) (*with frustration*) Mmgh… Mmgh… See? (*With great disappointment*) Cause I get [it] wrong…
33. Thom: (*Talking to Laura in an encouraging tone, and pointing to the 100th cell*) The answer is 100.
34. Laura: [One hundred] and one *(she insists she has the right answer although having noticed she made a mistake in her calculations*)

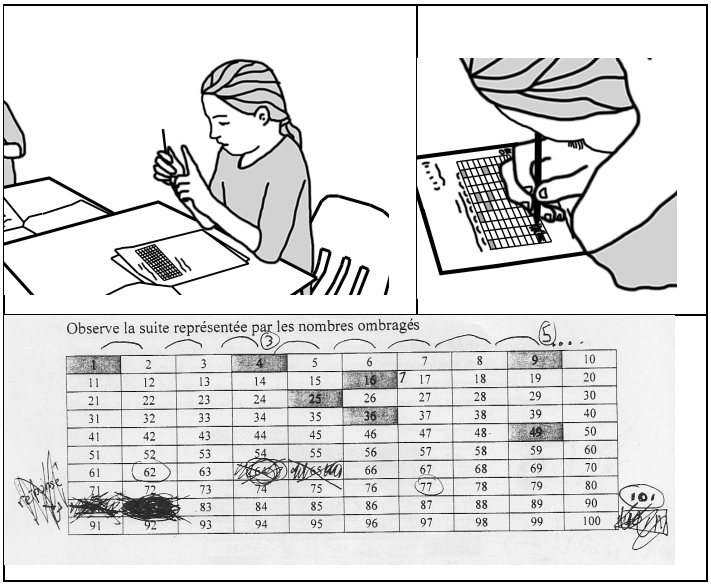

Figure 2. In pic 1 (top lef) Laura asks Thom to wait. In pic 2 (top right) she scratches number 82 during 4 seconds. Pic 3 (bottom) shows her activity sheet.

The episode starts with Thom and Jay inviting Laura to explain her result. The boys confess to not having understood her short explanation. Laura slightly rotates the page towards Jay and starts counting from cell 82, although she makes a mistake and includes 82 in the counting process; Thom and Jay watch her count attentively. Thom joins her while she is counting 13 and both count together the rest of the cells. To her dismay, she does not arrive at the expected cell 101. She hesitates and, in turn 20, with irritation, she moves the arms in front of her. Something went wrong and she still does not understand what or why. She has two options. She may try to get some feedback from

Thom and Jay, who have proven collaborative and willing to help, or she can try to sort out the problem by herself. She opts for the second option. When Thom volunteers an explanation, she asks him to wait. She is thinking in a very effortful way. Her body becomes rigid and tense (see Figure 2, pic 1). In turn 26, she concedes that she might have forgotten to pen cell 100. It is not a mistake. It is an accident, she says. She might not believe the reason she has offered to her teammates, as she starts counting again. The fact that she starts counting from cells 49 and 64 may suggest that she is now unsure of the correctness of her procedure. She needs to check it. Thom, who has been asked to wait, decides not to count aloud with her, but follows her counting with a series of rhythmic short nods of his head. Things become even more complicated as she lands on cell 81, and not 82 that she has marked on her sheet. She shows her disappointment by hitting the desk with the pen. In line 28, although she acknowledges the possibility that she has made an error, she eliminates 81, to the dismay of Jay and Thom, who hurry up to exclaim that 81 is good. She loses control and things escalate. She utters "great" in a heavily pronounced manner, showing confusion and deep frustration. In general, frustration refers to a sense of dissatisfaction associated with difficulties of encounter. In this case, frustration appears around the conceptual dilemma of whether the good cell is 81 or 82. Laura spends 4 seconds (which is a huge amount of time in the context) scratching cell 82 and voicing her frustration through a sequence of verbal "Mmgh" lamenting sounds. Thom tries to alleviate the tension, talking no longer about cell 81, and says "The answer is 100," while she still insists that the answer is 101.

Laura's and the other students' unfolding emotional experience is a key component of the process of subjectification they are all immersed in. A process of subjectification refers to the always evolving sense of the self that results from the manner we and others recognize and position ourselves socially. Students' sense of the self are to a large extent related to the manners in which they engage in activity and come to position themselves in cultural practices in the public space. Laura's positioning is mined by difficulties that she senses and interprets in ways that colour it rather negatively. She overcomes partially the frustration and starts counting again, although with less confidence. She starts counting with annoyance from cell 81 and, counting 18, she arrives at cell 99 and not to the expected cell 101. After reflecting for a moment, she restarts counting again, but instead of starting from cell 64, she starts from cell 65. This time she arrives at cell 81 and looks puzzled. The dialogue continues as follows:

35. Thom: The answer is 100, the three following numbers are 64…
36. Laura: (*Without listening to Thom, interrupting, she says with great distress*) Oh my god! (*She lifts her arms up*) I am, (*holding her head with her left hand and looking at the number*s) ugh...!
37. Thom : Because she has 64 right there.
38. Laura: (*With frustration*) I mixed myself up now!
39. Thom: (*Intervenes to try and help*) After
40. Laura: I mixed myself up (*still holding her head with her left hand, she hits repeatedly the desk with the pen in her right hand*).

41. Thom: (*Trying to help, he points to the numbers on Laura's sheet*) 81, and after 100.
42. Laura : I'm all messed up now!
43. Thom: Therefore
44. Laura: Maybe I made a mistake…
45. Jay: It's alright Laura, everyone makes mistakes
46. Laura: I'm all messed up now! (*She still holds her head with her left hand; see Figure 3*).
47. Thom: It's true.
48. Laura: I always lose my memory. What if I say more than 10?
49. Jay: (*Trying to help*) Laura, just, just do that, scratch that out (*suggesting to cross out 101*).

Figure 3. Laura deeply discouraged.

7. Madam, now I'm too mixed up…!

A few minutes later the teacher came to see the students work. The teacher hears the students' explanation and engages in a counting process with them. She counts the cells between the numbers of the sequence, starting from the first two terms (i.e., 1 and 4). She notices that Laura is working on her own and wants to include her in the discussion:

50. Teacher: (*Counting on Jay's page*) 1, 2, 3 (*Talking to Laura*) Laura do you agree with that?
51. Laura: 12, 13, 14, 15 (*Continues counting on her page; she lifts her finger up to signify "wait" as in Figure 2, pic 1*).
52. Teacher: (*In an inviting tone*) Count with me…
53. Jay: (*Laura continues working on her own; Jay and Thom count at the same time, while the teacher points to the cells on Jay's page*) 1, 2, 3, 4, 5
54. Teacher: And after that?
55. Jay: (*At the same time as Thom*) 1, 2, 3, 4, 5, 6, 7...
56. Laura: All my work is mixed up! […]

57. Teacher: Ok (*She use Laura's page to point to the cells*) So, Laura, did you arrive at 9 here?

58. Laura: Now I'm mixed up…!

59. Teacher: (*In an inviting tone*) Continue to count.

60. Laura: Madam, now I'm too mixed up!

61. Teacher: How many, what number should we have afterwards? After 9 we should have how many? We count up to what?

62. Laura: Madam.

63. Thom: 11 (*Laura makes a gesture of discouragement as she does not understand*).

64. Teacher: Ah! 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (*the teacher points to the cells on Laura's page; Laura watches her point and count*) Did we arrive at the correct number?

65. Thom: Yes.

66. Teacher: Ah, the next number that I have to count is how much?

67. Laura: Agh... (*Her body falls to the back of the chair, demonstrating a great confusion and frustration*)

68. Thom: 13!

69. Teacher: Why 13?

70. Thom: Because plus 2 […]

71. Teacher: Do you see Laura?

72. Laura: (*She makes a gesture of discouragement; see Figure 4*, *Pic1*) Madam now (*the upper part of her body falls down slowly towards the desk, Pics 2-4 and, crying, she says*) I scribbled on my page!

73. Teacher: Ok Laura, can we go out [of the classroom] for a minute?

Figure 4. Laura shows frustration. At the end she cries.

The efforts that the teacher made to include Laura in the discussion did not pay off. It would be a mistake, however, to think that Laura's emotional dimension has clouded her judgment and impeded her from thinking rationally. From the cultural-historical perspective here sketched, emotions, as we pointed out in previous sections, are always intertwined with thinking. Emotions, I suggested, are rather entrenched in physiological processes and historical conceptual and ethical categories through which individuals perceive, understand, reflect, and act in the world. In other words, it is not because Laura became emotional that she failed to think and calculate in an appropriate way[2]. Although unpredictable in its details, the emotional-cognitive process that she underwent unfolded shaped by the manner in which she perceives herself in her relationship to knowledge and to others. In the same way as love is practiced and felt culturally, so is the manner in which we experience and practice learning. And in the same way that love is differently instantiated by different lovers from the same culture, so is learning. What the previous excerpts intimate through our interpretative stance is that, drawing on cultural models of being (here modes of learning and learners), Jay and Thom position themselves differently from the manner in which does Laura. This general positioning affords specific ways through which to emotionally interpret the world and our actions within it. As the next episode shows, such a positioning is underpinned by what the students understand of what is ethically expected from them.

8. The ethical dimension of emotions: Cheating

The students moved to the next part of the task: they were finding the three next terms of the sequence after term 100. The table stopped at cell 100 (see Figure 2, pic 3), so now the students were expected to move into more abstract mathematical relationships. Instead of counting, they were supposed to add numbers or to come up with a functional relationship: $n \rightarrow n^2$ or $n \rightarrow n \times n$ (although not necessarily expressing the relationship in the alphanumeric symbolism).

[2] This doesn't mean, however, as one of my reviewers notes, "that emotions have to be seen only as the end of a (cultural) process." Emotions are already there, with us; they evolve as we evolve into cultural subjects through subjectification processes, appearing —as Vygotsky suggested—in new relations with other elements of our whole life.

74. Thom: (*Laura is mentally calculating; she is whispering numbers to herself*) Laura would you like to see our copy, (*Laura lifts her finger in her classical "wait" gesture*) we're going to explain to you what we have to do now. Jay, are we going to explain it to her now?

75. Jay: Yes!

76. Thom: Ok.

77. Jay: Laura?

78. Laura: (*Thinking with great concentration*) Wait! I'm right there, you're gonna mess up my calculations!

79. Thom: Oh I am going to do the same thing as you.

80. Laura: (*Loosing her train of thought*) Ok! you just messed them up! Ughhh! I was *so* close! I keep messing my stuff up, I'm trying to think, but you messed it up! (*Looks at Thom accusingly*). Ughh! (*She sighs and whispers something*) I tried to do my copy, I know, I know the answer; it's just I'm trying to get there, I know the answer.

81. Thom: Would you like us to explain to you how?

82. Laura: I don't want to… like cheat, Ok, I don't want to cheat.

83. Jay: That's not, that's not cheating! We're a group!

84. Laura: I know but still…

85. Thom: Yeah we work together.

86. Laura: I just want to, I just want to

87. Thom: If they… (*Points to another group*), that would be cheating from them, but if you look at our copy that would not be cheating because we're working together.

88. Jay: Because that's together.

89. Laura: I know, but I don't know.

90. Jay: Would you like some help?

91. Laura: I was so close to knowing what the answer was!

92. Jay: Would you like some help though?

Laura conceives of learning as something that she has to do by herself, despite the fact that the class discussed the meaning of group work before embarking in the mathematical content. She certainly heard the ideas about collaboration, understanding others, etc. But that does not mean that she felt concerned. In fact, to reach the cultural forms of being that we have been promoting in this class (see Radford 2012) and that Jay and Thom instantiated in the previous excerpts, requires a long and sustained endeavour. It is not sufficient to hear words or to utter them. The students have to engage in and practice intersubjective understanding, openness towards others, etc. These fostered

cultural forms of being at the heart of our didactic designs bring with them forms of ethical relationships and concomitant forms of sensing. They make students prone to *move* towards certain forms of actions rather than others. This is what emotion means etymologically. Emotion comes from the Latin *emovere*, that is "to move" or "to move out."

Within her conception of learning, Laura has tried systematically and honestly to answer the questions by herself. Her frustration, disappointment and other expressions of the emotional phenomena involved in the episodes appear now clearly comprehensible. So is her tireless refusal to get help from the students. To do otherwise would amount, according to her, to cheating.

The *Encyclopedia of Applied Psychology*, defines cheating as "any intentional action or behavior that violates the established rules governing the administration of a test or the completion of an assignment, [and] gives one student an unfair advantage over other students on a test or an assignment" (Cizek 2004, p. 308). The definition stresses the cultural censuring dimension through the legal governing apparatus of conduct and behaviour. Those regulatory devices frame Laura's motives, which clash however with those of Jay and Thom, moved (or "emotionned" if we continue using emotions in their etymological sense) by a Bakhtinian ethics of solidarity and intersubjective understanding (Radford 2008a, 2012)[3]. As Jay argues in turn 83, "that's not cheating! We're a group!" To explain the idea, in turn 87 Thom refers to another group. Cheating would be to look at the work of another group. But cheating cannot occur within the group as long as the group works together: "if you look at our copy that would not be cheating because we're working together." Emotions, as we can see, always reflect "phenomena, perceived and understood from the special point of view of the perspective of a person who is interested in them" (Zaporozhets 2002, p. 61). But emotions cannot be reduced to the panoramic view of the subject, as our analysis intimates. They are rather entrenched in ethical and other cultural categories through which emotions become personal and cultural at the same time.

9. Summary and concluding remarks

In mathematics education, McLeod's pioneering work has been very important to move the study of the affective domain from stable features of individuals (as in the case of beliefs and attitudes) to dynamic, contextual processes. McLeod's (1989, 1992) tremendous insight, however, remains bounded by the inherent limitations of Mandler's (1984, 1989) cognitive conception of emotions that influenced his views. According to Mandler emotions arise out of interruptions of plans that we carry out. Mandler's view is based on the idea that emotional behavior rests on two systems: arousal and meaning analysis. While the first is cast in behaviorist terms and the idea of stimulus, the second is formulated within the traditional rationalist framework that assumes a lonely individual coping with an ahistorical surrounding through schemas and representations. If Mandler's subject is formulated as *emotional + cognitive*, emotion is formulated as *arousal + meaning*. In the end, the account remains quite behaviorist. It is not surprising that, in

[3] The Bakhtinian character of the ethics that we foster rests indeed in the primacy of the Other (or Otherness or alterity) in our ways of being. This is why, for Bakhtin as for us, consciousness is always dialogical and intersubjective (see, e.g., Bakhtin, 1981, 1990; Radford, 2008b

drawing from Mandler's work, McLeod (1989, 1992) ended up picturing the affective domain as *repeated* experiences that depend on the magnitude (or intensity), direction (positive or negative), duration, and control of emotions.

The cultural-historical conceptualization of emotions that I have sketched here draws on previous research, but departs from it in several aspects. Within the cultural-historical conception of emotions that I articulated, emotions are not considered irrational forces or mere disruptions in our everyday life. Emotions are part of a worldview that, through our participation in cultural and social activities, we come to share. Emotions comprise a physiological component but cannot be reduced to it. They are shaped by conceptual and ethical cultural categories out of which we define our stance towards the world, and how we relate to people and events. To illustrate this idea, drawing on the work of Illouz (1997), Ratner (2000), and W. Leach (1980), I discussed the example of love (allegedly the most intimate and personal of our emotional life) and attempted to show that what is expressed through the term 'love' is culturally situated and produced. I contrasted the medieval ideas and feelings about love to the modern consumerist counterpart and tried to show that love is mediated by cultural conceptual and ethical categories.

The second part of the article was an effort to show how emotions are implicated in mathematical thinking. My argument is that it is misleading to believe that emotions obstruct thinking. Emotions and thinking are not separate entities. They are fused together. We cannot think without emotions. Emotions and thought come to constitute a unity in ontogenetic development. In the course of social life, emotions develop and "appear in new relations with other elements of mental life" (Vygotsky 1999, p. 244). They become related in particular to the students' motives, regardless of how they are expressed—e.g., "in the form of interest, desire or passion" (Leont'ev 2009).

In the classroom episodes here discussed, two contrasting forms of motives drove the students' actions. In the case of Laura, motives were cast in terms of an ethics of auto-sufficiency, where individuals come to conceive of themselves in terms of the origin of meaning, cognition, and intentionality. This conception of ethics is not spontaneous: it is cultural and has its specific history (Radford, 2012). It paves the way to emotionally engage in activity in certain ways. In this case, Laura felt compelled to work alone. She considered that attending to what the other students are doing is cheating. All that she felt during the episodes—the irritations, disappointments, frustrations, vexations— was felt in tune with her understanding of her own role and her role vis-à-vis others. Thom and Jay's motives, by contrast, were cast in terms of a "communal ethics" that promotes participation in the public space, openness, solidarity, a sense of belonging, and critical awareness (Radford 2012). Like the auto-sufficient ethics that underpins Laura's actions, this concept of ethics is not spontaneous. It has also its own history. Thom's and Jay's continuous attempts to connect with Laura were bounded by such an ethical project. Differences in the cultural ethical stances and the ensuing outlook of the world, people, and events, offer the raw material out of which thinking and their concomitant emotions unfold in activity-bound processes of subjectification. It is in this sense that I hope to have shown that cultures fill, infuse, and permeate our emotional life.

Taken together, the historical example and the classroom episodes remind us that in the same way as lovers and love are socio-cultural constructs, so are the students and

what they feel and sense when learning. In the Middle Ages, cultural ideas of love and lovers were conveyed by the songs of troubadours, by literature (written and oral), and by other media. Contemporary cultural ideas of learning and learners are conveyed by schools and other social institutions, family, and mass culture. They provide the elements out of which conceptual-emotional experience unfolds within processes of subjectification. I am not intimating, however, that love and learning are produced in some causal manner. Causality has been the paradigm of the natural sciences since Aristotle and Galileo. Yet, the human psyche seems to escape to mechanical explanations epitomized by causal relations. The relationship between culture and their individuals is one of mutual constitution in a complex dialectical way. They are not separated entities glued together by a third term. They co-evolve together: they mediate each other. Yet, with its persistent emphasis on the pole of the individual and the concomitant subjective outlook of psychological and conceptual phenomena, our longstanding Western philosophical and psychological traditions have enduringly posited the individual as the source of intellectual and emotional life—even if from time to time acknowledgment of the cultural dimension is made, as in the case of McLeod, who suggests that "The role of the culture that shapes [our] beliefs would seem to be particularly important" (1992, p. 578). By sticking to the view of the subject as 'cognitive' *plus* 'emotional,' it becomes practically difficult to understand the formation and transformation of motives and emotions in its relationship to culture and history. The cultural-historical perspective that I have presented here tries to avoid this pitfall. It sees emotions as part of the processes of subjectification, processes out of which we position ourselves as cultural subjects in social and political practices. As Menon contends,

> To me, this appears to be the distinguishing feature of cultural psychology—the idea that culture and psyche cannot be smoothly and easily disentangled one from the other, and it is this premise that gives cultural psychology the theoretical power to achieve a dense understanding of a people's emotional reality. (Menon 2000, p. 45)

The historical and classroom episodes also suggest some elements that might be useful to take into account in the teaching and learning of mathematics. The first insight points to the theoretical-methodological premise that the study of individuals—what they do, how they think and feel—cannot be divorced from the sociocultural contexts in which they live and grow. In other terms, the individuals' cognitive, volitional, and emotional dimensions cannot be disentangled from these contexts, for these contexts are not merely "backgrounds" but rather constitutive elements of the human psyche. Emotions in particular cannot be understood if they are abstracted from these historical, cultural, contexts that shape the individuals' motives. Second, emotions are not natural kinds; emotions are historically and culturally constituted. What people sense about guilt, anger or love is not something invariable in time (chromos) and space (topos). Emotions are chronotopical. Cultures offer a range of emotional possibilities of action and reaction that individuals dialectically actualize or instantiate as they learn, since birth, to interact with others and to engage in material and embodied activity.

Acknowledgments:

This article is a result of a research programs funded by the Social Sciences and

Humanities Research Council of Canada (SSHRC/CRSH). I wish to thank the reviewers of this article for their generous comments, help, and advice.

References

Bakhtin M (1981) The dialogical imagination. University of Texas Press, Austin

Bakhtin M (1990) Art and answerability. University of Texas Press, Austin

Blumer H (1933) Movies and conduct. Arno Press and The New York Times, New York

Cannon W(1922) Bodily changes in pain, hunger, fear and rage. D. Appleton and Company, New York

Cannon W(1927) The James-Lange theory of emotions: A critical examination and an alternative theory. The American Journal of Psychology 39(1/4): 106-124

CannonW, Lewis J, Britton S (1927) The dispensability of the sympathetic division of the autonomic nervous system. Boston Medical and Surgery Journal 197: 514-522

Capellanus A (1960) The art of courtly love. Columbia University Press, New York (Original work published ca. 1184)

Cizek G (2004) Cheating in academia. In: Spielberger C (ed) The encyclopedia of applied psychology, vol 1. Elsevier, Oxford pp. 307-312

Darwin C (1886) The expression of the emotions in man and animals. D. Appleton and Company, New York

de la Croix A (2013) L'érotisme au moyen âge. Le corps, le désir, l'amour [Erotism in the Middle Ages. Body, desire, love]. Tallandier, Paris

Engeström Y (1987) Learning by expanding: An activity-theoretical approach to developmental research. Orienta-Konsultit Oy, Helsinki

Evans J (2006) Affect and emotion in mathematical thinking and learning. In: New mathematics education research and practice. Sense Publishers, Rotterdam, pp 233-256

Evans J, Zan R (2006) Sociocultural approaches to emotions in mathematics education: Initial comparisons. In: Novotná J, Moraová J, Hrátká M, Stehlíková, N (eds) Proceedings of the 30th conference of the international group for the psychology of mathematics education, vol 3. PME, Prague, pp 41-48

Furinghetti F, Pehkonen E (2002) Rethinking characterizations of beliefs. In: Leder GC, Pehkonen E, Tôrner G (eds) Beliefs: A Hidden Variable in Mathematics Education. The Netherlands: Kluwer, Dordrecht, pp 39 – 58

Geertz C (1980) Negara: The theatre state in 19th century Bali. NJ: Princeton University Press, Princeton

Goldin G (2002) Affect, meta-affect, and mathematical belief structures. In: Leder G, Pehkonen E, Tôrner G (eds) Beliefs: A Hidden Variable in Mathematics Education?. The Netherlands: Kluwer, Dordrecht, pp 59–72

Hannula M (2011) The structure and dynamics of affect in mathematical thinking and learning. In: Pytlak M, Rowland T, Swoboda E (eds) Proceedings of the seventh congress of the European society for research in mathematics education. Erme, Rzeszów, pp 34-60

Hazm, Ibn (ca. 1022) The ring of dove. A treatise on the art and practice of Arab love. Downloaded from http://www.muslimphilosophy.com/hazm/dove/index.html

Holodynski M (2013) The internalization theory of emotions: A cultural historical approach to the development of emotions. Mind, Culture, and Activity 20(1): 4-38

Illouz E (1997) Consuming the romantic utopia: Love and the cultural contradictions of capitalism. The University of California Press, London
James W(1884)What is an emotion. Mind 9(34): 188-205
Leach E (1981, April 4) The poetics of power. The New Republic 30-33
Leach W(1980) True love and perfect union. Basic Books, New York
Leont'ev [or Lentyev] AN (2009) Activity and consciousness. Pacifica, CA: MIA. Retrieved August 29, 2009, from http://www.marxists.org/archive/leontev/works/activity-consciousness.pdf.
Lewin K (1935) Dynamic theory of personality. McGraw-Hill, New York
Mandler G (1984) Mind and body: Psychology of emotions and stress. Norton, New York
Mandler G (1989) Affect and learning: Causes and consequences of emotional interactions. In: Mc Leod D, Adams V (eds) Affect and mathematical problem solving. Springer, New York, pp 3-19
Markus G (1982) Langage et production [Language and production]. Éditions Denoël /Gonthier, Paris
McLeod D (1989) The role of affect in mathematical problem solving. In: McLeod D, Adams VM (eds) Affect and mathematical problem solving. Springer, New York, pp 21-36
McLeod D (1992) Research on affect in mathematics education: A reconceptualisation. In: Grouws D (ed) Handbook of research on mathematics teaching and learning. Macmillan, New York, pp 575-596
Menon U (2000) Analyzing emotions as culturally constructed scripts. Culture & Psychology 6(1):40-50
Murphy H (1978) The advent of guilt feelings as a common depressive symptom: A historical comparison of two continents. Psychiatry 41:229-242
Radford L (2008a) The ethics of being and knowing: Towards a cultural theory of learning. In: Radford L, Schubring G, Seeger F (eds) Semiotics in mathematics education: Epistemology, history, classroom, and culture. Sense Publishers, Rotterdam, pp 215-234
Radford L (2008b) Di Sé e degli Altri: Riflessioni su un problema fondamentale dell'educazione [The Self and the Other: Reflections on a Fundamental Problem in Education]. La Matematica e la sua didattica 22(2):185-205
Radford L (2012) Education and the illusions of emancipation. Educational Studies in Mathematics 80(1): 101-118
Radford L (2013a) Three key concepts of the theory of objectification: Knowledge, knowing, and learning. Journal of Research in Mathematics Education 2 (1): 7-44
Radford L (2013b) Sensuous cognition. In: Martinovic D, Freiman V, Karadag Z (eds) Visual mathematics and cyberlearning. Springer, New York, pp 141-162
Ratner C (2000) A cultural-psychological analysis of emotions. Culture & Psychology 6(1): 5-39
Rosaldo M (1983) The shame of headhunters and the autonomy of the self. Ethos 11(3):135-151
Roth MW(2007) Motive, emotion, and identity at work: A contribution to third-generation cultural historical activity theory. Mind, Culture and Activity 14(1-2):40–63

Roth WM, Radford L (2011) A cultural historical perspective on teaching and learning. Sense Publishers, Rotterdam
Sherrington CS (1900) Experiments on the value of vascular and visceral factors for the genesis of emotions. Proceeding of the royal society 66:3-97
Shusterman R (2012) Thinking through the body. Essays in somaesthetics. Cambridge University Press, Cambridge
Solomon R (1978) Emotions and anthropology: The logic of emotional world views. Inquiry 2:189-199
Varela F, Thompson E, Rosch E (1991) The embodied mind. MIT, Cambridge
Vygotsky LS (1987) Collected works (vol. 1). Rieber RW, Carton AS (eds). Plenum, New York
Vygotsky LS (1999) Collected works (vol. 6). Rieber RW(ed). Plenum, New York
Zaporozhets AV (2002) Towards the question of the genesis, function, and structure of emotional processes in the child. Journal of Russian and East European Psychology 40(2): 45-66