History and Epistemology in Mathematics Education

Proceedings of the 5$^{\text{TH}}$ European Summer University

Publisher: Vydavatelský servis, Plzeň
Editors: Evelyne Barbin, Naďa Stehlíková, Constantinos Tzanakis, 2008
ISBN 978-80-86843-19-3

Prague
July 19–24, 2007

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Beyond Anecdote and Curiosity

The Relevance of the Historical Dimension in the 21ST Century Citizen's Mathematics Education

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1 Introduction

Making recourse to a historical dimension in the teaching and learning of mathematics raises, from the outset, some practical and theoretical questions. On the practical side, we find questions related to e.g. the design of historically inspired classroom activities. For instance, how can we use history in a practical way? These are the "how" questions. "Why" questions pertain to the theoretical side. Of course, "why" and "how" questions are interwoven, for practice is always mediated by theory and theory is blind without practice.

The previous comment should make clear that, according to the line of thought that I am following, there is no privileged starting point from which to address the questions of the historical dimension of the 21st Century Citizen's mathematics education. Both practical and theoretical questions are important. Since, in this panel, Frank Swetz will deal with some aspects of the "how" questions, in what follows, I will focus on the "whys".

2 Why resort to history in our modern teaching of mathematics?

In past years, this question has been answered in several ways. One of the answers is: because history is useful for motivating students and teachers. Since many students (and teachers!) find mathematics esoteric and a nuissance, history, in the form of mathematicians' biographies, can play a motivational role. I have drawn from e.g. Charraud's (1994) interesting book on Georg Cantor and Astruc's (1994) *Évariste Galois* to highlight the human and social aspects surrounding creative mathematical thinking. But history, I want to argue, is much more than a motivational tool.

Another answer is the following: we can resort to history because history provides us with a panorama that goes beyond the mere technicalities of contemporary mathematics. Discussing the history of certain problems may indeed be an interesting way to make students sensitive to the changing nature of mathematics, allowing one to emphasize, at the same time, the contributions of different cultures (Commission Inter-Irem, 1992; Noël, 1985; Beckmann, 1971; Delahaye, 1997; Maor, 1994). But again, history is much more than that.

A third answer is that history can be a tool to deepen our understanding of the development of students' mathematical thinking. This was the view that I was defending some ten years ago (see e.g. Radford, 2000). Although such a view is mined with many difficult questions (Furinghetti and Radford, in press), I still feel comfortable with it. However, I consider it now to be terribly incomplete. History is not merely a tool to make mathematics accessible to our students. History is a necessity. Why? The answer was offered by the Russian philosopher Eval Ilyenkov. As he put the matter, history is a necessity, because “A concrete understanding of reality cannot be attained without a historical approach to it.” (Ilyenkov, 1982, p. 212).

Reality, indeed, is not something that you can grasp by mere observation. Neither can it be grasped by the applications of concepts, regardless of how subtle your conceptual tools are. The current configurations of reality are tied, in a kind of continuous organic system, to those historic-conceptual strata that have made reality what it is. Reality is not a thing. It is a *process* which, without being perceived, discreetly goes back, every moment, to the thoughts and ideas of previous generations. History is embedded in reality.

Let me illustrate this idea with a picture that comes from the influential book of Maturana and Varela, *The Tree of Knowledge* (1998). The picture in Figure 1 shows how myrmicine ants undertake an interchange of stomach substances. There is a continuous flow of secretion through the sharing of stomach contents each time that the ants meet. The ant on the left can be seen as representing history, while the ant on the right can be seen as representing the present. That which the right ant is acquiring would be — in the metaphoric comparison that I am suggesting — a kind of cultural-conceptual kit containing language, symbols, beliefs about how the world is, how it should be investigated, etc. More precisely, the ant on the left represents the phylogenetic development of the ethical, aesthetic, scientific, mathematical and other concepts and values that we encounter in the culture in which we live and grow. The ant on the right represents our own socio-cultural conceptual individual development over our lifetimes (i.e., ontogeny). In growing, we are continuously drawing on the past.

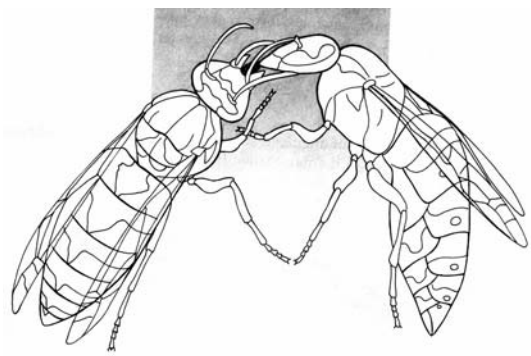

Figure 1 The link between phylogenesis and ontogenesis through the metaphor of the myrmicine ants. (Picture taken from Maturana and Varela (1998), p. 187)

Our ubiquitous drawings on cultural-historical knowledge do not occur, however, at a conscious level. The human brain and human consciousness are not capable of recording and recognizing the historical dimension of knowledge as we acquire it. We can just imagine the wisdom with which a being having such a capability would be endowed! How, then, can we recognize the ubiquitous (although not necessarily visible or evident) presence of history in knowledge, this presence whose understanding is a prerequisite for the understanding of reality? Since knowledge does not evolve randomly, the process of development of knowledge is such that it preserves history in itself in a sublated form. The problem, then, is, for a given object of knowledge “to find out in what shape and form the historical conditions of the object’s emergence and development are preserved at the higher stages of its development.” (Ilyenkov, 1982, p. 208).

The embedded dimension of history in knowledge can be unpacked or unravelled through a kind of critical epistemological archaeology (Foucault, 1966). The goal of the archaeology of knowledge is precisely to determine, for a certain historical period, the "constitutive order of things", that is to say, those chief elements that create (and are, at the same time, created in a dialectical movement) by a fluid order that constitutes the distinctiveness of the episteme of a historical epoch. Following Foulcault's insight, I consider the archaeological space of this order — its niche — to be the space of language and social practice.

Summing up the previous ideas, history is neither merely a motivational tool nor just a way to understand the students' mathematical thinking. History is a necessity. No history amounts to closing, on ourselves, the doors to a grasping of reality; that would amount to egocentrism and blindness. We must recognize that more often than not, in our teaching of mathematics, we have not been very successful in making the historical dimension of knowledge and its import in understanding our world evident. Mathematical knowledge has been reduced to a kind of commodity that bears in itself the fetishism of mass production and consumption. Mathematics has become the search for quick and good answers — two chief effects of a world where technological values (like the fast and the mechanical) have come to displace human ones. Of course, in saying this, I am not pleading for a return to pre-modern times. My point is rather to stress the separation that we have created between Being and Knowing. I firmly believe that the re-connection between Being and Knowing is one of the most important challenges for the historical dimension of the 21st Century Citizen's Mathematics Education. *Knowing something* should be at the same time *being someone.*

3 Being and Knowing

As I see it, the re-connection between Being and Knowing requires us to envision, in new terms, our ideas not only of knowledge but of the self as well. Since the Eleatics and Plato, classical theories of knowledge have envisioned the subject-object relationship as a movement along the lines of a subjectivity attempting to get a grip on the realm of Truth. Modernity did not modify the structure of this relationship, although it traded the substantialist idea of truth for a technological idea of efficiency (Radford, 2004). In the view that I am suggesting here, any process towards knowledge (in other words, all processes of *objectification*) is also a process of *subjectification* (or of the constitution of the "I"). Like poetry or literature, mathematics — as one of the possible forms of reflection, understanding and acting upon the world at a given moment in a culture — is not a mere repository of conceptual contents to be appropriated by a dispassionate observer of reality, but a producer of sensibilities and subjectivities as well (Radford and Empey, in press). The knowing subject does not exist in relation to the object of knowledge only; the subject-object relationship is also mediated by the I-Other (or, more generally speaking, the I-Culture) relationship, so that, as the philosopher Emmanuel Lévinas noted, the problem of truth raised by the Parmenides is posited in new and broader terms: the solution to the Parmenidean problem of truth now includes, in a decisive manner, the social or intersubjective plane (1989, p. 67).

Instead of defending against the potential critique of the cultural relativism that this non-substantialist epistemological view endorses (for a more detailed discussion, see Radford, 2006 and in press), I will rather end my participation in this panel with a comment on the importance of resorting to history in our modern teaching. Hopefully, this comment will help me dissipate some possible misunderstandings that could arise from my objective to include the subjective dimension in knowing. My position could, indeed, be interpreted as reducing mathematical knowledge to a kind of interpersonal exchange — a kind of negotiation of ideas, as knowledge production is often unfortunately conceived of in many contemporary educational theories. Resorting to history should rather be done while being fully conscious of the fact that these two ever-changing things — what we think and what we are — have only been made possible by the phylogenetic developments of the cultures that we live in. The meanings that we form about our world have a cultural history as pre-conditions. To rephrase the literary critic Mikhail Bakhtin, we can say that our meanings only reveal their depths once they have come into contact with past historical meanings: "they engage in a kind of dialogue, which surmounts the one-sidedness of [our] particular meanings." (Bakhtin, 1986, p. 7).

Mathematics, with its tremendously sophisticated conceptual equipment, should be a window towards understanding other voices and subjectivities, and understanding ourselves as historically and culturally constituted creatures.

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