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Epistemology as a research category in mathematics teaching and learning

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Abstract: In a seminal text, M. Artigue (1990) discusses the function of epistemological analysis in teaching. In 1995 she returns to this issue in her plenary conference delivered at the annual meeting of the Canadian Mathematics Education Study Group / Groupe Canadien d'Études en Didactique des Mathématiques. In my presentation, I draw on Artigue's ideas and inquire about the role of epistemology in mathematics teaching and learning. In particular I ask the question about whether epistemology might be an element to understand differences and similarities in different current mathematics education theories.

Introduction

As we know very well, mathematics came to occupy a predominant place in the new curriculums of the early 20th century in Europe. It is, indeed, at this moment that, in industrialized countries, the scientific training of the new generation became a social need. As Carlo Bourlet—a professor at the Conservatoire National des Arts et Métiers—noted in a conference published in 1910 in the journal *L'Enseignement des mathématiques*:

> Notre rôle [celui des enseignants] est terriblement lourd, il est capital, puisqu'il s'agit de rendre possible et d'accélérer le progrès de l'Humanité toute entière. Ainsi conçu, de ce point de vue général, notre devoir nous apparaît sous un nouvel aspect. Il ne s'agit plus de l'individu, mais de la société (Bourlet, 1910, p. 374)[1]

However, if the general intention was to provide a human infrastructure with the ability to ensure the path towards progress (for it is in technological terms that the 20th century conceived of progress and development), it remains that, in practice, each country had to design and implement its curriculum in accordance with specific circumstances. Curriculum differences and implementation resulted, indeed, from internal tensions over political and economic issues, as well as national intellectual traditions and the way in which the school was gradually subjected to the needs of national capitalist production. These differences resulted also from different concepts of

[1] "Our role [i.e. the teachers' role] is extremely serious, it is fundamental, because it is a matter of making possible and accelerating the progress of the whole of Humanity. Thus conceived of, from this general viewpoint, we see our duty in a new light. It is no longer a matter of the individual, but of society."

education. To give but one example, in North America, over the 20th century, the curriculum has evolved as it is pulled on one hand by a “progressive” idea of education— i.e., an education centered on the student and the discovery method— and, on the other, by ideas which organize the teaching of mathematics around mathematical content and the knowledge to be learned by the student. While proponents of the second paradigm criticize the first for the insufficiency of their discovery methods used to develop students' basic skills in arithmetic and algebra, proponents of the first paradigm insist that, to foster real learning, children should be given the opportunity to create their own calculation strategies without instruction (Klein 2003). We see from this short example that the differences that underlie the establishment of a curriculum are far from circumstantial. They are, from the beginning, cultural. Here, they relate to how we understand the subject-object relation (the subject that learns, that is to say the student; and the object to learn, here the mathematical content) as mediated by the political, economical, and educational context. And it is within a "set of differences" in each country that the increasingly systematic reflection on the teaching and learning of mathematics resulted, in the second half of the 20th century, in the establishment of a disciplinary research field now called "mathematics education", "didactique des mathématiques", "matemática educativa", "didattica della matematica", etc.

As a result of its cultural determinations (which, of course, cannot be seen through deterministic lenses: they are determinations in a more holistic, dialectical, unpredictable sense), this disciplinary field of research cannot present itself as something homogeneous. It would be a mistake to think that the different names through which we call discipline merely reflect a matter of language, a translation that would move smoothly from one language to another. Behind these names hide important differences, possibly irreducible, in the conception of the discipline, in the way it is practiced, in its principles, in its methods. They are, indeed, as the title of this panel indicates, research traditions.

The work of Michèle Artigue explores several dimensions of the problem posed by the teaching and learning of mathematics. I mention two in relation to what I just said.

The first dimension consists in going beyond the simple recognition of differences between the research traditions in mathematics teaching and learning. Artigue has played, and continues to play, a fundamental role in creating bridges between the traditions found in our discipline. She is a pioneer in the field of research that we now call connecting theories in mathematics education (e.g., Prediger, Bikner-Ahsbahs, & Arzarello, 2008). Artigue’s role in this field is so remarkable that there is, in this conference, a panel devoted to this field.

A second dimension that Artigue explores in her work is that of epistemology in teaching and more generally in education. She has also made a remarkable contribution to the point that there is also a panel on this topic at this conference. In what follows, I would like to briefly focus on the first dimension in light of the second. In other words, I would like to reflect on epistemology as a research category that provides insight to understand differences and similarities in our research traditions.

Epistemology and Teaching

The recourse to epistemology is a central feature of the main theoretical frameworks of the French school of *didactique des mathématiques* (e.g., Brousseau, 1983; Glaeser, 1981). The recourse to epistemology, however, is not specific to mathematics. There is, I would say, in French culture in general, a deep interest in history. An inquiry into knowledge cannot be carried out without also raising questions about its genesis and development. In this context, one could hardly reflect on mathematical knowledge without taking into account its historical dimension. I can say that it is this passion for history that attracted me in the first place when I arrived in France in the early 1980s. In Guatemala, my native country, and perhaps in the other Latin American countries, as a result of the manner in which colonization was conducted from the 16th century to the 19th century, history has a deeply ambiguous and disrupting meaning: it means a devastating rupture from which we will never recover and that keeps haunting the problem of the constitution of a cultural identity. In France, however, history is precisely that which gives continuity to being and knowledge—a continuity that defines what Castoriadis (1975) calls a *collective imaginary*. From this collective imaginary emanates, among other things, a sense of cultural belonging that not even the French revolution disrupted in France. Right after the French revolution men and women certainly felt and lived differently from the pre-revolutionary period. But they continued recognizing themselves as French. With the disruption of aboriginal life in the 15^{th} century (15^{th} century as reckoned in accordance to the European chronology, of course, not to the aboriginal one), the aboriginal communities of the “New World” were subjected to new political, economical, and spiritual regimes that changed radically the way people recognized themselves. One may hence understand why the passion for history that I found in France was something new for me, as was also the idea of investigating knowledge through its own historical development.

The function of epistemology, however, is not as transparent and simple as it may look like at first sight. And this function is even less transparent in the context of education. The use of epistemology in the context of education cannot be achieved without a theoretical reflection on the way in which epistemology can help educators in their research. It is precisely this reflection that Michèle Artigue undertakes in her 1990 paper in RDM and to which she returns in her plenary conference delivered at the annual meeting of the *Canadian Mathematics Education Study Group / Groupe Canadien d'Études en Didactique des Mathématiques* (Artigue, 1995). Indeed, in these papers she discusses the function of epistemological analysis in teaching and identifies three aspects.

Epistemology allows one to reflect on the manner in which objects of knowledge appear in the school practice. Artigue speaks of a form of "vigilance" which means a distancing and a critical attitude towards the temptation to consider objects of knowledge in a naive, *a*historical way.

A second function, even more important than the first one, according to Artigue, consists of offering a means through which to understand the formation of knowledge. There is, of course, an important difference when we confront the historical production of knowledge and its social reproduction. In the case of educational institutions (e.g., schools, universities), the reproduction of knowledge is achieved within some constraints that we cannot find in the historical production of knowledge.

> les contraintes qui gouvernent ces genèses [éducatives] ne sont pas identiques de celles qui ont gouverné la genèse historique, mais cette dernière reste néanmoins, pour le didacticien, un point d'ancrage de l'analyse didactique, sorte de promontoire d'observation, quand il s'agit d'analyser un processus d'enseignement donné, ou base de travail, s'il s'agit d'élaborer une telle genèse. (Artigue, 1990, p. 246)[2]

The third function, which is not entirely independent of the first, and which is the one that gives it the most visibility to epistemology in teaching, is the one found under the idea of *epistemological obstacle*. Artigue wrote in 1990 that it is this notion that would come to mind to an educator to whom we unexpectedly ask the question of the relevance of epistemology to teaching.

The historical-epistemological analysis has undoubtedly refined itself in the last twenty years, both in its methods and in its educational applications (see, for example, Fauvel and van Maanen, 2000; Barbin, Stehlíková, and Tzanakis, 2008). We understand better the theoretical assumptions behind the notion of epistemological obstacle, its possibilities and its limitations.

My intention is not to enter into a detailed discussion of the notion of epistemological obstacle that educators borrow from Bachelard (1986) and that other traditions of research have integrated or adapted according to their needs (D'Amore, 2004). I will limit myself to mentioning that this concept relies on a *genetic* conception of knowledge, that is to say a conception that explains knowledge as an entity whose nature is subject to change. Now, knowledge does not change randomly. Within the genetic conception that informs the notion of epistemological obstacle, knowledge obeys its own mechanisms. That is why, for Bachelard, the obstacle resides in the very act of knowing, it appears as a sort of "functional necessity". It is this need that Brousseau (1983, p. 178) puts forward when he says that the epistemological obstacles "sont ceux auxquels on ne peut, ni ne doit échapper, du fait même de leur rôle constitutif dans la connaissance visée".[3]

This conception of knowledge as a genetic entity delimits the sense it takes in the different conceptual frameworks of the French school of didactique des mathématiques. More or less under the influence of Piaget, knowledge appears as an entity governed by adaptive mechanisms that subjects display in their inquisitive endeavours. These mechanisms are considered to be responsible for the production of operational invariants: This is the case of the theory of conceptual fields (Vergnaud, 1990). As a result, this theory looks at these invariants from the learner's perspective. But the adaptive mechanisms can also be understood differently: they can be considered as forms of action that show "satisfactory" results in front of some classes of problems. "Satisfactory"

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[2] "The constrains that govern these [educational] geneses are not identical to those that governed the historical genesis, but the latter remains nonetheless, for the didactician, an anchoring point, a kind of observational promontory when the question is to analyze a certain process of teaching, or a working base if the question is to elaborate such a genesis."

[3] Epistemological obstacles "are those to which knowledge cannot and must not escape, because of their constitutive role in the target knowledge."

means here that they correspond to the logic of optimum or best solutions in the mathematician's sense. This is the case of the theory of situations that looks at these forms of actions under the epistemological perspective. Beyond the boundary point that defines the class of problem where knowledge shows itself satisfactory, these forms of action generate errors. That is to say, they behave in a way that is no longer suitable in the sense of optimal, mathematical adaptation. Knowledge encounters an obstacle. The crossing or overcoming of the obstacle ineluctably requires the appearance of new knowledge.

How far and to what extent do we find similar conceptions of knowledge in other educational research traditions? I would like to suggest that it is here where we can find a reference point that can allow us to find differences and similarities in our research traditions—sociocultural theories, critical mathematics education, socio-constructivist theories, and so on.

I mentioned above that in the genetic perspective on knowledge, the obstacle appears with a "functional necessity". However, there are several ways to understand this need. In what follows I give two possible interpretations.

The first interpretation, and perhaps the most common, is to see this need as internal to mathematical knowledge. This would involve conceiving of mathematical knowledge as being provided, in a certain way, with its own "internal logic." This interpretation justifies how, in the epistemological analysis, the center of interest revolves around the content itself. Social and cultural dimensions are not excluded, but they are not really organically considered in the analysis (D'Amore, Radford and Bagni, 2006). If I can use an analogy, I would say that these dimensions constitute a "peripheral axiom" which we can use or not, or use a bit if we will, without compromising the core theorems (or results) of the theory.

In the second interpretation, the development of knowledge appears intimately connected to its social, cultural, and historical contexts. So we cannot conduct an epistemological analysis without attempting to show how knowledge is tied to culture, and without showing the conditions of possibility of knowledge in historical-cultural layers that make this knowledge possible. It is here that we find Michel Foucault's conception of knowledge, whose influence in the French tradition of mathematics education has remained relatively marginal, to my great surprise.

What is important to note here is that behind these two interpretations of knowledge and its development are two different conceptions of the philosophy of history. In the first interpretation, history is intelligible in itself. In the second interpretation, history is not necessarily intelligible. To be more precise, in the first interpretation, whose the theoretical articulation goes back to Kant (1991), the conception of the history revolves around the idea of a reason that develops by self-regulation. *History is reasonable in itself*. There are aberrations and ruptures, of course, but if you look more closely, history appears intelligible to reason. Here, "history is a slow and painful process of improvement." (Kelly,1968, p. 362). In the second interpretation, in which theoretical articulation goes back to Marx (1982), history and reason are mutually constitutive. Their relation is *dialectical*. There is no regulatory, universal reason. The reason is historical and cultural. Their specific forms, what Foucault calls *epistemes,* are conditioned in a way that is not causal or mechanical, by its nesting in the social and political practices of the individuals. It is precisely the lack of such a nesting in the rationalist philosophies that Marx deplores in *The German Ideology*: "the real production of life appears as non-historical, while the historical appears as something separated from ordinary life, something extra-superterrestrial." (1998, pp. 62-63). He continues further on: those theoreticians of history "merely give a history of ideas, separated from the facts and the practical development underlying them" (1998, pp. 64-65). In the Hegelian perspective (Hegel, 2001) of history that Marx prolongs in his philosophical works, it is, indeed, in the socio-cultural practices that we must seek the conditions of possibility of knowledge, its viability and its limits. The reason is unpredictable and history, as such, is not intelligible in *itself*. It cannot be, because it depends on the reasons (always contextual and often incommensurable between each other) that generate it.

In this philosophical conception of history, what shape and role could the epistemological analysis have? And what could be its interest in different traditions of research on the teaching and learning of mathematics? Concerning the first question, one possibility is the use of a materialist hermeneutic (Bagni, 2009; Jahnke, 2012)) that emphasizes the cultural roots of knowledge (Lizcano, 2009; Furinghetti and Radford, 2008). Concerning the second question, the reasons already given by Artigue in the early 1990s seem to me to remain valid. These reasons can undoubtedly be refined. This refinement could be done through a reconceptualization of knowledge itself, reconceptualization that might consider the political, economical and educational elements that, as suggested by the introduction of this article, come to give their strength and shape to knowledge in general and to academic knowledge in particular. The topicalization of epistemology in the different theoretical frameworks and the different traditions of research would be an anchor point to better understand their differences and similarities.

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