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On the growth and transformation of mathematics education theories

Working Paper

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I would like to start by acknowledging how honoured I feel to have the opportunity to participate twice in this Colloquium through which we all celebrate the tremendous contributions that Michèle has made to our research field. This time I wish to talk about a field that has come to be known as "Connecting theories in mathematics education." Under the undeniable influence of Michèle, this field has gained a substantial impetus in the past few years. It has become a new research field of its own.

But before I go into my subject matter, I would like to suggest here that "Connecting" theories in mathematics education is important not only to those who are directly involved in this new disciplinary field but also to all mathematics educators. Indeed, the practice of connecting theories helps us to elucidate what theories are. For instance, to connect different research traditions, participants must make clear the ideas, principles, and assumptions of their own theoretical approaches.

The encounter with other theoretical approaches also offers participants the opportunity to recognize theoretical similarities and differences and to inquire as to what extent two or more approaches are opposed, similar, compatible, and so on.

Of course, the recognizance of differences and similarities between theories depends on what we mean by theory in the first place. In particular, it becomes important to clarify what we mean by theory in mathematics education. Naturally, directly or indirectly, this question has been asked by many math educators—for instance, Niss (1999), Sierpinska & Lerman (1996), Sierpinska & Kilpatrick (1998). Please let me add my two cents to the discussion.

I would like to start by going back to the etymology of the term theory. The word "theory" stems from the Greek verb *theōrein*, which comes from the merging of two root words, *thea* and *horaō*.

Thea (from which the term theatre derives) is the outward aspect in which something shows itself — what Plato called eidos.

The second root word in theōrein, horaō, means: to look at something attentively. Thus, it follows, as Heidegger (1977) suggested, that theōrein or theory is a form of seeing, to look at something attentively and to make it reveal itself to us through the spectacle of its appearance.

As we can see, a theory in the Greek sense is a kind of contemplative act. It is something to help us make sense of something already out there, by looking at it attentively. Classifications, like the botanical ones carried out by Aristotle, were the tools to do that. Finding the genus and its variants was the method to ascertain the limits of the species. But, in this line of thought, the observed objects were not forced to appear. They were there, accessible to be collected and inspected. We have to wait until the late Middle Ages and early Renaissance to find the idea that we can *force* the object to appear. That was the role of the scientific experiment.

But the idea of the scientific experiment led to a reconceptualization of the objects of investigation. That is, one was led to reflect on what was meant by a "fact" and how a fact was evident or constituted evidence of something more general.

We can distinguish at least two main trends. One in which, following the Greeks, facts are subjected to principles or universal propositions governing the theory. In an important sense, a fact illustrates a general principle. In *Posterior Analytics*, Aristotle claims that "sense perception must be concerned with particulars, whereas knowledge depends upon the recognition of the universal" (Aristotle, *Posterior Analytics*). Hence, for Aristotle and the Ancient thinkers, a fact embodies something that transcends it. By contrast, since the early 17th century, under the influence of Francis Bacon, facts were understood by some natural philosophers as theory-free particulars. As Mary Poovey notes in her *A History of the Modern Fact*, some scientists argued that "one could gather data that were completely free of any theoretical component" (1998, p. xviii). With Francis Bacon particulars gained an epistemological prestige.

The previous comments underline the idea that a theory includes assumptions about the "nature" of facts and how the facts of a theory relate to the theory's principles. In Aristotle's approach the fact refers to general principles; the fact is a particularisation of the general. In the Baconian approach, the fact generates the principle through an inductive process. In both cases, an understanding of the reality under investigation is achieved.

Of course, this is true of theories in Mathematics Education too. For instance, Mogens Niss (1999) contends that a theory in math education has two goals. First it entails a descriptive purpose, aimed at increasing understanding of the phenomena studied. Second, it has a normative purpose, aimed at developing instructional design. I shall come back to the second goal and focus now on the first goal—understanding.

The understanding of the phenomena under investigation can only be achieved against the background of general principles — it can be abstract principles in the Aristotelian sense, inductive principles in the Baconian sense, but it can also be something else. The understanding of the phenomena needs to be achieved against the background of general principles, for understanding, as Hegel noticed, is a form of theoretical consciousness that is beyond the fact as such. If you remain with the fact and the fact alone, without subsuming or relating it to something else, you have not yet understood.

So, a theory necessarily comprises a set of principles. Actually, it is not just a set in the sense of a bunch of items. The principles of a theory are conceptually organized. It is perhaps better to see them as a kind of graph, to emphasize the idea that principles are related.

Here is an example.

One principle of constructivism is the following:

knowledge is not passively received but built up by the cognizing subject

Here is a second principle.

the cognizing subject not only constructs her own knowledge but she does so in an autonomous way.

The second principle adds a requirement about how the building of knowledge stated in the first principle is supposed to be achieved.

But we have more than principles in a theory. A theory is a *heuristic* device used to make sense of the world. As such, it asks and tries to answer questions. For instance, to follow with the constructivist example, we can ask: How do children construct the concept of number?

So, in addition to principles, we have research questions. To answer them, we have to produce facts that support the answers to the questions. In order to do that, we still have to find the facts that will be bearers of evidence. And the meticulous way of doing that is what the *methodology* of a theory consists in. The methodology is what is going to force the realm of reality we are interested in to show up. To use Heidegger's (1977) description, the methodology is that which makes the realm of reality "reveal itself through the spectacle of its appearance." Once seen, the appearance or phenomena is amenable to interpretation, which may result in the understanding Niss (1999) is talking about.

Drawing on what has been said, I have suggested (2008) that a theory in math education can be considered as a triplet (P, M, Q).

Naturally, a theory evolves. Theories are not fixed entities; they evolve in time. There is indeed a dialectical relationship among the various components of a theory. The dialectical relationship is mediated by the *results* that a theory produces. What this means is that the three components P, M, and Q, of a theory change as the theory produces results. In other words, the results of a theory influence its components. For instance, with the development of more and more sophisticated digital technologies researchers are capable of producing more sophisticated facts and analyzing them in more complex manners. Digital technologies allow researchers to improve the methodology of their theories and produce new facts. These facts are then formulated, with the aid of the theories' principles, in theoretical terms, leading to new understandings of the phenomena under consideration. In turn, the fabrication or production of facts and their theoretical formulation in the manner of results allow researchers to refine more and more the theoretical principles and research questions of their theories.

Here is an example.

(The example about rhythm from the JRME article (Radford, Bardini & Sabena, 2007). We did not anticipate rhythm as playing a subtle and profound semiotic role in mathematics cognition. Watching the video clip over and over within the possibilities of frame-to-frame analysis, we evidenced a "fact" that was theorized through the principles of the theory: we realized that rhythm was a fundamental semiotic means of knowledge objectification. *Ptraat* software allowed us to carry out a pitch and prosodic analysis to confirm the role of rhythm. The new results required a refinement of the theoretical principles).

But theories also evolve by interacting with other theories. And it is here that the question of connecting theories in mathematics education comes in.

What I have said about theories is not an account of their emergence. Such an account, which is problematic on its own, should require a different approach. In the field of connecting theories what we have is two or more theories coming into contact. Although they are always changing, the theories are already there.

There are some interesting and very specific problems that arise out of the attempt of putting theories in some sort of relationship.

To investigate what happens when theories come into an explicit relationship —for instance, when a same piece of phenomena (a video clip for example) is analyzed by two or more theories— to investigate what happens with the theories, I suggested that it might be worthy to consider theories as positioned in something that semiotician Yuri Lotman (1990) calls a *semiosphere*.

Let me give you a pictorial metaphor of Lotman's concept.

Theories inhabit the semiosphere — a multi-cultural, heterogeneous and dynamically changing space of conflicting views and meaning-making processes generated by theories and their different research cultures.

It is in the semiosphere that theories live, move, and evolve. It is in the semiosphere that theories come into a relationship.

The relationship may have different goals. Prediger, Bikner-Ahsbahs, and Arzarello (2008) identified some of them in their ZDM paper. They include contrasting, combining and even ignoring!

The goal of the relationship makes the theories come close to each other. How close they come depends on the goal of their dialogue.

Understanding each other may not require the same proximity as when one wants to combine or synthesize them. But the kind of relationship that can exist between theories depends also on how *compatible* theories are.

Now, how can we have a sense of how far or close or compatible theories are?

A theory can be stretched so as to come close to another one. But there are limits. One interesting historical example of a relationship between theories resulted from the dialogue that the North-American constructivism and the German interactionism carried out in the 1990s. Those theories are certainly different in many important respects, in particular in their theoretical principles, as shown for instance by their different concepts of meaning. In Constructivism meaning is a psychological construct. In Interactionism, meaning is a socio-relational or interactional notion — it is not something that is in the head but in the interaction. The different theoretical principles of those theories define the contours of what is theoretically achievable in terms of combining them. Constructivists realized that they could incorporate something that was missing in their theory: the social dimension. But this incorporation of the social, they knew very well, had to be done in a way that is consistent with their general theoretical principles. As we all know, in the end, the social dimension of knowing was integrated in a way that kept intact the epistemic exigencies of their postulates, such as the autonomy of the learner in the act of learning. This is why in the North-American constructivism, as Martin Simon (2012) reminded us, it is impossible to run a social and individual analysis at the same time. You cannot predicate of the individual and the social simultaneously. For the North-American constructivism, the social and the individual are like those quantum entities that you cannot see simultaneously.

This interesting problem is not specific to constructivism. It appears in the theory of Didactic Situations (Brousseau, 1997) as well. The constructs of devolution, a didactic situation, and milieu are indeed attempts at addressing the question of the social and the individual. I don't have time here to comment on the tensions that are produced in this theory by the integration of the social in the account of leaning. The point that I want to make is rather that theoretical principles offer possibilities but also set limits to what can be incorporated without becoming inconsistent.

Let me come back to the general idea of linking theories. I think that most theories —perhaps all of them — are different. There is always a gap that you will find between theories if you dig deep enough. If such a gap did not exist, theories would be reducible to a single Grand Theory and mathematics education would be a tautological discourse.

Now, the fact that two theories can *be* different, that there is always a gap, is not a reason to imagine that a dialogue between them cannot be fruitful. A dialogue between theories, however, is not easy to achieve. I shall mention here two reasons why.

The first one has to do with the polysemy or coexistence of many possible meanings for a word or phrase. *Epistemic action* or *social interaction* may have one meaning in one theory and a different meaning in another theory.

The second reason is that theories in mathematics education reflect and refract implicit and specific national-cultural "world views". They are unavoidably immersed in those symbolic systems of cultural significations that Cornelius Castoriadis, Ernst Cassirer and others have pinpointed in their investigation of the symbolic structures of society—structures from where (implicitly or explicitly) our theories draw their views of what constitutes a good student, a good teacher, a good math lesson, and so on.

Boundaries

As I have just suggested, theories can be put into some sort of relationship. We can always try to connect them in some way. Now, there is a limit to what can be connected.

This limit is determined by the goal of the connection, but also by the specificities of the components (P, M, Q) of the theories that are being connected. This limit has to do with the boundary of each theory under consideration.

For Lotman (1990), a boundary is one of the primary mechanisms of semiotic individuation, something that marks the limits of a first-person form (''I,'' ''us'') in opposition to non-first person forms (''you,'' ''them'').

Drawing on this idea, I suggest calling the boundary of a theory the ''edge'' that a theory cannot cross without a substantial loss of its own identity. The boundary sets the ''limit'' of what a theory can legitimately predicate about its objects of discourse; beyond such an edge, the theory conflicts with its own principles.

Thus, the manner in which constructivism theorizes learning can be stretched to a certain point, but we cannot make it coincide with the manner in which Vygotskian approaches theorize learning. Constructivism would not to give up its idea of the learner as an autonomous, adaptive, and self-regulating agent. If it does, then it is no longer constructivism. Constructivism would have transmuted into something else.

Growth and transformation

The existence of a hard kernel in a theory does not prevent the theory from growing. Boundaries are continuously growing and changing. And actually, one of the most interesting effects of connecting theories is that it makes theories grow.

For instance, in a previous experiment in connecting theories, reported in the 2010 PME (see Bikner-Ahsbahs, Dreyfus, Kidron, Arzarello, Radford, Artigue & Sabena (2010)), Abstracton in Context and Interest Dense Situation theories entered into a semiospheric relationship. As a result some peripheral conceptual entities, that is, entities that were not organic parts of each one of these theories, ended up gaining a more central role. This was the case of the *general epistemic need* concept. This marginal entity made its entrance through the theories' interaction.

Another example: the connection of the Semiotic Bundle and Interest Dense Situation approaches brought forward a peripheral construct, the *epistemological gap* construct.

It seems then that when two (or more) theories position themselves towards each other to enter into a semiospheric dialogue, a halo of new conceptual possibilities is formed. Potential entities appear. But they remain in the periphery of the cluster that the theories constitute. They remain "revolving around", as the etymological sense of *periphery* intimates. An effort of objectification is required to bring the peripheral entities into attention. And, in this objectifying movement, in order to accomplish the crossing of the peripheral threshold, we need someone else. For in the end, it turns out, as Bakhtin was suggesting that "every internal experience occurs on the border, it comes across another, and in this tension-filled encounter lies its entire essence." (Bakhtin, 1984, p. 287, adapted from Todorov, 1984, p. 96).

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