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TOWARDS AN EMBODIED, CULTURAL, AND MATERIAL CONCEPTION OF MATHEMATICS COGNITION

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In this paper I sketch an embodied, cultural, and material conception of cognition and discuss some of the implications for mathematics education. The basic idea is that cognition is a feature of living material bodies characterized by a capacity for responsive sensation. The sketched approach, which I term sensuous cognition, refers precisely to this view where sensation is considered to be the substrate of mind, and of all psychic activity (cognitive, affective, volitional, etc.). I argue that human cognition can only be understood as a culturally and historically constituted sentient form of creatively responding, acting, feeling, transforming, and making sense of the world. I briefly refer to a classroom episode involving 7–8-year-old students dealing with pattern recognition.
Key Words: Cognition, sensation, multimodality.

SENSUOUS COGNITION

Customarily, gestures, body posture, and other embodied signs have been conceptualized as playing a secondary role in mathematical cognition. Traditional cognitive psychology and other similar psychological trends inspired by idealist philosophy have not paid attention to embodied signs and tactile-kinesthetic activity. In traditional cognitive psychology, for instance, the mind is conceived of as a computational device; consequently, the research focus is on the linguistic formats through which information is transmitted and decoded (e.g., declarative vs. procedural sentences). A new research trend, however, offers a different approach to the understanding of human cognition. In this trend our tactile-kinesthetic bodily experience of the world and our interaction with artifacts are considered to be much more than merely auxiliary or secondary elements in our cognitive endeavours (Bautista & Roth, 2011; Borba & Villareal, 2006; Edwards, Radford, & Arzarello, 2009; Lakoff & Núñez, 2000; Sheets-Johnstone, 2009). Dwelling upon Vygotsky's (1987-99) and Leont'ev's (1978, 2009) work on enactivism (Maturana and Varela, 1998), in this paper I elaborate on what I have previously termed *sensuous cognition* (Radford, 2009a). The idea of sensuous cognition that I would like to advocate here rests on a non-dualistic view of the mind. In dualistic accounts, the mind is conceived of as operating through two distinctive planes, one internal and one external. The internal plane is usually considered to include consciousness, thought, ideas, intentions, etc., while the external plane refers to the material world—which includes concrete objects, our body, its movements, and so on. I adopt here a monistic position according to which mind is a property of matter. More specifically, mind is conceptualized as a feature of living material bodies characterized by a capacity for *responsive sensation.*

Sensation is a phylogenetically evolved feature of living organisms through which they *reflect* and *respond to* or *act on* their environment. Since the organism is itself a part of the material world, any reflection of reality is strictly a function of a material, corporeal organism (Leont'ev, 2009, p. 12). As a result, reflection and action do not occur in two *separate* planes. They occur in a same plane—the plane of life.

Now, reflection cannot be considered a passive act of receiving sensorial impressions, as 17th and 18th century empiricists hold. Reflection involves both: (1) something that transcends the organism as such (something that, in order to differentiate it from the subject itself, we can call *objective,* namely *the object of reflection*), and (2) the *reflected object*, something that is *subjective* (in the sense that reflection depends on the specific organism reflecting the environment).

Reflection arises from, and becomes entangled with, the various modes of sensation, which in their development become integrated and specialized. Thus ontogenetically speaking, through a tactile experience, the child can feel the weight of an orange; through a perceptual one, she can have a sense of its relative chromatic characteristics. Later, she can feel its porous skin even if it is out of her actual tactile reach. Knowing hence appears to occur through a multi-modal sensorial experience of the world. This multi-sensory characteristic of cognition is not specific to humans; it is shared by insects (Wessnitzer and Webb, 2006) and other primates as well. However, compared to the case of insects and other primates, human sensorial organs collaborate to a greater extent (Gómez 2004; Köhler, 1951), so that what we perceive or touch is endowed with a variety of sensuous coordinated characteristics. For instance, the human hand does not only feel the trace of the object. We can say that the hand also "perceives its colour, its volume, its weight" (Le Breton 2007, p. 151).

To sum up, instead of being something purely "mental," reflection and its products remain, one way or another, intertwined with the environment that is been reflected and with the organism's capacities for sensation. Mind, in this context, is the ability of organisms to reflect, and act on, the reality around them. Thinking, memory, imagination, and other cognitive functions are directly and indirectly related to a large range of sensorimotor functions expressed through the organism's movement, tactility, sound reception and production, perception, etc. What I term *sensuous cognition* refers precisely to this view where sensation is considered to be the substrate of mind, and of all psychic activity (cognitive, affective, volitional, etc.).

THE CO-EVOLUTION OF THE SENSES AND CULTURE

A chief characteristic of the human senses is the manner in which they co-evolve with culture. Indeed, our senses are not merely part of our biological apparatus. The raw range of orienting-adjusting biological reactions we are born with is transformed into complex, historically constituted forms of sensing. As we live in society, interact with others, and participate in more or less specialized forms of training, the biological orienting-adjusting reactions undergo cultural transformation and are converted into complex historically constituted forms of sensing, leading to specific features of human development and the concomitant forms of cultural reflection. This is why in the process of development the child not only matures, but is also equipped with sophisticated ways of seeing, touching, hearing, tasting, and so on.

The ontogenetic process of the cultural transformation of the senses has been investigated in great detail in the past few years. To mention but one example, Zaporozhets (2002) reports research with three- to five-year old preschoolers who were learning to discriminate between variants of two geometric figures: triangles and quadrilaterals. At first, the preschoolers were making a substantial number of errors. Then, they were invited to trace systematically with a finger the outline of the figure, paying attention to directional changes of the motions at angles, and accompanying the tactile exploration with side counting (one, two three…). The investigator reports that at this stage perception was accomplished through the tactile experience, while the eye performed an auxiliary role. "Later," Zaporozhets says, "the eye developed the ability to solve these types of perceptual tasks independently, consecutively tracing the outline of a figure, as it was earlier done by a touching hand" (2002, p. 31). During this process, the eye undergoes a transformative change: "initially, the eye motions have an extremely extensive nature, consecutively tracing the entire outline of the perceived figure and simulating its specifics in all details" (p. 32). In a subsequent stage, the eye's motions "gradually begin to decrease and to focus on the individual, most informative attributes of the object" (p. 32).

THE ARTIFACTUAL DIMENSION OF SENSUOUS COGNITION

A closer look at the previous example shows that the new cultural forms of sensation are deeply interrelated with the use of *artifacts*. In Zaporozhets' example, preschool children develop a mathematical form of perception that allows them to distinguish between cultural categories of geometrical figures. In the course of this developmental process, the children have recourse to the material objects whose contours they cover with a finger while using numbers to count aloud. What this example shows is that our individual senses evolve intertwined not only one with the other senses, but also with the *materiality* of the objects in our surroundings. The materiality that shapes our senses is not, however, reduced to inert matter, but, as the example shows, matter already endowed with meaning (e.g., 'triangularity,' 'quadrilarity,' etc.).

It is this key role of artifacts in the constitution and evolution of forms of sensing and reflecting that Luria and Vygotsky underlined in their work. The use of artifacts, they contended, constitutes the first phase in cultural development (Luria & Vygotsky, 1998; Vygotsky, & Luria, 1994). Such a phase marks the emergence of new forms of actions and reflection and the concomitant appearance of psychic functions.

All in all, the previous discussion amounts to making a point about the embedded nature of artifacts in the evolution of our ways of sensing and reflecting. Luria and Vygotsky stress the fundamental cognitive role artifacts and material culture play in the ways we come to know. The claim that I am making, hence, goes beyond the conceptualization of artifacts as merely *mediators* of human thinking and experience, or as prostheses of the body. Artifacts do much more than mediate: they are a *constitutive part* of thinking. Behind this view lies, of course, the general concept of mind as a property of matter. This property expresses the enactive relationship between materiality and mind that inspired Vygotsky's, Luria's, and Leont'ev's work and that Bateson (1973) illustrates so nicely in his example of the blind person's stick. It is in this context that anthropologists Malafouris and Renfrew (2010) claim that we can speak of things as having a cognitive life. They say: "things have a cognitive life because minds have a material life" (p. 4).

Sensuous cognition is hence a perspective that highlights the role of sensation as the substrate of mind and of all psychic activity. But in contrast to other approaches where the focus remains on the individual's body, sensuous cognition offers a perspective where sensation and its cultural transformation in sensing forms of action and reflection are understood to be interwoven with history, cultural artifacts, and materiality at large. Sensuous cognition calls into question the usual divide between mind and matter and casts in new terms the classical boundaries of mind. It offers a new perspective in which to conceive of students' and teachers' actions in teaching-learning processes. In particular, sensuous cognition invites us to pay attention to perception, gestures, kinesthetic actions, sign- and artifact-use in different ways.

A CLASSROOM EXAMPLE

In what follows, I would like to discuss an example from a Grade 2 class (7–8-year-old students) involving the generalization of an elementary figural sequence (see Figure 1).

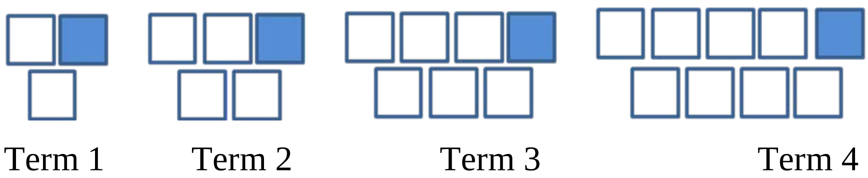


Fig. 1. The first terms of a sequence that 7–8-year-old students investigate in a Grade 2 class.

To become sensitive to the cultural-historical algebraic forms of perceiving terms in sequences like the one discussed here, students engage in processes that are far from mental. They engage with the task of exploring the sequence in a sensuous manner. I would like to illustrate this point by discussing the way in which the teacher and a group of students reflect on Term 8 of the sequence. The first question of the mathematical activity consisted in extending the terms of the sequence up to Term 6. Then, in questions 2 and 3, the students were asked to find out the number of squares in terms 12 and 25. In question 4, the students were given a term that looked like Term 8 of the sequence (see Figure 2). They were told that this term was drawn by Monique (an imaginary Grade 2 student) and encouraged to discuss in small groups to decide whether or not Monique's term was Term 8. The trained eye would not have difficulties in noticing the missing white square on the top row. The untrained eye, by contrast, may be satisfied with the apparent spatial resemblance of these terms with the other terms of the sequence and might consequently fail to note the missing square.

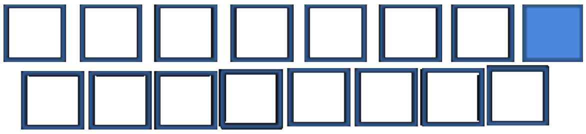


Fig. 2. The students were requested to discuss whether Monique's term is Term 8 of the given sequence.

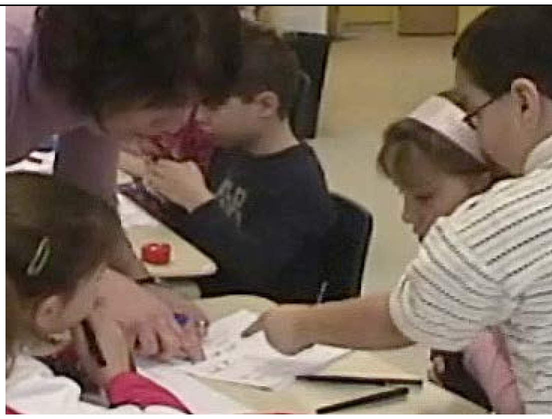
Let me focus on the discussion that a group of students had with the teacher—a group formed by James, Sandra and Carla. When the teacher came to see their work the students had already worked for about 32 minutes together. They had finished drawing Terms 5 and 6, tried (unsuccessfully) to find the number of squares in Term 12 and 25, and answered the question about Term 8 (which they considered to be Term 8 of the sequence). The teacher engaged in collaborative actions to create the conditions of possibility for the students to perceive a general structure behind the sequence:

1. Teacher: We will just look at the squares that are on the bottom (*while saying this, the teacher makes three consecutive sliding gestures, each one going from bottom row of Term 1 to bottom row of Term 4; Pics 1-2 in Fig. 3 show the beginning and end of the first sliding gesture*). Only the ones on the bottom. Not the ones that are on the top. In Term 1 (she points with her two index fingers to the bottom row of Term 1; see Pic. 3), how many [squares] are there?
2. Students: 1!
3. Teacher: (Pointing with her two-finger indexical gesture to the bottom row of Term 2) Term 2?
4. Students: 2! (James points to the bottom row of Term 2; see Pic 4).
5. Teacher: (Pointing with her two-finger indexical gesture to the bottom row of Term 3) Term 3?
6. Students: 3!
7. Teacher: (Pointing with her two-finger indexical gesture to the bottom row of Term 4; see Pic 5) Term 4?
8. Students: 4!
9. Teacher: (Making a short pause and breaking the rhythmic count of the previous terms, as if starting a new theme in the counting process, she moves the hand far away from Term 4 and points with a two-finger indexical gesture to the place of hypothetically one would expect to find Term 8; see Pic 6) How many squares would Term 8 have on the bottom?
10. Sandra: (hesitantly, after a relatively long pause) 4?

Pic 1

Pic 2

Pic 3



Pic 4

Pic 5

Pic 6

Figure 3. The teacher's and students' sensuous (perceptual, gestural, tactile, aural, vocal) engagement in the task.

In Line 1, the teacher makes three sliding gestures to emphasize the fact that they will count the bottom row of the four given terms. The gestural dimension of the teacher-students' joint activity is somehow similar to the tactile experience of the students who, in the aforementioned experiment reported by Zaporozhets (2002), follow the contour of shapes with their fingers. Here, the tactile dimension is carried out instead with gestures through which the teacher suggests a cultural form of perceiving the terms of the sequence—one in which the mathematical ideas of variable and relationship between variables are emphasized.

Now, the teacher does not gesture silently. Gestures are coordinated with utterances. This is why it might be more useful to consider the teacher's utterance as a *multimodal utterance*: that is to say as a bodily expression that resorts to various sensorial channels and different semiotic registers (Arzarello, 2006). In this case, the teacher coordinates eye, hand, and speech through a series of organized simultaneous actions that orient the students' perception and emergent understanding of the target mathematical ideas. In our previous work we have termed *semiotic node* this complex coordination of various sensorial and semiotic registers (Radford, 2009b). The investigation of semiotic nodes in classroom activity is a crucial point in understanding the students' learning processes. The concept of semiotic node rests indeed on the idea that the understanding of multi-modal action does not consist in making an inventory of signs and sensorial channels at work in a certain context. From a methodological viewpoint, the problem is to understand how the diverse sensorial channels and semiotic signs (linguistic, written symbols, diagrams, etc.) are *related, coordinated,* and *subsumed* into a new thinking or psychic *unity* (Radford, 2012). Such a methodological problem makes sense only against the background of a conception of the mind that overcomes the dualistic view of internal-external processes. In our case, the methodological problem makes sense against the background of a concept of the human mind as *sensuous* through and through.

Yet, as Line 10 intimates, the passage from Term 4 to Term 8 was not successful. The objectification (that is, the becoming aware; see Radford, 2010) of the algebraic manner in which sequences can be algebraically perceived has not occurred yet. The teacher hence decided to restart the process, with some important modifications, as we shall see.

As mentioned previously, Term 8 of the sequence was not materially drawn. In the previous excerpt, the teacher *pretends* that Term 8 is on the empty space of the sheet, somewhere to the right of Term 4. She points to the empty space, as she pointed to the other terms, to help the students imagine the term under consideration. During the second attempt, the teacher does not go from Term 4 to Term 8; this time she goes term after term until Term 8.

11. Teacher: We will do it again...
12. Teacher: (*Pointing to Term 1 with a two-finger indexical gesture*) Term 1, has how many?
13. Carla: (*Pointing with her pen to the bottom row*) 1, (*without talking to the teacher points to Term 2 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term 2)* 2, (again without talking *the teacher points to Term 3 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term* 3), 3, (*same as above*) 4, (*now moving to the hypothetical place of Term 5 would be expected to be and doing as above*) 5.
14. Teacher: Now it's Term 8! (*The teacher comes back to Term 1.* She points again *with a two-finger indexical gesture to the bottom row of Term 1*) Term 1, has how many [squares] on the bottom?
15. Students: 1.
16. Teacher: (*Pointing with a two-finger indexical gesture to the bottom row of Term 2)* Term 2?

17. Students: 2!
18. Teacher: (*Pointing with a two-finger indexical gesture to the bottom row of Term 3)* Term 3?
19. Students: 3!
20. Teacher: (*Pointing with a two-finger indexical gesture to the hypothetical place where bottom row of Term 4 would be)* Term 4?
21. Students: 4!
22. Teacher: (*Pointing as above*) Term 6?
23. Students: 6!
24. Teacher: (*Pointing as above*) Term 7?
25. Students: 7!
26. Teacher: (*Pointing as above*) Term 8?
27. Students: 8!
28. Sandra: There would be 8 on the bottom!

The teacher and the students counted together the squares on the bottom row of Monique's term and realized that the number was indeed 8. At this point the relationship between variables started becoming apparent for the students. The relationship started being objectified. The teacher then moved to a joint process of counting the squares on the top row. The students were perplexed to see that contrary to what they believed, Monique's Term 8 did not fit into the sequence (for details, see Radford, 2010).

Later on in the lesson the students were able to quickly answer questions about remote terms, such as term 12 and Term 25, which were not perceptually accessible. They refined the manner in which the terms of the sequence could be perceived. The number of squares on the bottom row was equated to the number of the term in the sequence, while the number of squares on the top row was equated to the number of the term plus one. Here is an excerpt from the dialogue of Sandra's group as they discuss without the teacher:

Sandra: (*Referring to Term 12*) 12 plus 12, plus 1.

Carla: (*Using a calculator*) 12 plus 12 … plus 1 equal to …

James: (*Interrupting*) 25.

Sandra: Yeah!

Carla: (*looking at the calculator*) 25!

SYNTHESIS

In the first part of this paper, drawing on Vygotsky and his cultural-historical psychological school, as well as on the work of Maturana and Varela, I sketched a theoretical approach to cognition that highlights the role of sensation as the substrate of mind and all psychic activity. In the second part of the article, I presented a short example that, I hope, gives an idea of the manner in which sensuous cognition may help us understand teaching-learning activity. Sensuous cognition, I argued, does not amount to claiming that our various senses come into play in classroom interaction. This is no more than a banal statement. The real question, I argued, is about understanding how, through classroom activity, our forms of sensing and reflecting are culturally transformed. The episode suggests how the target cultural knowledge is objectified as a new ideational-material psychic unity is forged. The students no longer need to see the terms of the sequence. What could only be made apparent through an intense interplay between various sensorial modalities and different signs is now contracted, subsumed and reorganized in a new complex psychic unity where no reference is made to top or bottom rows.

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