

## Chapter 4

### ON CULTURE AND MIND:

### A POST-VYGOTSKIAN SEMIOTIC PERSPECTIVE

### WITH AN EXAMPLE FROM GREEK MATHEMATICAL THOUGHT

Luis RADFORD

*Claude Janvier In Memoriam*

#### 1. Introduction

Vygotsky's "Genetic Law of Cultural Development"—which constituted Vygotsky's answer to the age-old question of the role of society in the formation of the mind—links, in a decisive manner, human cognition to the individuals' use of signs in activity. The ethnographic fact that the motives underpinning human activity and the use of signs which mediate them are culturally situated, suggests that the actual form of such activity takes in a culture constrains and, in turn, is constrained by its own "Modes of Acting", which are generated by the dialectical interaction of Activity and what we call *Cultural Semiotic Systems* (CSSs)— i.e., those socio-historically constituted systems in which we find the beliefs of a culture, the generative patterns of meaning-making, and so on. Furthermore, it is argued that the very semiotic nature of Activity and the epistemological role of signs, in the interplay between Activity and the CSSs, account for the "Modes of Knowing" (or *épistèmes*, to use Foucault's expression) encompassing the specific forms that the system of ideas (or *ideologies*, in Voloshinov's and Bakhtin's terms) may take. In contrast to social behaviorism and symbolic interactionism, Modes of Acting and Modes of Knowing, as described here, remain attached to the social, historical, and economical dimensions and the concrete life of the individuals. This theoretical perspective provides an alternative to the conceptualization of Reason as offered by the Enlightenment, and suggests that "mathematical thinking" can be anthropologically conceived as a semiotic expression of the rationality of the culture in which the mathematical activity is carried out. This point and the role played by the construct of CSSs are illustrated through a case-study taken from the Euclidean theory of the Odd and Even Numbers—a theory that is examined here in light of the Modes of Acting and Knowing of Classic Greece.

## 2. Vygotsky's Genetic Law of Cultural Development

In Plato's dialogue *Protagoras*, Socrates maintains that virtue (*aretê*) cannot be taught; *aretê*, for him, is something that comes from the interior of the individual. Protagoras, in contrast, argues that virtue is learned by habit, by seeing how others practice it; we learn virtue as the child learns how to speak (see *Protagoras*, 324d-327e). Protagoras' idea leads us directly into a question that has been raised again and again from different perspectives through time, and that, in the 1920s, Lev Vygotsky—like the psychiatrist Pierre Janet, the social psychologist George Herbert Mead, and the philosopher and sociologist James Mark Baldwin, among others—stated in terms of the role of society in the constitution of mind. Writing against pure spiritualistic approaches, and embedded in the intellectual trends of the first decades of the century (e.g., Freudianism, Gestalt Psychology, Behaviorism, Evolutionism), Vygotsky was interested in creating a theoretical basis for the understanding of the role played by society in the formation of the psychological processes carried out by the individuals. Thus, two of the questions that framed his research—stated in the very beginning of *Mind in Society*—were that of the relations between human beings and their physical and social environment, and that of the psychological consequences produced by the activities arising from the human enterprise of the mastering of nature. While the first question runs against the simplistic idea that the social environment is the cognitively innocuous exterior scene where human actions are achieved, the second question stresses the importance that human actions have in the psychological realm.

In accordance with the evolutionary point of view that became frequently adopted at the turn of the century in the study of the human mind and human behavior, Vygotsky paid particular attention to human and animal psychology, and found in their comparison some cues to the elaboration of his theory of human psychological functions. For instance, Köhler's book, *The Mentality of Apes*, first published in 1925, had a paramount importance for Vygotsky's ideas (see Vygotsky 1997). In one of Köhler's experiments, a banana was placed on the floor outside of a chimpanzee's cage. Tschego—the chimpanzee in this experiment—unsuccessfully stretched her arm out to reach the fruit. Then Tschego saw a stick in the cage, placed in such a way that she could see both the stick and the fruit at the same time. She then used the stick to reach the fruit. However, when the stick was put in a position where the chimpanzee could not see both the banana and the tool at the same time, the stick was no longer used to get the fruit. Köhler says:

I have used every means at my disposal to attract Tschego's attention to the sticks in the background of her cage [...] and she did look

straight at them; but, in doing so, she turned her back on the objective, and so the sticks remained meaningless to her. Even when we had introduced her, in the course of one morning's test, to seize and use one of the sticks, she was again quite at loss in the afternoon, although the sticks had not been removed from their former position, and she stepped on them in the course of her movements to and fro, and repeatedly looked straight at them. (Köhler 1951: 37)

Vygotsky suggested that one of the differences between apes' and children's strategies to solve similar problems is found in the role played by tools (see Vygotsky and Luria 1994: 100-106). Instead of considering tools and symbolic activity as independent of each other as his predecessors had done, Vygotsky considered that, in the case of the child, the use of tools and the symbolic activity give rise to a complex psychological unit: "the practical use of tools and the symbolic forms of activity . . . form a complex psychological entity..." (Vygotsky and Luria 1994: 112).

Referring to the apes' strategies in Köhler's work, he stressed the fact observed by Köhler himself, that in the case of apes, the entire process of problem-solving is essentially determined by perception (Vygotsky 1978: 31). In contrast, he noticed that during problem-solving processes, children of four and five years of age use language to speak, and that instead of being superfluous, or simply being "an invaluable technical aid", as Köhler himself suggested (Köhler 1951: 267), speech becomes interwoven with actions. Speech, Vygotsky noted, serves first to organize the child's actions, and later it acquires an anticipative role that eventually results in the replacement of some of the actions. Taken metaphorically, words (and other physical objects) as psychological signs, he investigated how perception (as well as memory and attention) is profoundly modified by the use of tools. This led him to the conclusion that while the ape remains constrained to the sensorial field, the child perceives the world not only through his or her eyes and hands but through language too (Vygotsky 1978: 26; Vygotsky and Luria 1994: 109). In one of the many paragraphs devoted to perception Vygotsky and Luria say:

The natural laws of perception most clearly observed in the receptive processes of the higher animals undergo basic changes due to the inclusion of speech in human perception, and human perception thus acquires an entirely new character. (Vygotsky and Luria 1994: 126)

Indeed, for Vygotsky, a fundamental distinction between the psychological functions of humans and animals was due to the fact that humans were not only able to use natural signs and produce artificial ones but, rather, was due to a certain semiotic plasticity of the human

mind—that is, a specific semiotic capacity that makes mind *alterable* by the actual use of signs:

By being included in the process of behavior, the psychological tool alters the entire flow and structure of mental functions. It does this by determining the structure of a new instrumental act just as a technical tool alters the process of a natural adaptation by determining the form of labor operations. (Vygotsky 1981:137)

The analogy between concrete labor tools and signs alluded to in the previous quotation was worked out in several essays (e.g., “The internalization of the higher psychological functions,” included as Chapter IV in Vygotsky 1978; Vygotsky 1981; Vygotsky and Luria 1994). Given that labor tools and psychological tools belong to two different human phenomena, he argued that tools and signs cannot be equated. In Vygotsky’s account they are *similar* in that both allow individuals to act and interact with their surroundings, not in a direct form but in a mediated one. But they are different in terms of the way they *orient* human behavior. On the one hand, the tool is oriented towards the object of the activity (e.g., the mastery of nature). In this case the tool serves to externally orient human behavior. On the other hand, the sign serves as a pivotal point in the accomplishment of a psychological process that internally orients human behavior. “These activities are so different from each other”, concluded Vygotsky, “that the nature of the means they use cannot be the same in both cases.” (Vygotsky 1978: 55).

However, although different in nature, the mediating role of signs and tools become tied, according to Vygotsky, when one considers that the actual activity in which an individual is engaged is encompassed by the sociocultural history of the activity. This point is well illustrated in the following passage that echoes some of the ideas of *The German Ideology* (Marx and Engels 1982; see for instance page 103): “The mastering of nature and the mastering of behavior are mutually linked, just as man’s alteration of nature alters man’s own nature.” (Vygotsky 1978: 55).

According to Vygotsky, although the creation and use of signs as auxiliary psychological tools to solve problems was a fundamental distinction between the psychological functions of humans and other animals, this distinction cannot account for the whole psychological difference. In fact, signs are but a part of a general process, specific to human beings, which links individual psychological processes to a social setting—a process sustained by the *internalization* of what happens in the social (or external) plane to the individual (or internal) plane. He wrote:

The internalization of socially rooted and historically developed activities is the distinguishing feature of human psychology, the basis of

the qualitative leap from animal to human psychology. (Vygotsky 1978: 57)

By internalization, Vygotsky did not mean a unidirectional cultural transmission act that comes with prepared packages of knowledge, in order to fill the empty container of the supposedly "not-yet-knower" (Lawrence and Valsiner 1993). Rather, he saw internalization as a dynamic transformational mechanism whereby social interactions become interiorized by the individual (for a more detailed discussion see Wertsch 1985: 61 ff). This idea, known as the *genetic law of cultural development*, is formulated as the transformation of an interpersonal process into an intrapersonal one. Vygotsky said, "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level" (Vygotsky 1978: 57).

The mediating role of signs in the processes of internalization is made clear in the following passage (see also Vygotsky and Luria 1994: 109-110):

The internalization of cultural forms of behavior involves the reconstruction of psychological activity on the basis of sign operations. (Vygotsky 1978: 57)

The *genetic law of cultural development* is Vygotsky's answer to the question of the relations of society and the psychological functioning of the individual.

Although Vygotsky did not give a general, comprehensive theoretical account of internalization, he and his collaborators provided some concrete examples (e.g., pointing, inner speech and memory). Piotr Ya. Gal'perin, who was interested in children's intellectual development in school instructional settings (e.g., Gal'perin 1989a), was led to examine in further detail the idea of internalization, and to better understand the steps that such a process undergoes. Gal'perin carried out extensive experimental research that covered different domains (arithmetic and writing, for instance; see Gal'perin 1989b, where he gives an alternative explanation of the Piagetian results obtained in the tasks of conservation of quantities), and identified some general characteristics of internalization. He pointed out that during the process of internalization, significant changes occur. The concrete action to be internalized undergoes a semiotic *contraction* and a process of *automatization* to the extent that the mental action becomes an object of awareness only on particular occasions, as for instance, when unexpected results or conditions are encountered by the individual. The internal plane—i.e., that plane which includes the products of internalization—is not, he insisted, "an empty vessel where anything can be put" (Gal'perin 1967: 30). On the contrary, the internal plane has a constructive nature. New actions are

performed only on the basis of an already developed plane that the new actions come to join. During this process, the previously developed plane is modified. When he tackled the question of the uniqueness of private thinking, he noticed that, although internalization is a social construct, it is the very constructive nature of the internalization of actions which affords variability in mental constructions and makes internalization a private construct as well. An action, for instance, has a contextual orienting component that, although remaining culturally defined, is intimately related to the subjective understanding of the action (Gal'perin 1989b). In other words, for him, the malleability of the bi-directional constructive nature of the internal plane renders the mental action both a private (hence unique) and a social phenomenon.

Gal'perin was completely aware of the fact that the intellectual plane was more than the ideas it may contain. "[M]entality", he said, "also exists in non-verbal creatures because this transformation [of external to internal actions] is not confined to speech or the intellectual level" (Gal'perin 1967: 31). However, the formation of higher mental functions was, for him, exclusively linked to speech. Thus, he continues the previous phrase as follows: "But higher mental functions are formed in this way alone, and in this sense Vygotsky is perfectly right and perhaps even more so than he was able to demonstrate in his lifetime" (1967: 31). Gal'perin considered that speech remains the objective carrier of actions when the latter is divorced from things. This is why he considered that internalization remains, to a large extent, embodied in speech.

### 3. The Outer Mind

In his efforts to go beyond the debate held during the first decades of the 20th century between psychology (e.g., as understood by Wundt) and anti-psychology (e.g., as represented by Husserl), Valentin Nikolaevich Voloshinov (1844/5-1936) proposed that psyche would not be studied within the paradigm of natural-scientific studies but within a sociological one. For him, psychology is necessarily social psychology. One of the cornerstones of his account consists in noticing that we do not inhabit a mere concrete, material world, but a world full of meanings, and that meaning belongs to the order of signs. To this, he added the remark that the functioning of the psyche can neither be reduced to, nor be located in, the physiological processes underlying it, and that, consequently, the psyche must be located somewhere else. "[S]lide by side with the natural phenomena," he wrote, "with the equipment of technology, and with articles for consumption, there exists a special world—the *world of signs*." (1973: 10; italics as in the orig-



inal). And Voloshinov insisted that such a world cannot be reduced to the material world: "Without ceasing to be a part of material reality, such an object (i.e., the object converted into a sign—L.R.), to some degree, reflects and refracts another reality." (1973: 9). So far, his approach seems to be one of those that we find in idealistic accounts. Nevertheless, in contrast to these, he overtly condemned the idea that the signs are but the expression or the manifestation of inner life. For him, it is not the psyche that explains the sign but the other way around. Where, then, is the mind if it is not in the realm of the private, inaccessible interior life of the individual from where signs would supposedly be emanating as carriers of ideas? It is not *within* the organism. To be more precise, the mind, suggested Voloshinov, is in this geographical place that he called the *territory of the sign*.

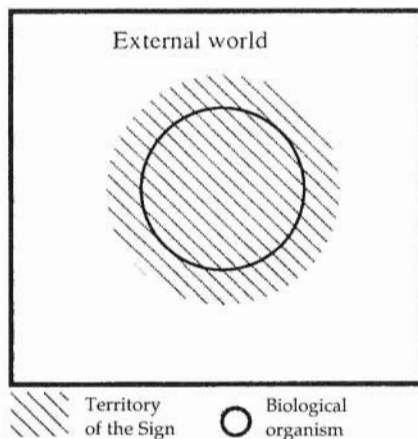
In order to understand this, we need to start with the actual place of the individual in his or her own environment. As a biological organism, the individual is placed *in* the external world. But since the psyche cannot be reduced to its physiological aspect (otherwise we would fall into a kind of unsustainable "vulgar materialism") or to its symbolic aspect (the sin of idealism) the psyche must intersect the interior as well as the exterior of the biological organism. Such an intersection or encounter occurs in the territory of the sign. And Voloshinov specified that this "encounter is not a physical one: *the organism and the outside world meet here in the sign. ... This is why the inner psyche is not analyzable as a thing but can only be understood and interpreted as a sign*" (1973: 26; italics as in the original).

For the very same reason, the territory of the sign is inhabited by the different systems of ideas that humans produce concerning laws, scientific knowledge, religion, literature, aesthetics, and so on (see Voloshinov 1973: 9; 1976: 50)—systems of ideas that he, as well as Bakhtin, called *ideologies*. (As noted by Morris (1997: 249), the Russian word *ideologiya* has a less colored political meaning than its corresponding English translation).

In one of the passages of *Marxism and the Philosophy of Language*, Voloshinov says:

The psyche enjoys extraterritorial status in the organism. It is a social entity that penetrates inside the organism of the individual person. Everything ideological is likewise extraterritorial in the socioeconomic sphere, since the ideological sign, whose locus is outside the organism, must enter the inner world in order to implement its meaning as sign. (Voloshinov 1973: 39)

To better understand Voloshinov's account, let us provide a topological panorama of the relations between the external world, the territory of the sign and the biological organism. In Figure 1 we have illustrated



**Figure 1. A Topological representation of Voloshinov's Territory of the Sign.**

of signs. Of course he did not mean "palpable" signs. The inner signs he was referring to as the actual content of psyche and consciousness were sensibly inner speech: "Inner sign is, after all, preeminently the word, or inner speech" (Voloshinov 1973: 37). Inner speech appears for him as a somewhat contracted form of outer, multivoiced speech: the units of inner speech "resemble the *alternating lines of a dialogue*" (1973: 38, italics as in the original).

But if ideology and psyche exist in signs and both inhabit the same space, what is their difference? The difference between them is one of *orientation*:

...any outer sign expression, an utterance, for instance, can also be organized in either one of two directions: either toward the subject himself or away from him toward ideology. In the first instance, the utterance aims at giving outer sign expression to inner signs, as such, and requires a purely psychological kind of understanding. In the second instance, a purely ideological, objective-referential understanding of the utterance is required. (Voloshinov 1973: 36)

In Figure 2 we provide a topological panorama of the psyche and ideology, stressing their difference in orientation with respect to the biological organism. Although Figures 1 and 2 do not appear in Voloshinov's work, they may help us to better understand Voloshinov's ideas.

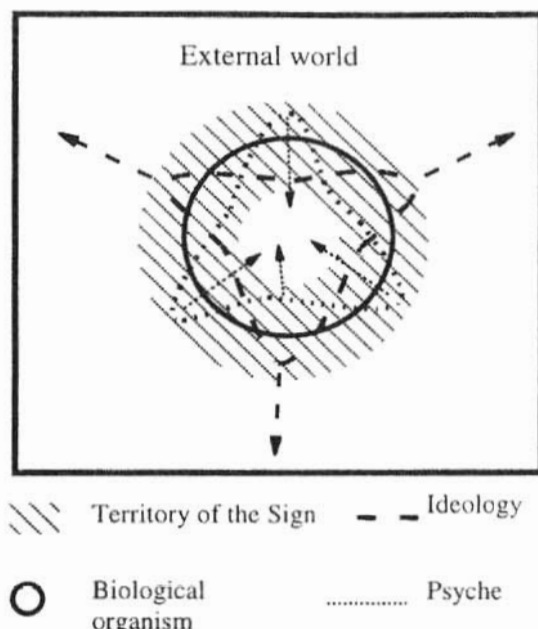
Voloshinov then raised the question of how inner speech is implemented, and the answer was provided in terms of the appropriation, by the individual, of the speech that she or he encounters in social life, an

the aforementioned relations, stressing the fact that the territory of the sign in which the psyche and ideology lie has the dual status of extra-territorial and intra-territorial domain with respect to the biological organism.

But Voloshinov went further and tried to specify the boundary between ideology and psyche. In order to do so, he had to explicitly say what the real content of the psyche is, and his answer was: *the inner sign*.

For him, psyche is made up





**Figure 2. A Topological representation of Voloshinov's distinction between psyche and ideology.**

idea that is similar in many aspects to Vygotsky's "genetic law of cultural development". Voloshinov wrote:

Speech had first to come into being and develop in the process of the social intercourse of organisms so that afterward it could enter within the organism and become inner speech. (1973: 39)

The appropriation of speech and, in general, of all forms of human communication from whence the human psyche arises, is not a process *per se*. "[A] psychic phenomenon becomes explainable solely in terms of the social factors that shape the concrete life of the individual in the conditions of his social environment" (Voloshinov 1973: 25-26). By this move, Voloshinov distanced himself from other accounts that presented psyche or mind as a context-free construct. He considered the "pure epistemological subject" to be mere fiction.

#### 4. Cultural Semiotic Systems

By linking the system of ideas to signs and couching the mind in the world of signs, Voloshinov was able to provide a monolithic picture of the individual and his or her society. In this picture, signs lose the

somewhat technological flavor of the Vygotskian notion of tools to master "natural" or "animal" behavior and gain a symbolic component. Of course, Vygotsky was aware that his idea of human behavior (that is, the "natural behaviour" humanized through the use of signs) is embedded in a universe of symbols other than the "material" ones. Nevertheless, he seems to have considered that the view of a symbolic universe encompassing human actions may have easily led to forgetting the contextual nature of cognition, and thereby ended up with a description of the high psychological processes with no link to the reality of the individuals and the actions that individuals carry out in this reality. Thus, when discussing the role of play in the development of children, Vygotsky (1967; 1978: 92 ff.) insisted that children's activity in play is neither to be considered as independent of the context nor is it to be thought of as independent of the particular motives of the child.

[I]f play is understood as symbolic, there is the danger that it might come to be viewed as an activity akin to algebra; that is, play, like algebra, might be considered a system of signs that generalize reality, with no characteristics that I consider specific to play. (1978: 94; 1967: 9)

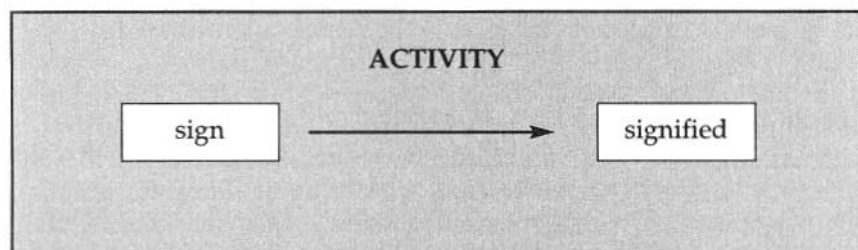
Vygotsky's point was that both the *motives* underpinning the actions in play and the children's plots come from their cultural reality. In a play, the child who wants to be a banker or a priest takes his role from what is socially expected of a banker or a priest. The dialogue and actions that he will display will be coherent with what he believes fits those social agents. Hence the symbolic activities always remain related to their reality.

Concerning Voloshinov's account of the mind, it is clear that such an account does not afford the risk of seeing symbols out of their own reality and fall into an idealistic perspective. In fact, signs emerge, he said, "only in the process of interaction between one individual consciousness and another." And he added, regarding consciousness, that it "becomes consciousness [...] only in the process of social interaction" (1973: 11).

Voloshinov and the Nevel-Vitebsk-Leningrad intellectual circle—a circle that operated during the 1920s and included, among others, Bakhtin—did not elaborate a theoretical description of social interactions. In contrast, A. N. Leontiev and his disciples devoted their efforts to the theorization of such interactions and their epistemological dimension through a detailed elaboration of the category of activity first introduced in psychology by Vygotsky himself. Activity, in Leontiev's view, appears embedded in a larger system of social interactions and rooted in the material forms of production of the individuals. In his last book he wrote:

With all its varied forms, the human individual's activity is a system in the system of social relations. It does not exist without these relations. The specific form in which it exists is determined by the forms and means of material and mental social interaction (*Verkehr*) that are created by the development of production and cannot be realized in any other way other than in the activity of concrete people. (Leontiev 1984: 92; see also Leontiev 1981: 47)

We shall not enter here into a discussion of the Leontievan category of activity (see e.g., Leontiev 1984). Suffice it to say that concerning signs, Leontiev agreed with Vygotsky in that tools mediate activity (e.g., Leontiev 1981: 54-58). This previously led us to suggest (Radford 1998) that a sign always remains framed by the practical activity of the individuals and to conceive the sign as a semiotic object functioning in a map or environment where the specific characteristics of the activity has to be taken into account (see our Figure 3).



**Figure 3. The semiotic map of the sign.**

Moreover, Leontiev considered that the main characteristic of activity is its goal-oriented aspect, that is, its *motive* or *object orientation* (Leontiev 1981: 48)—something that could be material or ideal (Leontiev 1981: 59) and not only encompassed by cold cognitive processes but by desires and emotions too. Furthermore, in his theoretical description of activity, he took into account the fact that motives, desires, and emotions are not objects on their own but are caught in a web of social significations. Those social significations, he said, "are created by the society and they have their own history in the evolution of language and in the forms of the social consciousness; they reflect [...] the ideological representations of the society—religious, philosophical, political." (Leontiev 1984: 163).

The fact that activity is mediated by signs and that its motive is in one form or another related to the "supra-individual" significations (Leontiev 1984: 162) belonging to the realm of the "social consciousness" or "collective consciousness"—terms used by Leontiev (1984: 146) in his effort to relate the cultural ideas (e.g., beliefs) to the individual

consciousness via activity—prevent us from considering the individual's use of signs as independent of the signifying forms of the culture. A. A. Leontiev (A. N. Leontiev's son) put this very clearly:

the sign [...] emerges as a constitutive part of the system of conventional signifying forms and means for external expressions and the consolidation of ideal phenomena. (1981: 244)

Seen from the "supra-individual" sphere, signs do not lie randomly in the cultural space of the individuals. Given that culture is not homogeneous, signs are not equally distributed nor used in an indifferent manner. Rather, signs are culturally patterned and socially distributed (Cole 1996a, 1996b). They are embodied by what we want to call different *cultural semiotic systems*; that is, those cultural systems which make available varied sources for meaning-making through specific social signifying practices.

The important fact that signs are embodied in cultural semiotic systems make it impossible for signs to be merely a substitute for something else, as medieval scholars had conceived it. Indeed, upon closer examination, the sign and its signified are not in a sole relationship of substitution. This relationship, conveyed in Figure 3 by the arrow, is already embedded in a cultural semiotic system (CSS) that provides the practical activity of the individuals with meaning. Thus, for instance, the Mesopotamian token representing a cow is a sign (an "arithmetical" one) framed by the symbolic system of the "archaic" economy without which it would be impossible to imagine the division of professions, the distribution of land, and the political and religious role of the Temple. As Castoriadis, one of the most acute critics of Troskianism says:

Everything presented to us, in this socio-historic world, is indissociably woven with the symbolic. Nor will it ever be exhausted. Real actions, individual or collective - work, consumption, war, love, childbirth - the innumerable material products without which no society could survive an instant, are not (not always, not directly) signs. But each is impossible outside of a symbolic system. (1975: 162)

According to this, the arrow of semiotic connection between the sign and its signified in the semiotic map of a sign depends on the specific CSS in which such a connection arises, so that this connection may be written as follows:

Sign  $\xrightarrow{\text{(CSS)}}$  Signified

The concept of Cultural Semiotic System (CSS) that we want to submit is hence a system conveying contextual *significations* embodying the use of signs. Those significations sanction rules of sign use, making the pro-

duction and understanding of a sign "inextricably tied in with the situation in which the sign is implemented." (Voloshinov 1973: 73). But this is not all. Besides its influence at the level of the territory of the sign, the CSS is a structural element in the organization of the activities of the cultural group as a consequence of the specific sign-mediated semiotic nature of the actions of which an activity is made up. Hence, topologically speaking, a CSS appears twice: (i) in interaction with the territory of the sign, on the one hand, and (ii) in interaction with activity, on the other. Thus, for instance, a stone intended for an axe has a certain value for the Eipo of New Guinea—a value linked to the forms of production of the Eipo economy. As a representation of this value, the stone becomes a sign. In the typology of signs employed by Cole (1996a, 1996b), the stone itself belongs to the "primary level." Another sign used to represent this stone (a drawing or any other mark, for instance) would be a sign belonging to the "secondary level." But the ideological signs belonging both to the primary and secondary levels are embodied in a CSS which provides a signification for the symbolic "act of gifting" in which the stone will be included. Eibl-Eibesfeld *et al.* (1989) observed that the Eipo behave in the course of gift-giving and gift-accepting settings with feigned calmness. On those occasions, they give away valuables and also accept them. After the exchange of gifts, the Eipo secretly spread out their gifts and check them. They see how many axe stones they were given as well as other goods and figure out the balance. Those who entered into an unfavorable exchange of gifts become angry and throw themselves on the floor of their house. From our theoretical perspective, Eipo's actions and stone-sign use appear underpinned by the idea of a fair exchange of goods.

Of course, the idea of a fair exchange of goods is not an Eipo characteristic. Such an exchange has been observed again and again in cross-cultural research. For example, while in the Eipo setting the exchange includes a "calculation" of the values that they give to the goods, that is, a calculation that we may term as an Eipo mathematical calculation even though it does not acquire an explicit form; a similar exchange, in commercial settings, was called *baratto* (barter) by the Italian Medieval merchants (e.g., Franci and Toti Rigatelli 1982: 78, or Swetz 1989) and was arithmetically dealt with in terms of the "rule of three". But the particular structure of the Eipo gift exchange activity (the calmness they show in front of the others, the secret calculation of the given and received gifts, the expression of joy or anger) is typified and normed by the Eipo's own cultural semiotic system which provides the necessary significations for such a gift exchange setting to occur.

We can put the above remarks in a more general form as follows: through their interaction with activity, and given the semiotic nature of

the latter, cultural semiotic systems account for rules of sign use. Cultural semiotic systems convey a normative dimension which sanctions the production and understanding of signs and actions.

As we will see in the next section, in their interaction with the territory of the sign, cultural semiotic systems account also for the basis of the generation of modes of knowing (or *épistèmes*, to use Foucault's expression) which, in turn, provide ideologies with specific contents. At this point of our discussion we must say a word about the origin of CSSs and their epistemological implications.

It is worthwhile to recall Cassirer's investigations about the relation between language and the ideas and significations that the individuals form about their world. Cassirer, well known for writing against the theory of reflection, a theory according to which our ideas are but the mirror of external objects, rejected materialism arguing that

knowledge can never reproduce the true nature of things as they are, but must frame their essence in "concepts." But what are concepts save formulations and creations of thought, which, instead of giving us the true forms of objects, show us rather the forms of thought itself? (Cassirer 1953: 7)

And trying to overcome the dualism object/subject of Kantianism, he adhered to the idea that the world is created by language. What one can call "reality" is, for him, that which the *forms* of sensibility and representation give to us. Those forms—that he called *symbolic forms* and which comprise language, the mythical and religious universes, the arts—are conceived as an energy of the spirit linking the sign to the spiritual signification. In the objects that they produce, the symbolic forms enclose their own truth and meaning. He says:

Instead of measuring the content, meaning, and truth of intellectual forms by something extraneous which is supposed to be reproduced in them, we must find in these forms themselves the measure and criterion for their truth and intrinsic meaning . . . we must see in each of these spiritual forms a spontaneous law of generation; an original way and tendency of expression... (Cassirer 1953: 8)

The system of social significations conveyed by our idea of CSSs can neither afford any spontaneous law of generation of intellectual forms nor any neo-Kantian intuition of meaning. The system of social significations that we have in mind is unalienable from the practical activities of the individuals and the reality that they co-construct. It is a well known fact that Leontiev insisted on the effective correlation between activity, its motives, and the actual relationships of the individual with reality (see Kozulin 1996). This leads us to understand cultural semiotic systems as socio-historically constituted and arising from practical

activities and from the culture that those activities create on their way. As we suggest in Figure 4, social significations are created by activities and vice-versa.



**Figure 4.** The dialectical relation between activity and CSSs.

We shall later come to a more detailed discussion about the relationship between activity and CSSs. For the time being, let us note that the consequences for cognition of the role of cultural symbolic systems have been largely ignored in cognitive individual-centered accounts. As long as cognition is conceived as a set of private processes occurring in the head, and the exterior world is seen as merely a space where the individual expresses those processes, there is no reason to look at cultural symbolic systems. In contrast, if mind and activity are seen as constitutive of each other, if mind is seen as intertwined with the practical activity of a society, and intellectual activity is seen as an ideal reflection of it (Ilyenkov 1977: 260; Davydov 1990: 237), then the task of investigating the cultural symbolic systems becomes urgent.

For the purposes of our discussion let us now give a brief example. It is clearly documented that the Inca used a system of knots in fabric to count—the quipus. Knots hence appear as signs, as psychological tools, to remember and to execute some elementary arithmetical calculations. The Inca's choice of the actual material support cannot be attributed to a pure coincidence. Colored fabrics had, certainly, a symbolic value that served to convey the idea of material wealth and high social position. Counting, of course, emerged as in other cultures from practical activities, and was a primal necessity for the control of local and individual economies. Counting was done on culturally valuable objects. However, the actual choice of the Incas had many restrictions if we compare their counting technology with the clay tablets-based technology as found in Mesopotamia at the end of the 4th millenium BC. As we can see, the semiotic systems where counting signs were placed offered different cognitive possibilities. They were not cognitively neutral. Of course, by this we do not mean that if the Incas had had the marvelous idea of using clay tablets, they would have undergone the same development as the Mesopotamians! It would be a mistake to see cultural symbolic systems from a tempting causative viewpoint.



## 5. The Euclidean and Pre-Euclidean Theory of Even and Odd Numbers

We want to discuss here, in accordance with the ideas presented in the previous sections, a brief example of how cultural semiotic systems issued from the practical activities of a culture open possibilities for the emergence and legitimization of those modes of knowing (or *épistèmes*), which in turn provide ideologies with a definite and specific content. In order to do so, we will turn our attention to the pre-Euclidean and Euclidean theory of Even and Odd Numbers and see Euclid's mode of proving as an ideological instance of the general classic Greek *épistème* and its own cultural semiotic system.

In 1999 Netz published a remarkable book dealing with the concept of deduction in Greek mathematics. Netz's book provides an insightful epistemological discussion of how deduction arose and took form in Greece. Even though he devotes a chapter to the historical setting inclusive demography and social classes are discussed, his account is not quite successful in making the connection between the analysis of the mathematical content and the Greek cultural background. Indeed, the historical setting constitutes the last chapter of the book, almost appears as an appendage to the mathematical and epistemological analysis. The reason for Netz's difficulty in relating culture and mind may be due to the fact that key concepts such as activity and practice remain theoretically undeveloped. As a result, it is not clear how cognition relates to practice.

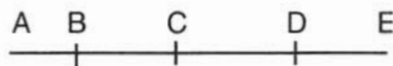
Here we shall start by looking at a proposition contained in Book IX of Euclid's *Elements*. The proposition bears the number 21 and reads as follows:

If as many even numbers as we please be added together, the whole is even.

And the proof is the following:

For let as many even numbers as we please, AB, BC, CD, DE, be added together; I say that the whole AE is even.

For, since each of the numbers AB, BC, CD, DE is even, it has a half part; [VII. Def. 6] so that the whole AE also has a half part.



But an even number is that which is divisible into two equal parts [id.]; therefore AE is even. (Heath 1956: 413)

Proposition 21 was already very well known prior to Euclid's time and formed part of the theory of Even and Odd Numbers (of which

some propositions are contained in *Elements* IX, 21-34). In all likelihood, the theory dates back to the early Pythagoreans, who had investigated it through the use of stones, as they did for the theory of polygonal numbers; that is, those numbers whose units can be arranged in polygonal forms (i.e., triangles, squares, pentagons, and so on). Aristotle mentions that "...Euritus found out what was the number of the things (for example of a man or a horse), imitating the figure of the live things with stones, as some persons have put the numbers in forms of triangles or squares" (*Metaphysics* 1092<sup>b</sup>10). For instance, 3, 6 and 10 are triangular numbers; two square numbers are 4 and 9.

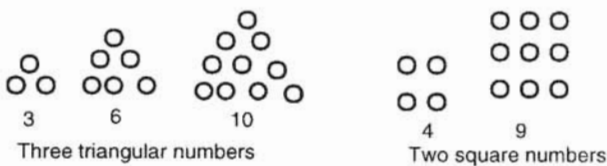
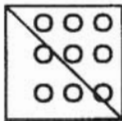


Figure 5. Triangular and square numbers.

Stone-manipulation techniques allowed the Pythagoreans to realize that a square number is composed of two consecutive triangular numbers:



A square number is made up of two triangular numbers.

Figure 6. Pebbles technique to investigate numbers.

Philolaus, a philosopher of the 5th century BC, who, as the story goes, was the first to have broken the oral teaching methods of the Pythagorean brotherhood by writing down some of their ideas, said "The number has two different forms, the even and the odd, and a third composed of both, the even-odd" (Freeman 1956: 74, fragment 5).

Within the Pythagorean non-deductive, pre-Euclidean concrete arithmetic, some propositions about even and odd numbers seem to have been elaborated (e.g., Becker 1936; Lefèvre 1981). According to Becker, proposition 21 (like others) was proven through the use of concrete examples. The central idea in the proof is the property of even numbers as those that can be halved, allowing for a geometrical representation of this kind:

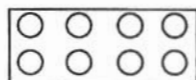


Figure 7

The proof could then be *displayed* as follows:

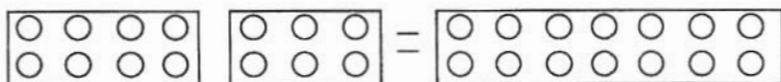


Figure 8

Proposition 22 was stated by Euclid in these terms (Heath 1956: 413):

If as many odd numbers as we please be added together, and their multitude be even, the whole will be even.

In this case, the proof seems to have been based on the idea that odd numbers differ from even numbers by a unit, so that a typical geometrical representation of odd numbers is as follows:

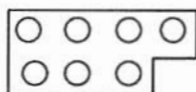


Figure 9

The proof then could be displayed in the following form:



Figure 10

The cultural symbolic systems and their corresponding general epistemic structure will provide us with an understanding of all the trouble that Euclid inflicted upon himself to prove something that everybody already knew and that would have been easier to see with the pebbles technique. In short, we want to understand that which Taisbak, in a radiant expression, referred to as "Euclid's neurotic obses-

sion", i.e., the obsession of proving things that were not disputed by anybody anyway (Taisbak 1971: 13). *Grosso modo*, what we want to suggest is the following: as the Eipo's activities encompassing the negotiation of gifts are completely coherent within the realm of its own *ideology* and *mode of acting*, so is Euclid's mode of proving.

The first point to be stressed is that Euclid's mode of proving cannot be understood within the field of mathematics itself. This mode of proving was transposed from other activities to mathematics. Szabó, through a philological analysis, has shown that the key mathematical terms required in a deductive system, like hypothesis, thesis and axiom, were first used in the reflections about dialectics. In fact, he went further and showed an impressive similarity between the mathematical methods and those of the dialectics (Szabó 1977: 262 ff.), which led him to suggest that initially mathematics was but a part of dialectics as developed by the Eleatan philosophers. As Szabó suggests, the passage from the empirical (pebbles- and geometric drawing-based) Greek mathematics to the deductive mathematics was underpinned by:

1. the very distinction between "real" knowledge and "opinion" drawn by Parmenides in his famous didactic poem and
2. the distinction between the Being and non-Being.

The two aforementioned points gave rise to an ontology and an epistemology of a very particular nature (something that we do not necessarily notice given that our own way of thinking has many of its fundamental roots in the ontology and epistemology sketched in the Parmenidian poem). Whereas, on the one hand, the distinction between "real" knowledge and "opinion" led to the refusal of sensations as producer of knowledge, and, on the other, to the legitimization of "real" knowledge as something attainable by the reason only, the emergence of the concept of hypothesis and the indirect proof (or proof by *reductio ad absurdum*) was related to the distinction between the Being and non-Being.

The relevance of these points in the formation of Greek *épistème* can be better understood in reference to the following two aspects: Firstly, the fundamental opposition between Being and non-Being made it possible to generate a mode of knowing where the fertile principle of a third excluded term was made available. Indeed, the Being and non-Being opposition created a borderline in the ontological space elaborated by the post-Parmenidean philosophers, and authorized Reason to posit itself on only one of those sides, on the enlightened side of the Being. It is on this side that one will find all that is thinkable. It is impossible for any real thing to be and not to be, or even still to be something else. But there is another important point concerning the real

objects. They are metaphorically conceived as occupying a certain space on the side of the Being. They cannot be indefinite; otherwise they can fall on the dark side of the changing things, on the side of the things which do not keep an identity with themselves, on the side of the non-Being. A real object in the classic Greek *épistème* is clearly delimited. Since any real thing is immovable and eternal, a real thing can only find linguistic expression in the form of the verb *to be*. "We say that it [the physical world] was and is and shall be," says Plato; "but 'is' alone really belongs to it [the eternal world]" (see *Timaeus* 38c2-3, 37e-38a). The necessary delimitation of objects was accomplished by clear definitions compulsively based on the use of the verb *to be*. "A number *is*," says Euclid, "a multitude composed of units"; "An even number *is* that which is divisible into two equal parts" (Book 7, definition 2 and 6; Heath 1956: 277; emphasis added). Without delimitating the space of the object (something achieved by the definition), Reason cannot take the object in its hands. Very different was the case of the Babylonian ontology: we do not find a definition of number or of polygonal figures or angles. The objects are there in front of the scribe, accessible to him through his senses. As we noticed elsewhere (Radford 1997a), Babylonian *épistème* was triggered by a peculiar realism where the mathematical objects were mediated by generalizations of the surveyors' practical activities.

Secondly it is worth noting that the refusal of appearances—another factor decisively underpinning the configuration of the mode of knowing of objects in Greek thought—was a recurring theme in the whole Greek intellectual tradition. Truly, since Homer's epic poems, appearances were linked to deception. In the *Iliad*, Troy was vanquished by the Greeks through deception by hiding some Greek warriors in a wooden horse that the Trojans brought into their city. In the *Odyssey*, Athena appeared to Ulysses in the form of a young shepherd to tell him about Penelope, his wife, and her suitors. To take vengeance upon them, Athena metamorphosed Ulysses into an unsightly beggar so that he would not be recognized. From Parmenides onwards, as we pointed out previously, Greek thought takes on a new position, sets a clear distinction between true knowledge and opinion, and clearly refuses to allow appearances of the sensible world to inform us about the true knowledge. This does not mean that after Parmenides, the role of appearances in Greek thought vanished. Certainly, this role split into two parts: a *positive* one, which found expression in the artistic domain (masks, for instance, will continue to be used in the comedies and tragedies of artistic festivals) and a *negative* one, consisting in the refusal to build true knowledge on appearances. But this negative role was, in fact, a producer of knowledge, too. Greek scientific *épistème* did not

merely shy away from or ignore the sensible world: Greek scientific *épistème* was certainly built as a subtle and wonderful expression against appearances.

Placed within the Eleatan-Platonic mode of knowing, Euclid could not prove the already well-known propositions about the odd and even numbers in the visual explanatory way that the early Pythagoreans did. As we just saw, after Parmenides, visualization fell into disgrace in scientific discourse. Explanation or justification, as we suggested in a previous work (Radford 1996a), is a social affair and not a "natural" pre-given event (for if it were pre-given, the Nietzschean question would be *by whom?*). Certainly, the anti-visualist category of explanation used by Euclid accords itself dialectically with the "Being/non-Being" structure of the ontological space. It is here, in this ontological structure, that we find the possibility of legitimization of the deductive proof. As Aristotle says in *Posterior Analytics*, "we think that we know when we know the causes". That which causes a proposition to be true can come from two sources only: either a proposition is true because it is inferred from the postulates (in which case the truth descends, as water in an aqueduct, from the pure source, carried by the secure canals of Reason), or a proposition is true owing to the fact that its negation leads us to the side of the non-Being, by concluding that something is and is not. In the latter case, a chain of deductive reasoning aiming to prove "p" and assuming momentarily "non-p", is not carried out with the intention of *explaining the contradiction* (in the sense of explaining that which ultimately causes the contradiction) but to ensure that we have trespassed into the territory of the non-Being. The contradiction finds its own mode of explanation in the Eleatan-Platonic ontology. Thus the incommensurability of the diagonal with the side of a square (as reported by Aristotle) is not explained in terms of the very reason that makes incommensurability possible, but because the hypothesis of commensurability leads us to say that the side of the square is both an odd and a non-odd (or even) number. This is the reason for which the Euclidean answer to the question "why?" is not only very different to the Pythagoreans' but to the one that we can expect from a child in a modern school too.

To sum up, given the context of the previous discussion, Euclid's "neurotic obsession" may be understood. Euclid's use of symbols as well as his adoption of methods of proving appears clearly framed by the accepted mode of knowing afforded by a cultural semiotic system, articulated, as we have seen, in two important beliefs: the distinction between the Being and non-Being and the refusal of the sensual realm as repository of true knowledge. The semiotic cultural system legitimizes some signs and their use and excludes others. Hence, pebbles or

stones are forbidden while lines and segments are permitted (see the previously quoted Euclid's proof of proposition 21). In *The Republic* 510<sup>d</sup>, when talking about the mathematicians, Plato says: "You know too that they make use of and argue about visible figures, though they are not really thinking about them, but about the originals which they resemble". In the same way, some methods of investigation and proof are accepted and others are condemned. For instance, the use of mechanical instruments (as used by Eudoxus and Architas, for example) in the study of the two means in proportion is criticized by Plato, while the use of compass and straightedge is allowed:

But Plato took offense and contended with them that they were destroying and corrupting the good of geometry, so that it was slipping away from incorporeal and intelligible things towards perceptible ones and beyond this was using bodies requiring much wearisome manufacture. (Plutarch, *Lives: Marcellus*, xiv; quoted by Knorr 1986: 3)

## 6. Modes of Acting: The Symbolic Component

That some reasons for the actual shape taken by the classic Greek *épistème* may be related to the socio-economic and political arena is beyond any doubt. Restivo (1992) has already stressed the traces left by the distinction between manual and intellectual tasks and the corresponding difference between slaves and masters in the actual form of Greek mathematics. We can find, in this distinction, a reason for the rejection of mechanical drawing instruments. By the same token, the aesthetic perfection that the Greeks found in the circle and the straight line may account for the acceptance of the compass and the straight-edge (Parmenides, for instance, praises the sphere as a geometrical form for its uniformity and equally balanced shape. Obviously, those elements are typical of the circle and the straight line, too). But I believe this does not suffice. Division of labor and slavery already existed in pre-Greek societies (since the Uruk period in Mesopotamia, for instance). The astonishing and marvelous level reached by Greek mathematics cannot be attributed to an economic factor either, for the Greece of the golden age was poorly industrialized, mainly sustained by fishery and an agriculture limited by the hard conditions of its cultivable soil (e.g., Bolkenstein 1958). Along with all this, we need to look closer at the relationship between activity and the cultural semiotic systems mentioned in Figure 4. This bi-directional relationship *Activity*  $\leftrightarrow$  *CSS* is neither a naked nor a neutral relationship. As noted in section 3, such a relationship plays a structuring role in the activities that the individuals carry out. In fact, we have more. The relationship between activity and CSS is modulated, on a more general level, by a *symbolic structure*



that accounts for the *modes of acting* from where groups of activities find a generic expression and become socially institutionalized, one example being that of the institutionalization of activities related to planned education and its social space—the school. Choices in terms of admittance, curriculum (whose norms indicate what should and should not be taught), pedagogy (which says how to teach), and so on, cannot be understood without reference to the way in which knowledge is refracted by the symbolic structure and the cultural modes of acting. To pursue our example of Greek thought, let us mention the fact that the Greeks believed that human nature was such that a bad action was caused by a lack of knowledge. They saw in (true) knowledge a path to counter the low aspect of human nature (e.g., passions) and to become a good man. (They considered passions as an illness of the soul; Galen himself wrote a treatise entitled *On the cure of the passions of the soul*.) For them, knowledge, as well as mathematical knowledge (see Roochnik 1994) had an ethical value.

Of course, in our contemporary society, knowledge has an ethical value, too. All the bases from whence modern thought arose were embedded in the ethical idea that knowledge, and particularly scientific knowledge, will provide the whole of humanity with a better world in which to live (Lyotard 1979). But the expression of those beliefs belonging to the Greek CSSs encountered in Greek society is completely different from ours. Truly, in Plato's time, to a large degree, the activity of learning mathematics was seen as a propedeutic tool in the methodological investigation to contemplate the Ideas. As an instance of the Greek modes of acting, the students came to the sophists in order to be instructed in such matters (for some details on the curricular and pedagogical choices see Chapters 4 and 6 of Fowler 1987, Mueller 1991 or Pfeiffer 1968). While the first task of the sophists was then to take the students' mind away from the business of the perishable world and to initiate them in the only life worth living, we, in our contemporary societies, make Herculean efforts to convince students' of the utility of mathematics in "real life"! Hence, as in our case, although with clear differences due to the discrepancy in terms of the contents of the corresponding cultural semiotic systems, the actual form of the sophists' activities found semiotic expression in the relationship between the general structure of the activities of the society and the beliefs belonging to the CSSs.

The different semiotic expressions which a symbolic structure and its modes of acting may afford go beyond the structure of activities around knowledge. In general terms, any symbolic structure accounts for the specific link between knowledge and power. Regardless that Plato's systems of government were not actually implemented, in *The*

*Republic* and *Laws*, the wiser and more learned people were those called to govern (something that contrasted with other cultural conceptions based on blood affinity and succession). The distinctively Greek appraisal of knowledge thus appears as the manifestation of the relationship of activity and cultural semiotic systems in the symbolic structure and modes of acting that they produce. Such a distinctive appraisal of knowledge hence finds explanation when it is compared, for instance, to the case of the Mesopotamian societies, where very few kings were able to read and write—tasks that culturally were considered bureaucratic and that consequently were relegated to the scribes (Radford 2001).

Education as the path to knowledge and the appropriation of power consequently acquires a different expression depending on the specificities of the symbolic structure—a semiotic expression that Foucault termed *la volonté de vérité* (the will of truth):

This will of truth, like other exclusive systems, relies on an institutional support: it is both strengthened and reinforced by a whole weight of practices, like pedagogy, certainly, like the system of books, of editing, of libraries, like the learned societies of the past, the laboratories of today. But it is also reinforced, doubtless more deeply, by the manner in which knowledge is power in society, how it is valued, distributed, shared, and in some ways, attributed. (Foucault 1971: 19-20)

## 7. Synthesis

Let us now try to put together the different threads of our discussion about culture and mind. We first discussed Vygotsky's genetic law of cultural development as the Vygotskian answer to the general question about the relationship between culture and society in the constitution of the mind—a question that was investigated by some contemporaries of Vygotsky, like Janet (see van der Veer and Valsiner 1988) and Baldwin. While Baldwin considered that the relation between the individual and the social resided in a process of bi-directional projective interpretative reading, affording, by differentiation (e.g., through imitation and practice), the emergence of the child's self-consciousness (see Baldwin 1911: 24 ff and 124), Vygotsky, in contrast, elaborated a semiotic account in terms of internalization through signs of the activities that individuals carry out in the external plane. The Marxist orientation of his account was an asset, as it went beyond behaviorist accounts, at the same time as it demarcated limits. Indubitably, his account was underpinned by a particular technological idea often shared by Marxist anthropology (e.g., Bloch 1985) concerning the relationship of human beings and

nature: human beings act together towards the mastering of nature. Van der Veer and Valsiner (1991: 221) have pointed out that the decisive role this vision of technology played in Vygotsky's thought and his idea of sign, led him to a psychological account that they term as "psychotechnology"—something van der Veer (1996) credits to Vygotsky's limited use of the concept of culture.

In a critique of Vygotsky's approach, Zinchenko addressed similar concerns. He wrote that for Vygotsky

[t]he central characteristic of the human mind was thought to be mastery of the natural or biological mind through the use of auxiliary psychological means. Vygotsky's fundamental error is contained in this thesis, in which he misconstrued the Marxist conception of the historical and social determination of human mind. (Zinchenko 1984: 66)

According to Zinchenko, Vygotsky failed to include the history of society's social and economical development in the history of cultural development (1984: 70).

Be this as it may, Voloshinov, as we saw, presented a less technological view. Like Vygotsky and Gal'perin, he, too, found the central point in the socialization of the mind in language. Although in both Vygotskian and Voloshinovian approaches, the sign has a central epistemological role, Voloshinov offers us an exotic blend of ideas from whence consciousness emphatically emerges as the multi-voiced sign with all its social accents, through a specific space that he called the *territory of the sign*—that space where ideologies and mind encounter each other.

Voloshinov and Vygotsky coincided in their emphasis of the important role of activities, from which reflection about the world emerges. The systematic study of activity, we noted, was carried out by Leontiev and his school. This led Leontiev to tackle the difficult concept of meaning—something that he tried to do without contradicting the Vygotskian concept of internalization. That this task was not undertaken without theoretical difficulties has been stressed by K. A. Abul'khanova (1973), one of Leontiev's critics. In any case, to go further Leontiev probably could not avoid the problem of linking actions to motives and beliefs.

Within the context of social systems of significations, we submitted our notion of cultural semiotic systems as systems comprised of beliefs (ontological, epistemological, esthetic, and so on), which generate patterns of meaning-making through activities and sign use, as structured according to the individuals' concrete mode of existence.

One of the characteristics of a CSS resides in the *relationships* that it keeps, on the one hand, with the ideologies (and the territory of the sign in general) and, on the other hand, with the individuals' activities.

While the former accounts for the modes of knowing (or *épistèmes*) of a cultural group and the actual content of ideologies, the latter accounts for the modes of acting in which activities are embedded. All those relations may be better understood by reference to the graph shown in Figure 11.

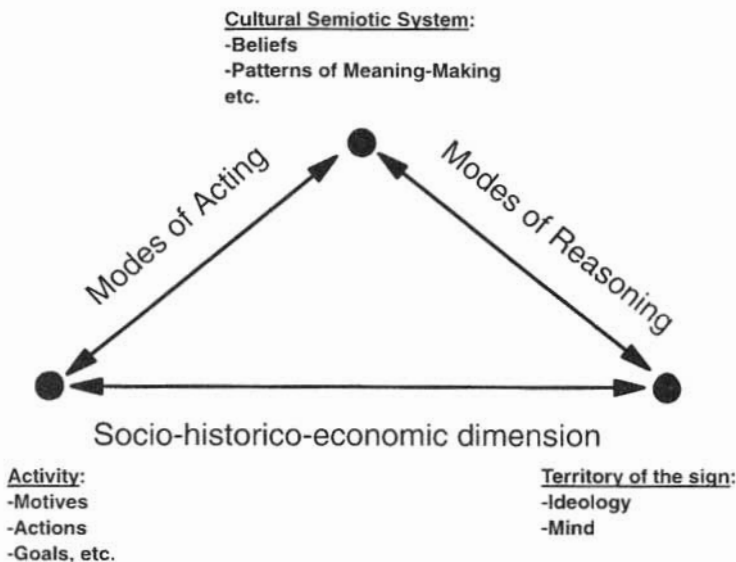


Figure 11

The dialectical nature of the relationships linking CSS, activity, and the territory of the sign shown in Figure 11 leads us to the dual of the first graph shown in Figure 12.

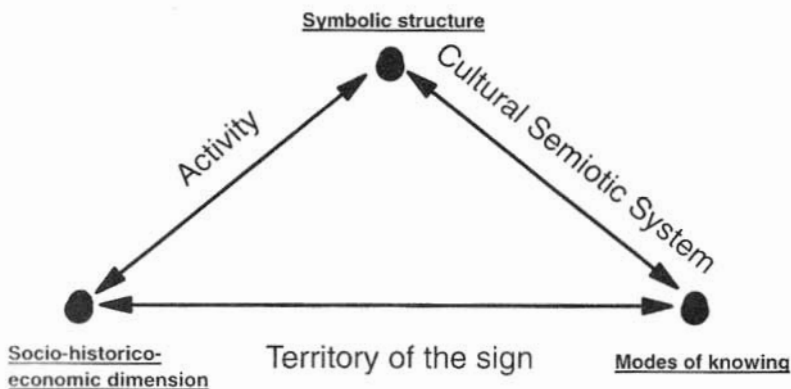


Figure 12

The dual graph shows that rather than merely terminal points of a dynamic process, these terminal points become, in turn, agents of the whole process. For instance, the CSS becomes the link between the symbolic structure and the modes of knowing. In developmental terms, primal and dual graphs are always alternating.

With the previous theoretical constructs, we attempted to provide an explanation of some relevant aspects of Euclid's theory of Even and Odd Numbers. We saw that the corresponding Euclidean mathematical methods and objects were couched in some beliefs originating in the Eleatan-Platonic ontology, and which penetrated all sorts of cultural activities, shaping attitudes to mathematical, philosophical, artistic and intellectual inquiry. Notice that those beliefs do not exhaust the sources of Euclidean thought. Euclid's mode of thinking was also influenced by an Aristotelian way of conceptualizing things, which resides in the central conviction that things (*species*) can be defined and organized by categories (*genders*), according to certain common particular sensual characters—something which Ortega y Gasset (1992) referred to as "sensual communism." We saw how, with the help of the verb *to be*, an object was rendered as something that is clearly delimited in accordance with the exigencies of the Greek *épistème*. Lizcano (1993) has clearly shown how the Greek mode of knowing, based on the conceptualizations of objects as delimited things on the side of the Being, excluded a reflection on what in one way or another could be related to negative numbers. Let us add here that this same relation to the Being and the resulting necessity of delimitating objects is, it seems to us, one key element in the answer to the following question that we raise in a specific Gadamerian sense (Radford 1997b):

Why could the Greeks of the classic era not work with unknown numbers and come up with something that might look like algebra?

(Evidently, in the previous question, we are not presuming the so-called "Greek Geometric Algebra" to be a numerical algebra disguised in geometric robes: Radford 1996b.) The post-Vygotskian perspective adopted here and the insertion of the theoretical construct of Cultural Semiotic System, although evidently applied incompletely in the analysis of Greek mathematical thought, allows us nevertheless to appreciate the incredible step taken by Diophantus, at the end of Antiquity, encapsulated in his term "undetermined" when he defined the arithmo (which in some way played the role of our "x") in his *Arithmetika* as "a undetermined quantity of units" (Radford 1996b). Although there is some conclusive evidence of an earlier Greek numerical-algebraic activity prior to Diophantus (Høyrup 1990; Radford 1996b, 2001), it is with him that such an activity became a legitimized mathematical practice. Dipohantus

divided his *Arithmetika* in "books" which organization resembled that of Euclid's *Elements* in that it contained a set of definitions and some porisms.

Of course, the formation of the Euclidean theory of Even and Odd Numbers can be seen as an instance of internalization (in Vygotsky's sense) of the Pythagorean pebbles-techniques. The example from Greek mathematics with which we dealt here shows that the process of internalization is not—as A. N. Leontiev (1981: 57) noticed when referring to internalization in general—merely the mimetic transferal of an external activity to a pre-existing internal plane. Rather, our example unveiled internalization as a complex process embedded in a cultural symbolic structure that can only be understood with reference to its own cultural semiotic system (in this case, the Euclidean one).

In general terms, what our discussion suggests is that internalization goes beyond the strict realm of sign use, and that a theoretical account of internalization requires a larger and richer concept of semiotic mediation capable of relating internalization to the symbolic structure of the society, as Leontiev's work indirectly hinted at. Our concept of cultural semiotic system, and the primal and dual graphs which functionally characterize it, is an attempt to approach, from a post-Vygotskian perspective, the problem of the social formation of mind which we mentioned in the very beginning of this paper.

## REFERENCES

- Abul'khanova, Kseniia Aleksandrovna. (1973). *O sub'ekte psikhicheskoi deiatelnosti* [About the Subject of Psychological Activity]. Moscow: Nauka.
- Baldwin, James Mark. (1911). *The Individual and Society, or Psychology and Sociology*. Boston: The Gorham Press.
- Becker von, Oskar. (1936). *Die Lehre vom Geraden und Ungeraden im Neunten Buch der Euklidischen Elemente*, Quellen und Studien zur Geschichte der mathematik, astronomie und physik, abt. B (Volume 3), 533-553.
- Bloch, Maurice. (1985). *Marxism and Anthropology*. Oxford, New York: Oxford University Press.
- Bolkestein, Hendrick. (1958). *Economic life in Greece's Golden Age*. Leiden: E. J. Brill.
- Cassirer, Ernest. (1953). *Language and Myth*. New York: Dover.
- Castoriadis, Cornelius. (1975). *L'institution imaginaire de la société*. Paris: Seuil.
- Cole, Michael. (1996a). Interacting minds in a life-span perspective: a cultural-historical approach to culture and cognitive development. In Paul B. Baltes and Ursula M. Staudinger (editors.), *Interactive minds*, 59-87. Cambridge University Press.

- Cole, Michael. (1996b). *Cultural Psychology*. Cambridge and London: The Belknap Press of Harvard University Press.
- Davydov, Vasilii Vasil'evich. (1990). Types of generalization. In Jeremy Kilpatrick (editor) *Instruction: Logical and Psychological Problems in the Structuring of School Curricula*. Reston, Virginia: National Council of Teachers of Mathematics.
- Eibl-Eibesfeld, Irenaus, Wolf Schiefenhövel, and Volker Heesch. (1989). *Kommunikation bei den Eipo. Eine humanethologische Bestandsaufnahme*. Berlin: Reimer.
- Franci, Raffaella, and Laura Toti Rigatelli. (1982). *Introduzione all'arimetica mercantile del Medioevo e del Rinascimento*. Siena: Quattro Venti.
- Freeman, Kathleen. (1956). *Ancilla to the Pre-socratic Philosophers*. Oxford: Basil Blackwell.
- Foucault, Michel. (1971). *L'ordre du discours*. Paris: Gallimard.
- Fowler, David H. (1987). *The Mathematics of Plato's Academy*. Oxford: Clarendon Press.
- Gal'perin, Piotr. (1967). On the notion of internalization. *Soviet Psychology* 5 (3): 28-33.
- Gal'perin, Piotr. (1989a). Organization of mental activity and the effectiveness of learning. *Soviet Psychology* 27 (3): 65-82.
- Gal'perin, Piotr. (1989b). Study of the intellectual development of the child. *Soviet Psychology* 27 (3): 26-44.
- Heath, Thomas L. (1956). *Euclid, The Thirteen Books of the Elements*, volume 2. New York: Dover.
- Ilyenkov, Evald V. (1977). *Dialectical Logic*. Moscow: Progress Publishers.
- Knorr, Wilbur. (1986). *The Ancient Traditions of Geometric Problems*. New York: Dover.
- Köhler, Wolfgang. (1951). *The Mentality of Apes*. New York: The Humanities Press / London: Routledge and Kegan Paul.
- Kozulin, Alex. (1996). The concept of activity in Soviet psychology. In Harry Daniels (editor), *An Introduction to Vygotsky*, 99-122. London and New York: Routledge.
- Lawrence, Janette A., and Jaan Valsiner. (1993). Conceptual roots of internalization: From transmission to transformation. *Human Development* 36: 150-167.
- Lefèvre, Wolfgang. (1981). Rechensteine und Sprache. In Peter. Damerow and Wolfgang Lefèvre (editors), *Rechenstein, Experiment, Sprache. Historische Fallstudien zur Entstehung der exakten Wissenschaften*, 115-169. Stuttgart: Klett-Cotta.
- Leontiev, Aleksei Alekseevich. (1981). Sign and activity. In James V. Wertsch (editors), *The Concept of Activity in Soviet Psychology*, 241-255. New York: M. E. Sharpe.
- Leontiev, Aleksei Nikolaevich. (1981). The Problem of activity in psychology. In James V. Wertsch (editors), *The Concept of Activity in Soviet Psychology*, 37-71. New York: M. E. Sharpe.



- Leontiev, Aleksei Nikolaevich. (1984). *Activité, Conscience, Personnalité*. Moscou: Éditions du Progrès.
- Lizcano, Emmánuel (1993). *Imaginario colectivo y creación matemática*. Barcelona: Editorial Gedisa.
- Lyotard, Jean-François. (1979). *La condition postmoderne*. Paris: Les éditions de minuit.
- Marx, Karl, and Friedrich Engels. (1982). *L'idéologie allemande*. Paris: Éditions sociales-Messidor.
- Morris, Pam. (1997). *The Bakhtin Reader*. London / New York / Sidney: Arnold.
- Mueller, Ian. (1991). Mathematics and education: Some notes on the Platonic Program. *Apeiron* 24: 85-104.
- Ortega y Gasset, José. (1992). *La idea de principio en Leibniz*. Madrid: Alianza Editorial.
- Netz, Reviel. (1999). *The Shaping of Deduction in Greek Mathematics*. Cambridge: Cambridge University Press.
- Pfeiffer, Rudolf (1968). *History of Classical Scholarship*. Oxford: Clarendon Press.
- Radford, Luis. (1996a). An historical incursion into the hidden side of the early development of equations. In Joaquim Giménez, Romulo Campos Lins, and Bernardo Gómez (editors.), *Arithmetic and Algebra Education*, 120-131. Tarragona, Spain: Universitat Rovira I Virgili.
- \_\_\_\_\_. (1996b). The roles of geometry and arithmetic in the development of elementary algebra: Historical remarks from a didactic perspective. In Nadine Bednarz, Carolyn Kieran, and Lesley Lee (editors), *Approaches to Algebra: Perspectives for Research and Teaching*, 39-53. Dordrecht /Boston/ London: Kluwer.
- \_\_\_\_\_. (1997a). *El pensamiento matemático como expresión semiótica sociocultural: el caso del "álgebra" de Mesopotamia*, invited paper delivered at the XI Reunión Latinoamericana de Matemática Educativa, Universidad Michoacana de San Nicolás de Hidalgo, Morelia, Michoacán, México, July 14-18, 1997.
- \_\_\_\_\_. (1997b). On psychology, historical epistemology and the teaching of mathematics: Towards a socio-cultural history of mathematics. *For the Learning of Mathematics* 17 (1): 26-33.
- \_\_\_\_\_. (1998). On signs and representations: A cultural account, *Scientia Paedagogica Experimentalis* 35 (1): 277-302.
- \_\_\_\_\_. (2001). The historical origins of algebraic thinking. In Rosamund Sutherland, Teresa Rojano, Alan Bell, and Romulo Lins (editors), *Perspectives in School Algebra*. Dordrecht, Boston, London: Kluwer.
- Restivo, Sal. (1992). *Mathematics in Society and History. Sociological Inquiries*. Dordrecht/ Boston/ London: Kluwer Academic Publishers.
- Roochnik, David. (1994). Counting on number: Plato on the goodness of Arithmos. *American Journal of Philology* 115: 543-563.
- Swetz, Frank J. (1989). *Capitalism and Arithmetic*. La Salle, Illinois: Open Court.
- Szabó, Árpád. (1977). *Les débuts des mathématiques grecques*. Paris: Vrin.

- Taisbak, Christian Marinus. (1971). *Division and Logos. A Theory of Equivalent Couples and Sets of Integers*. Odense University Press.
- van der Veer, René. (1996). The concept of culture in Vygotsky's thinking. *Culture and Psychology* 2: 247-263.
- van der Veer, René, and Jaan Valsiner. (1988). Lev Vygotsky and Pierre Janet: On the origin of the concept of sociogenesis. *Developmental Review* 8: 52-65.
- van der Veer, René, and Jaan Valsiner. (1991). *Understanding Vygotsky*. Oxford: Blackwell.
- Voloshinov, Valentin Nikolaevich. (1973). *Marxism and the Philosophy of Language*. London: Harvard University Press.
- \_\_\_\_\_. (1976). *Freudianism, A Critical Sketch*. Bloomington and Indianapolis: Indiana University Press. (Translation of *Frejdzizm*, published in 1927).
- Vygotsky, Lev Semenovich. (1962). *Thought and Language*. Cambridge, Massachusetts: MIT Press.
- \_\_\_\_\_. (1967). Play and its role in the mental development of the child. *Soviet Psychology* 5 (3): 6-18.
- \_\_\_\_\_. (1978). *Mind in Society*. Cambridge, Massachusetts: Harvard University Press.
- \_\_\_\_\_. (1981). The instrumental method in psychology. In James V. Wertsch (editor), *The Concept of Activity in Soviet Psychology*, 135-143. Armonk, New York: Sharpe.
- \_\_\_\_\_. (1997). *Collected Works*, Edited by R. Rieber and J. Wollock, New York and London: Plenum Press, volume 3.
- Vygotsky, Lev Semenovich, and Alexander Luria. (1994). Tool and symbol in child development. In René van der Veer and Jaan Valsiner (editor), *The Vygotsky Reader*, 99-174. Oxford: Blackwell Publishers.
- Wertsch, James V. (1981). *The Concept of Activity in Soviet Psychology*. Armonk, New York: M. E. Sharpe.
- \_\_\_\_\_. (1985). *Vygotsky and the Social Formation of Mind*. Cambridge, Massachusetts: Harvard University Press.
- Zinchenko, P. I. (1984). The problem of involuntary memory. *Soviet Psychology* 22(2): 55-111.

## ACKNOWLEDGEMENTS

This paper is part of an ongoing research program funded by the Social Sciences and Humanities Research Council of Canada, grant number 410-98-1287.

I want to thank Jens Høyup for the enlightening comments that he made on a previous version of this paper and Eva Firla, Rossella Dorigo and Jodie McDonald for their invaluable help in the preparation of this article.