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SYNCHRONIZING GESTURES, WORDS AND ACTIONS IN PATTERN GENERALIZATIONS

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In this paper we focus on the role of signs in students' perceptive processes underpinning the generalization of numeric-geometric patterns. Based on a video-taped Grade 9 classroom group activity undertaken by three students and framed by a cultural-semiotic theoretical perspective, we carry out a microgenetic analysis of an elementary form of mathematical generalization termed "factual". We present a detailed analysis of the dynamics between oral speech and gestures. Two main results are: the detection of intra-personal and inter-personal synchronizations between different semiotic systems; and the individuation of the key role played by objectifying iconic gestures.

INTRODUCTION AND THEORETICAL FRAMEWORK

The crux of the generalization of patterns lies in the fact that it predicates something that holds for *all* the elements of a class based on the study of a *few* of them. One question that has to be asked in this context is, hence, the following: What is it which enables the generalization to be accomplished? In other words, what is that process that allows the students to see the general through/in the particular? (Mason, 1996). In the case of geometric-numeric patterns, one of the crucial aspects of this process is perception. To *perceive* something means to endow it with meaning, to subsume it in a general frame that makes the object of perception *recognizable*. Because the perceptive process is interpretative, what one student sees in a pattern can be different from what another student sees in the *same* pattern.

The actual possibility of generalization therefore rests on perception and interpretation. In this paper, we are interested in better understanding the role of signs in students' perceptive processes underpinning the generalization of numeric-geometric patterns. As previous research suggests (Radford, 2002), perception as an active ongoing process of adjustments and refinements —a process in which the perceived object takes a progressive shape— is significantly dependent on the use of signs. With a pointing gesture, for instance, a student may indicate a specific part of a particular perceptual object to a colleague and enable him/her to *attend* to something that until then had remained unnoticed. The gesture here plays a specific role: the role of *objectification*, i.e., etymologically speaking, of making something apparent. Naturally, there are also other resources through which to accomplish an objectifying purpose: deictic words (e.g. "this", "that", "top", "bottom"), letters, diagrams, body movements, etc. All such resources that play the aforementioned phenomenological role in knowledge formation have been termed *semiotic means of objectification* (Radford, 2003).

We are interested here in investigating the microgenesis of an elementary form of mathematical generalization —a generalization termed *factual* (Radford, 2003). For example, in the pattern shown below, a factual generalization enables the students to find the number of circles in *any particular figure* (e.g. fig 100, fig 900) without counting the circles one after the other. Factual generalizations differ from more complex forms of generalization (e.g. contextual and symbolic) in that their level of generality remains confined to the numeric realm. Because of its limited scope, young students using factual generalizations only, cannot answer questions to explain how to find out the number of circles in *any* figure or to find a formula to calculate the number of circles in *Figure n*.

It is our contention that a microgenetic analysis of factual generalizations can shed some light on the way in which perception becomes refined in those crucial moments of the students' mathematical experience leading to the accomplishment of a generalization. In carrying out the microgenetic analysis, we will focus the students' deployment and coordination of semiotic means of objectification. In particular, we shall investigate the dynamics between oral speech and gestures.

METHODOLOGY

Our videotaped data comes from a 6-year longitudinal study, collected during classroom activities. In these activities, which are part of the regular school teaching lessons, the students spend a substantial period working together in small groups of 3 or 4. At some points, the teacher (who interacts continuously with the different groups during the small group-work phase) conducts general discussions allowing the students to expose, confront and discuss their different solutions. In addition to collecting written material, tests and activity sheets, we have three or four video-cameras each filming one group of students. Subsequently, transcriptions of the video-tapes are produced. Video-recorded material and transcriptions allow us to identify salient short passages that are then analysed using techniques of qualitative research in terms of the students' use of semiotic resources (details in Radford, 2000).

We will focus here on the introductory question of a problem in a Grade 9 math lesson. This problem dealt with the study of an elementary geometric sequence (see Fig. 1). In the question that we will discuss, the students were required to continue the sequence, drawing figures number 4 and number 5 and then to find out the number of circles on figures number 10 and number 100.

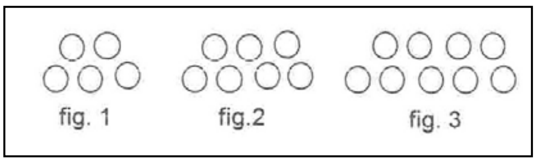

Fig. 1.

PROTOCOLS

We will analyse the microgenesis of a factual generalization of one group of students, formed by Jay, Mimi (sitting side by side) and Rita (sitting in front of them). In the videotaped episode, Jay and Mimi keep the worksheet; they begin counting the number of circles in the figures, and realize that it increases by two each time. Then, Jay is about to draw figure 4, with the worksheet and a pencil in his hands:

1. Rita: You have five here… *(pointing to figure 3 on the sheet)*
2. Mimi: So, yeah, you have five on top *(she points to the sheet, placing her hand in a horizontal position, in the space in which Jay is beginning to draw figure 4)* and six on the... *(she points again to the sheet, placing her hand a bit lower)*

Fig 2. Mimi's first gesture on line 2.

3. Jay: Why are you putting...? Oh yeah, yeah, there will be eleven, I think *(He starts drawing figure 4)*
4. Rita: Yep
5. Mimi: But you must go six on the bottom … *(Jay has just finished drawing the first row of circles)* and five on the top *(Jay finishes drawing the second row)*

Although Jay materially undertakes the task of drawing figures 4 and 5, each student is engaged in the action. In line 1, Rita is not merely informing her group-mates that figure 4 contains a row of 5 circles. In fact, through a deictic gesture she is suggesting a qualitative and quantitative way to apprehend the next figures. Pointing to a specific part of figure 3, which is given on the sheet, but referring in her speech to figure 4, Rita provides a link between the two figures. Through her *gesture-speech mismatch* (i.e. through a gesture that refers to something while she talks about something else; see Goldin-Meadow, 2003), she is certainly suggesting a specific way to build figure 4. This is an example of a process of *perceptual semiosis*, that is, a process in which perception is continuously refined through signs.

This apprehension of the figure is easily adopted by Mimi, and properly described through the spatial deictics "top" and "bottom" (lines 2 and 5). It amounts to shifting from blunt counting to a *scheme of counting*. To notice this scheme is the first step towards the general.

In line 2, Mimi's words are accompanied by two corresponding deictic gestures, which accomplish a number of functions: (1) participating in the drawing process, by entering Jay's personal space to offer guidance in carrying out the task; (2) depicting the spatial position of the rows in an iconic way, and (3) clarifying the reference of the uttered words. In line 5, Mimi does not make any gestures; rather, her *words* are perfectly *synchronized* with Jay's *action*, almost directing him in the action of drawing: in fact, to complete her sentence with the description of the second row, Mimi waits until Jay finishes drawing the first row of circles.

Later, the group work is interrupted by an announcement to the class about a forthcoming social activity. While Mimi and Rita pay attention to the announcement, Jay keeps on working, writing "23" and "203" as the answers for the question on the number of circles in figures 10 and 100. So, when the girls return to the task, they ask Jay for an explanation of his results:

6. Mimi: (*Talking to Jay*) I just want to know how you figured it out.
7. Jay: Ok. Figure 4 has five on top, right? *(with his pencil, he points to the top row of figure 4, moving his pencil from the left to the right)*
8. Mimi: Yeah…
9. Jay:…and it has six on the bottom *(he points to the bottom row using a similar gesture as in line 7).* […]
10. Mimi: *(pointing to the circles while counting)* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. (*Pause*) […] Oh yeah. Figure 10 would have …
11. Jay: 10 there would be like …
12. Mimi: There would be eleven *(she is making a quick gesture that points to the air. Jay is placing his hand in a horizontal position)* and there would be ten *(she is making the same quick gesture but higher up. Jay is shifting his hand lower down)* right?
13. Jay: Eleven (*similar gesture but more evident, with the whole hand*) and twelve *(same gesture but lower).*
14. Mimi: Eleven and twelve. So it would make twenty-three, yeah.
15. Jay: 100 would have one-hundred and one and one-hundred and two *(same gestures as the previous ones, but in the space in front of his face).*
16. Mimi: Ok. Cool. Got it now. I just wanted to know how you got that.

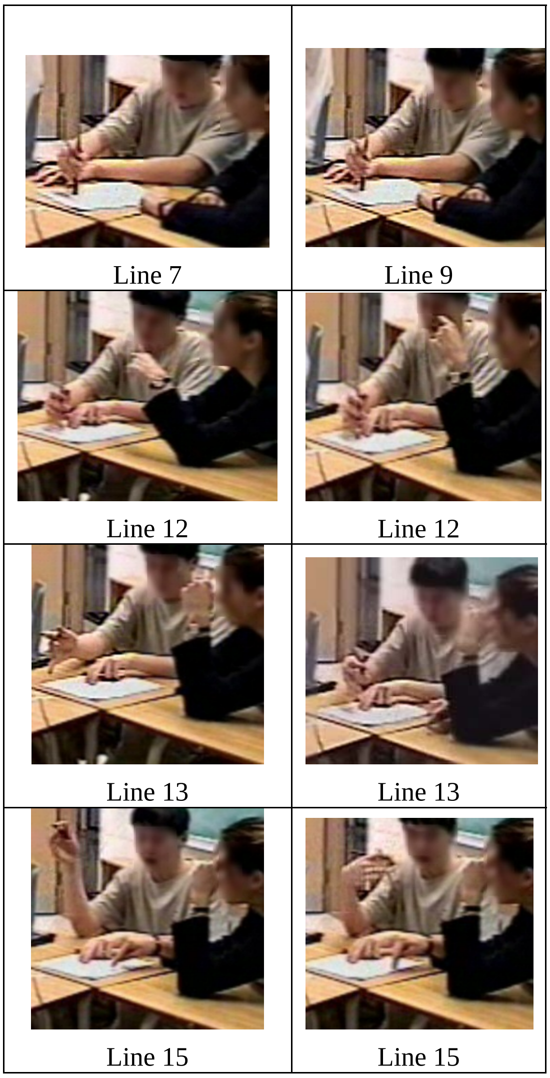

Fig 3. Some gestures occurring in the lines of the dialogue.

To account for his results about figures 10 and 100, Jay (lines 7 and 9) starts talking about figure 4, already drawn on the sheet, and uses the speech-gesture combination previously introduced by Mimi (lines 2 and 5): the same deictic terms "top" and "bottom", and analogous deictic gestures. Turning to figure 10, Mimi (line 12) matches her words with two gestures that refer to the two rows of the geometrical configuration. The same kind of gesture and uttered speech is then used by Jay, who corrects Mimi's answer (line 13). Even if the figure is referred to in two slightly different ways by the two students, starting from the top (Jay) or from the bottom

(Mimi), the words-gesture match is perfectly accomplished in both cases in a very natural way. The same is true for Jay's inference about figure 100 (line 15).

The relevance of the previous remarks is that through a coordination of gestures and speech, the students are accomplishing an *objectification of knowledge* (Radford, 2003), i.e., through signs of different sorts, the students are making *apparent* key traits of figure 100 —a figure that is not directly perceivable. The tight coordination between gestures and speech takes place in a particular segment of the students' mathematical activity. These segments of mathematical activity, characterized by the crucial coordination of various semiotic systems leading to the objectification of knowledge, constitute what have previously been termed *semiotic nodes* (Radford et al. 2003). An index of its presence is the perfect coordination of time, words and movement reached in line 12 (see Fig. 4).

12. Mimi: There would be eleven and there would be ten right?

Fig. 4. Synchronization between the gestures of the two students

Indeed as Fig. 4 illustrates, Mimi's words are rhythmically beaten not only with her own gestures, but also with those of Jay. In fact, even if the students are not looking at each other, Jay's hands are synchronized with Mimi's words, and, as a consequence, with Mimi's hands.

Let us now focus on the internal dynamics of this semiotic node (which indeed involves the whole episode, going from line 6 to line 16) to disentangle the different specific semiotic components and describe how their mutual relations pave the way for students' generalization.

In Jay's first utterance (lines 7 and 9), the deictic gestures appear endowed with a dynamic feature that clearly depicts the geometric apprehension of the figure as made up of two horizontal rows. Its goal is to clear away any ambiguity about the referent of the discourse, in order to explain a strategy. The figure (number 4) is perceptively present on the scene, and indeed materially touched by Jay through his pencil, a tool that can be actually considered part of his peripersonal space, that is the space immediately surrounding his body. Talking about figure 10, Mimi (line 12) performs two gestures that keep certain specific aspects of those of Jay, i.e. one gesture for each row, and the vertical shift. But now, because the referred figure is not available in the perceptual field, the gestures are made *in the air*. Also Jay's last gestures (line 14), referring to figure 100, appear in the air in the space in front of him, as if pointing to the rows of a non visible figure; indeed, if we pay attention to the position of his hands when he refers to the different figures, we can notice a progressive detachment from the sheet:

Referring to fig 4 | Referring to fig 10 | Referring to fig 100

Fig. 5.

The indexicality of the deictic gesture undergoes a gradual shift from an *existential signification* (referring to figures 4, materially present on the sheet) to an *imaginative* mode of signification (referring to figures 10 and 100). These gestures that mime or "iconize" the referent pinpoint and depict in an iconic way the essential features of the new referent, thus making it apparent. We term *objectifying iconics* these kinds of gestures which, thanks to their iconic features, play an important part in the process of knowledge objectification. Their role is in some way analogous to that of the deictic words previously termed *objectifying deictics* (Radford, 2002).

Notice that the objectifying iconic gestures undergo a process of simplification that involves the loss of movement (along the rows of the figure) and a shortening of their duration. A progressive simplification is also evident in the uttered words: from line ten onward, the deictic terms disappear, leaving a barely numerical semantic content, organized by the conjunction "and". Even if figures 10 and 100 are not materially present, the students can *imagine* them very precisely and would be able to draw them; but, having reached a certain stage in the process of objectification and socialization of the objectified knowledge, they do not need to specify all the details, and the reference to the form of the figure can smoothly remain implicit in their speech. This is also possible due to the role played by gestures. Let us focus on lines 12, 13 and 15 (Fig. 3).

The perfect synchronization between words and gestures allows the students to successfully cope with two intertwined aspects of the problem: one is numerical, discrete, and linear; the other is visual, geometrical, and analogical. The students handle the former through language, and the latter through gestures. They correspond to what Lemke (2003) terms the two fundamental types of meaning-making: meaning-by-kind or "typological meaning" (language) and "the meaning of continuous variation," the meaning-by-degree or "topological meaning" (motor gestures or visual figures). He identifies in this inherent and unavoidable difference a main source of difficulties in learning mathematics, since "In general mathematical expressions are constructed by typological systems of signs, but the values of mathematical expressions can in general vary by degree within the topology of the real numbers" (ibidem, p. 223).

CONCLUDING REMARKS

In this paper we focused on the genesis of a factual generalization. In investigating this kind of elementary form of mathematical generalization our goal was to unravel the semiotic activity that underpins its objectification. Previous research suggested the crucial role of signs in the students' progressive process of apprehension of the pattern. However, the detailed micro study of this process of perceptual semiosis — the interpretative process that enables the students to go beyond the particular and to attain the general— still needs to be better understood. As it has been suggested in earlier work (Radford, 2003), factual generalizations constitute, to an important extent, the basis for more sophisticated forms of generalizations.

Our microgenetic analysis intimates that the process of perceptual semiosis here studied was underlined by two kinds of meanings. On one hand, there is a typological meaning that emphasizes the dimension of "quantitas". On the other hand, there is a topological meaning that highlights the dimension of the "qualitas" induced by the geometric nature of the figures of the pattern. We saw here that the two aspects are inherently merged. Because the goal of the factual generalization is precisely to spare one from counting the circles in a figure one after the other, the numerical and the geometrical dimensions have to be harmonized. To harmonize them, the students activate a number of semiotic systems: oral speech, drawn figures, and gestures whose coordination by the students constitutes a semiotic node of their ongoing activity. Our analysis suggests the occurrence of gesture-speech match and mismatch and the critical role of gestures in the objectification of knowledge. Thus, based on the particular figures (e.g. figures 3 and 4), the students started talking about non-present terms such as figure 10 and figure 100. The latter were *objectified* (i.e. made apparent) thanks to couples of iconic gestures that represent the geometrical components that are essential in a particular figure apprehension —e.g. two horizontal rows of circles (see Fig 5). These objectifying iconic gestures bear the analogical aspect of the problem and allow the students to pair it with its correspondent typological meaning (expressed in the uttered speech), to successfully accomplish the given task.

In addition to this, our results also point to another aspect of the problem. As previously discussed, (Radford et al., 2004), there can be synchronization between different semiotic systems activated by the same individual: lines 3, 7 and 13 show examples of this. However, we also found evidence here of synchronization between individuals. In this case, it can occur between different semiotic systems, as in line 5, where Mimi is almost directing Jay's drawing action, or between different enactments of the same semiotic means by different students, as in line 12, where we observe Mimi's and Jay's simultaneous gesturing actions perfectly coordinated with Mimi's utterance. Thus, besides an intra-synchronization (i.e. an intra-subject or intra-personal synchronization) —of which the gesture speech-match is a particular case— an inter-synchronization (or inter-subject or inter-personal synchronization) also appears.

The hints provided by our micro-analysis need to be investigated further. In particular we need to better characterize the dynamics of semiotic nodes in factual, as well as in other more sophisticated forms of generalization, related not only to the context of patterns but to other domains of mathematics.

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