1 Introduction

Galileo opens his *Discourses and Mathematical Demonstrations Relating to Two New Sciences* with a remark about the famous 16th century Venetian arsenal, which he praises for its impressive amount of instruments and machines; this arsenal, he says, offers an opportunity to wonder and think. With their unprecedented variety of tools and artifacts, contemporary classrooms may have looked like the Venetian arsenal to Galileo. True, some of the artifacts that are part of our educational settings have been there for a long time now – for example, textbooks. Others, however, made their appearance with the digital technological progress during the 20th century. And, like the instruments and machines of the Venetian arsenal, they offer new possibilities for thinking and learning.

Now, for these possibilities to be materialized in the classroom, the conditions surrounding the use of artifacts in processes of teaching and learning need to be clearly understood. Indeed, since artifacts are artificial devices, neither the understanding of their use nor the best exploitation of their epistemic possibilities is self-evident. This is why investigating the proper conditions of artifact use in educational settings constitutes an important research problem. The various chapters in this part of the book tackle this problem and offer interesting theoretical and methodological contributions to current debates in the field. Thus, seeing the chapters from a general viewpoint, the various authors inquire about the manner in which teachers adapt and use specific resources in their own practice – for example, CAS (Kieran, Tanguay, and Solares), Enciclomedia (Trigueros and Lozano), a digital-based algebra environment (Drijvers), material objects and symbolic artifacts (Forest and Mercier), and textbooks (Rezat). Naturally, the authors tackle the

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1 I use the term artifact in its most general sense: as “an object made by a human being, typically an item of cultural or historical interest,” as defined by *The New Oxford American Dictionary*. The category of artifact (or its synonymous term tool) includes the one of didactic resources.
general research problem from different perspectives and ask questions of different kinds. Kieran et al. inquire about the adaptations of researcher-designed resources by teachers. Trigueros and Lozano move along similar lines and try to detect what they call the ‘operational invariants’ in the teachers’ use of resources. Drijvers attempts to elicit the kind of ‘instrumental orchestrations’ to which the teachers resort in their classrooms, while Rezat explores the forms of textbook use undergone by both teachers and students.

Taken together, the various case studies presented in these chapters show some of the difficulties that teachers face in the integration of resources in the classroom. They pinpoint, to various degrees of explicitness, some aspects of a more general nature that are imbricated in the educational use of artifacts. One of them relates to changes in our conceptions of classroom practices that result from the use of digital technologies. Indeed, traditional conceptions of what a good classroom practice is need to be revisited in light of the teachers’ and students’ use of artifacts. Thus, in Kieran and coworkers’ study, one of the teachers fails to use CAS to promote a deep mathematical understanding. The teacher does use the digital artifact, yet the artifact use seems to remain within the confines of traditional forms of teaching centered on direct content presentation. The teacher, it seems, fails to notice that the use of artifacts in the classroom introduces a new division of labor and that, in this new digital context, his or her role is thereby modified. To be properly exploited, the cognitive potential that an artifact brings with it requires not only a suitable understanding of the artifact itself but also of how it modifies the roles of the teacher and the students.

The manner in which we understand the division of labor that artifacts induce in the classroom depends on our own theoretical views about cognition. In fact, the possible roles that we attribute to artifacts or resources derive from the manner in which we conceive of cognition in the first place. It is only within a specific view of cognition that artifacts are endowed with particular cognitive, epistemic and ontological roles. Let me briefly dwell on these roles in the following sections.

2 The Cognitive Role of Artifacts

There seems to be a consensus around the idea that artifacts are mediators of activity. But what do we mean by the mediating nature of artifacts? There are several ways in which this question can be answered. One way is to understand the artifact as something that allows us to do something. It is from this perspective that artifacts are seen as a possible extension of the individual. Artifacts are considered here as something like prostheses or amplifiers: they are aids to accomplish actions. They help us without changing our cognitive landscape. What they do is to make accessible to us realms of reality that remain hidden because of our human sensorial limitations. The microscope and the telescope are good examples. By allowing the students to visualize and decompose three-dimensional figures, the Cubícula software mentioned in Trigueros and Lozano’s chapter could be seen in this way. But Cubícula and other artifacts could also be seen as playing a deeper cognitive role. In
this view, artifacts are not only facilitators of knowledge acquisition. They become part of the way in which we come to think and know.

The first meaning of mediation has been put forward in cognitive psychology (Cole & Griffin, 1980). The second meaning of mediation is at the heart of Vygotsky’s view of cognition, where tools are seen as psychological. Within this conception of cognition, artifacts are considered cultural devices that modify our cognitive functioning. As Vygotsky put it in one of the foundational texts of the historical–cultural school of psychology:

> By being included in the process of behavior, the psychological tool alters the entire flow and structure of mental functions. It does this by determining the structure of a new instrumental act just as a technical tool alters the process of a natural adaptation by determining the form of labor operations (Vygotsky, 1981, p. 137).

Within the historical–cultural Vygotskian conception of cognition, an artifact is considered to be a bearer of historical intelligence (Pea, 1993). It is a bearer of historical voices that need to find an interactional space in the classroom to enter into a dialogue with the teacher’s and the students’ voices. Now, how to promote dialogical spaces susceptible to including the artifact’s sediminated voices and intelligence is a problem in its own right. It entails a reflection on both the epistemological and ontological roles that we attribute to the artifacts.

3 The Epistemological Role of Artifacts

The aforementioned Vygotskian artifactual mediated view of cognition has epistemological implications that we still have to explore, for it changes the traditional view of what we mean by learning and knowing. To make this point clearer, let me go back to the discussions that originated from the introduction of calculators in primary schools a few decades ago. Calculators were seen as an object of interference and even an inhibitor to the development of students’ arithmetic thinking. Students were supposed to be able to carry out calculations without the help of the calculator. Once, one of my students told me that in his Grade 1 class he was even forbidden to count with his fingers or to make any gestures. Within this epistemology, knowing was understood as something purely mental.

If we consider artifacts as more than aids, their epistemic status changes. Knowing becomes knowing-with-tools as opposed to knowing via the tools. Artifacts become imbricated in the way we think and come to know. The epistemic status of artifacts can be summarized as follows: As artifacts change, so do our modes of knowing.

However, this view of artifacts needs further development. Otherwise, it risks remaining anchored in the traditional knowing subject – object of knowledge epistemic schema. Indeed, the only modification to this schema is the insertion of the artifact. The schema becomes: subject + artifact – object of knowledge. In this case, the Piagetian research question about schema formation is barely modified: instead of the subject’s purely mental schemas it becomes the subject’s schemas of artifact
use. This account fails to make explicit (at the theoretical and methodological levels) the fact that artifacts embody particular forms of cognition and communication, and that thinking emerges not out of patterns of actions with artifacts but in joint activity, out of actions with artifacts carrying social and historical meanings. What is missing in this account is the fact that knowing is a social and cultural practice. More specifically, knowing is a historical collective act. As a result, knowing is accomplished not only through invariant patterned actions with signs and artifacts but also in interaction with other individuals against the background of historical and cultural modes of thinking and communicating (Radford, 2010). The question is not, hence, how artifacts become appropriated or mastered, but how they mediate joint activity. Naturally, in the case studies presented in the various chapters, this question emerges either implicitly or explicitly. It appears in particular when the authors focus on the way the teachers mediate or orchestrate for the students the historical intelligence deposited in the artifacts. Cubícula, for example, conveys ideas of decomposing figures to think mathematically about their measurements. These are historical ideas that have been refined through centuries of human cognitive activity, from sand sketches in ancient Greece to 21st century digital representations.

4 Mathematics and the Ontological Role of Artifacts

In his chapter, Drijvers distinguishes three elements of didactic orchestration: a didactical configuration, an exploitation mode of the didactical configuration and a didactical performance. The latter corresponds to the actual classroom activity. It is in the last part that artifacts come to be used and that teachers, through the use of certain techniques, have the opportunity to guide the students in their processes of learning. Drijvers invites us to see the teacher’s actions as a form of didactical performance, involving expected, and unexpected aspects that take into account the students’ inputs. I would like to argue that the didactical performance is part of a more general activity – the activity of mathematics making.

Let me explain. In the previous section I suggested that thinking and knowing are social practices. In this section, I want to extend the idea to mathematics. My argument is not that mathematics is governed by social and cultural norms. This, of course, is true. But what I have in mind is something of an ontological nature, something about what mathematics is. Let us start by noticing that, ontologically speaking, mathematics is not really different from music. Both are cultural forms of expression, action, and interpretation. Naturally, there are obvious differences. But there are some important similarities as well. The most important similarity is this: musical and mathematical ‘objects’ share the same ontological nature. Thus, in the same way as music does not reside in musical scores, mathematics does not reside in written theorems. Mathematical objects do not coincide with mathematical written texts. Texts and other artifact are embodiments of the existence of their objects. As Lektorsky (1995, p. 193) put it, ‘man-made objects are in [a] certain sense modes of the embodiment and existence of knowledge’. Thus, I want to argue, in the same way that music resides in its performance, mathematics resides in the activity of
its enactment. In this sense, mathematics is always new and different, in the sense that each event is always unique and singular. But, at another level, it is similar to other contemporary and past events, without which we would not distinguish an activity about geometry from one about algebra. This similarity of events does not, however, preclude mathematics from living – in an ontological sense – in the event of its execution.

Considering mathematics from this viewpoint has some implications on classroom practice and on the ontological role of artifacts and resources. Artifacts can no longer be considered as a means to access mathematical objects and mathematical forms of reasoning, as these are not conceived of as transcendental entities. Artifacts, rather, are considered part of mathematics as material practice. Within this context, mathematics appears as a collective activity, spatially situated, which unfolds in a certain span of time, where the historical voices embedded in artifacts and the voices of students and teachers merge. Let us note, en passant, that in this perspective, the discussions about mathematical proofs assisted by computers (Devlin, 1992) take a different turn. The computer is not helping the mathematician carry out some calculations. Both become part of one chorus singing a polyphonic song.

This conception of mathematics as enactment or performance does not mean, however, that all performances are equally good. Each will be more or less successful depending on the historical–cultural understanding of mathematics. But because mathematics is something that is in the making, performances will also be considered to be more or less good depending on how teachers and students understand and coordinate their coemerging and evolving sense of involvement in the collective endeavor in which all of them participate. It is against this polyphonic context that the question of the artifacts and the division of labor that they induce reappear. If thinking mathematically is an artifactual mediated collective endeavor where each participant learns to critically situate herself within cultural and historical constituted modes of thinking (Radford, 2008), the question of responsibility and orchestration must then be seen in a new light. It appears as a voix à trois: the teacher’s, the students’, and the artifacts’.

To end this short commentary, I come back to Rezat’s interesting chapter. Rezat’s chapter shows the tensions that are caused in some classrooms by the presence of a textbook, particularly when the textbook brings a perspective that is different from the teacher’s. If the teacher considers her voice as the official one, the artifact has little room to operate. If, in contrast, the teacher considers her view as one of various possible views on a same problem, she can take advantage of the textbook to add its differing and subverting voice to hers and invite the students to reflect on the differences and nuances so that they can end up with a more polyphonic understanding of the matter under scrutiny. The making of mathematics would consist precisely in the understanding of differences and similarities that are brought to the fore by the students’ understandings as they are interwoven with the voices of the teacher and the historical intelligence deposited in artifacts.
References


