This paper was published in: E. Simmt and B. Davis (Eds.) (2005).
***Proceedings of the 2004 Annual Meeting of the Canadian Mathematics Education Study Group*, pp. 111-117.**

Body, Tool, and Symbol:
Semiotic Reflections on Cognition

Luis Radford
École des sciences de l'éducation
Université Laurentienne
Sudbury, Ontario

ABSTRACT: Although 20th century psychology acknowledged the role of language and kinesthetic activity in knowledge formation, and even though elementary mathematical concepts were seen as being bound to them (as in Piaget's influential epistemology), body movement, the use of artefacts, and linguistic activity, in contrast, were not seen as *direct* sources of abstract and complex mathematical conceptualizations. Nevertheless, recent research has stressed the decisive and prominent role of bodily actions, gestures, language and the use of technological artefacts in students' elaborations of elementary, as well as of abstract mathematical knowledge (Arzarello and Robutti 2001, Nemirovsky 2003, Núñez 2000). In this context, there are a number of important research questions that must be addressed. One of them relates to our understanding of the relationship between body, actions carried out through artefacts (objects, technological tools, etc.), and linguistic and symbolic activity. Research on the epistemological relationship between these three chief sources of knowledge formation is of vital importance for a better understanding of human cognition in general, and of mathematical thinking in particular. In the first of this paper, I discuss the roots of the reluctance in Western Thought to include the body in the act of knowing. In the second part, echoing current debates in mathematics education, I discuss, from a semiotic viewpoint, the importance of revisiting cognition in such a way as to think of cognitive activity as something that is not confined to mental activity. In the third part, I present a developmental overview of the theoretical foundations of my research program and of the research questions that my collaborators and I are currently investigating.

Résumé : Même si la psychologie du 20e siècle a pris en compte le rôle du langage et de l'activité kinesthésique dans la formation du savoir, et même si on a reconnu leur importance dans l'émergence des concepts mathématiques élémentaires (comme c'est le cas dans l'épistémologie génétique de Piaget), le corps, l'utilisation d'artefacts et l'activité linguistique, par contre, n'ont pas été considérés comme sources *directes* des conceptualisations mathématiques abstraites. Cependant, des nouvelles recherches ont mis en évidence le rôle central du corps, des gestes, du langage et des artefacts technologiques dans l'élaboration du savoir mathématique tant élémentaire qu'avancé (Arzarello et Robutti 2001, Nemirovsky 2003, Núñez 2000). Il y a, dans ce contexte, un certain nombre de questions qui doivent être étudiées. L'une de ces questions a trait à la relation entre le corps, les actions médiatisées par des artefacts (objets concrets, outils

technologiques, etc.) et l'activité linguistique et symbolique. La recherche sur la relation épistémologique entre ces trois sources principales de la formation du savoir est d'une importance vitale pour comprendre la nature de la cognition humaine en général et la pensée mathématique en particulier. Dans la première partie de cet article, je me pencherai sur les racines qui ont amené la pensée occidentale à exclure le corps dans l'acte de la connaissance. Dans la deuxième partie, je discuterai, du point de vue sémiotique et à la lumière des débats actuels en éducation mathématique, de l'importance de repenser ce qu'on entend par cognition. Dans la troisième partie, je présenterai une courte vue historique des fondements théoriques de mon programme de recherche et des questions que nous sommes en train d'étudier présentement.

1. Introduction

In one of the first episodes of Star Trek, Captain Kirk arrives at a strange planet. He does not see any signs of life. However, his detector keeps telling him that there are some forms of life in the surroundings. After a while, he realizes that life's signs come from the interior of some small transparent containers placed on a table. It turns out that these containers hold pure life: small brains that emit some color as they speak. They tell Captain Kirk that they are sophisticated forms of life that, to evolve, gave up body and became pure brains.

This science fiction story encapsulates in a clear way one of the cornerstone ideas of Western Thought, one in which the body is merely a hindrance with no relevance for our endeavours to attain knowledge. Since we humans have not yet found a way to divest ourselves of the body, we have created characters who give body to this idea. One of Captain Kirk's crew illustrates this point very well: Mr Spock is indeed the most clear-cut example of a logical thinker; emotions and body do not play any role in the way he thinks and calculates.

The Star Trek story is a futuristic version of the kind of rationality that Plato envisioned in the period of turmoil that followed the Peloponnesian war. The defeat of Athens led to a questioning of its traditional values and Plato's epistemology is indeed a response which attempts to salvage the aristocratic values. Politically, it was formulated as a kind rationality that opposes change. What is knowable is only that which does not change – something that Plato designated by the word *eidos* (essence). And to know it, we have to give up the body. In the *Phaedo* (65a-65b, p 47) Simmias is asked to determine who, among all sorts of men, would be able to attain true knowledge. Is it not him –Plato has Socrates ask– who

> pursues the truth by applying his pure and unadulterated thought to the pure and unadulterated object, cutting himself off as much as possible from his eyes and ears and virtually all the rest of his body, as an impediment which by its presence prevents the soul from attaining to truth and clear thinking? Is not this the person, Simmias, who will reach the goal of reality, if anybody can? (Phaedo, 65e-66a)

He then continues: "we are in fact convinced that if we are ever to have pure knowledge of anything, we must get rid of the body and contemplate things by themselves with the soul by itself." (*Phaedo*, 66b-67b)

Knowledge, for Plato, was only possible through reasoned discourse, through *logos*. The 17th century rationalists changed logos for a mind endowed with "powers" or "faculties", such as the faculties of understanding, memory and imagination –faculties that God granted to men (Descartes, *Meditation*, IV.9). When, in the First Mediation, Descartes asks the question: "What am I?" he answers: "A thinking substance". "I am anything but mind" (Descartes, *Mediation*s, II.15).
For Descartes, to know something amounted to having a distinct apprehension of the thing to be known. "I cannot be deceived in judgments of the grounds of which I possess a clear knowledge." (Descartes, *Mediations*, V.15). And apprehension and the distinctiveness of things were not ensured by the senses. Thus, to explain how bodies and external things become known, Descartes says that "bodies themselves are not properly perceived by the senses nor by the faculty of imagination". True knowledge is ensured, Descartes continues, "by the intellect alone; … [things] are not perceived because they are seen and touched, but only because they are rightly comprehended by the mind" (Descartes *Meditations*, II.16).
The mind or the spirit apprehends and knows things by the Rules of Reason, by rules expressed in the rules of logic, so that rationalists like Leibniz claimed that what we know is known not through the senses but by reason alone. Thus, all truths contained in arithmetic and geometry can be known by considering what we already have in our mind through reason, without having recourse to truths learnt by experience. This is why we can make these sciences in our own office, even with our eyes closed, for we do not need the eyes or the other senses (Leibniz, *New Essays concerning Human Understanding*).
The conception of a thinking mind governed by the cold rules of logic has served as the Western paradigm of thinking. All the attempts to reduce thinking to logical calculations belong to this paradigm. Empiricism, of course, has been a traditional contender of rationalism. Thus, opposing the rationalist trend, Hume argued that ideas are impressions that we receive from external objects.

To a large extent, the history of the 20th century pedagogy of mathematics is the history of a pedagogy that aimed to develop either a rationalist or an empiricist epistemology. The rationalist pedagogy of the early 20th century was a pedagogy focused on the development of logical thinking, abstraction, and rigor. Euclid's *Elements* was the text book and the model to follow. The rationalist epistemology re-emerged many decades later, embracing, with Bourbaki, a structural view of mathematics. In opposition to this pedagogy we find, also in the early 20th century, an empiricist one attached to the belief that the origin of our knowledge starts with our senses. Instead of focusing on rigor and proof, geometry, for instance, was taught as an experimental discipline, following Compte's positivism. But the pedagogy of mathematics was also based on the belief in a *continuity* between the sensual and the intellectual. Reason, it was assumed, picks sensual knowledge up and transforms it into abstract thinking. This mix of empiricist and rationalist pedagogy was drawing on the anti-dogmatic posture of the Enlightenment, which put the individual at the very core of knowledge. If something can be known, it can neither come from authority nor from what someone else says. It has to be known by the individual directly. Between the object of knowledge and the individual nothing could be interposed –except his/her sensual impressions. Several years later, theorizing the role of the senses along the lines of logic-mathematical structures, Piaget continued the Enlightenment tradition.

Indeed, following Kant –who attempted to achieve reconciliation between empiricist and rationalist trends– Piaget emphasized the role of sensorial-motor actions. If, however, body and artefacts played an epistemological role in his genetic epistemology, it was only to highlight the logical structures that supposedly underlay all acts of knowledge. The semiotic function, as Piaget called it (which includes *representation*, i.e situations in which one object can stand for another; *imitation* where sounds are imitated, evocation, etc.) was the bridge between the sensual and the conceptual, between concrete schemas and their intellectualized versions. This is why "operations [i.e. reflective abstracted actions] can sooner or later be carried out symbolically without any further attention being paid to the objects [of the actions] which were in any case 'any whatever' from the start." (Beth and Piaget 1966, p. 237-238). As a result, for Piaget, signs and symbols were in the end merely the carriers and the expressions of a thinking measured by its rational structural features. The emphasis on the rationalist part of Piaget's work is well articulated by one of his collaborators, Hermine Sinclair, who, after explaining Piaget's reasons for avoiding empiricism and rationalism and presenting his genetic epistemology as a third possibility, says: "His [Piaget's] proposal of a third possibility is nearer to the rationalist than to the empiricist hypothesis" (Sinclair, 1971, p. 121).

Is there another way in which to conceptualize the relationship between sensorial-motor actions and signs? In the next section I deal with this question.

2. Revisiting Cognition

Last summer, when in the introductory talk of a PME research forum held at the University of Hawaii, Nemirovsky presented a list of research questions and argued that mathematics educators should tackle them soon, some participants had the impression that the questions to which Nemirovsky was referring were already answered by Piaget's epistemology. Two of the questions on Nemirovsky's list were the following:

> What are the roles of perceptuo-motor activity, by which we mean bodily actions, gestures, manipulation of materials, acts of drawing, etc., in the learning of mathematics? How does bodily activity become part of imagining the motion and shape of mathematical entities? (Nemirovsky, 2003)

Nemirovsky's questions are motivated by recent research which has stressed the decisive and prominent cognitive role of bodily actions, gestures, language and the use of technological artefacts in students' elaborations of elementary, as well as of abstract mathematical knowledge (Edwards, Robutti, and Frant, 2004; Arzarello and Robutti, 2001; Núñez, 2000). To give but one example, Susan Goldin-Medow (2003), Kita (2003) and Roth (2001) have shown how gestures become key elements in mathematics, the sciences and ordinary intellectual activity.

In fact, what I find new in Nemirovsky's list is not the questions therein included, but rather the concomitant invitation to revisit our own conceptions about cognition. To say it in Piagetian terms, there is concrete evidence emphatically suggesting that the semiotic function is much more than a bridge between sensorial-motor and intellectual activities. Or to say it in other terms, it is no longer possible to conceive of intellectual activity as the "natural" *prolongation* of practical sensorial-motor intelligence. From an educational perspective, it then becomes urgent to cast intellectual activity in new conceptual terms, in terms capable of including body, tool, and symbol –even in its more “advanced” manifestations. Intellectual and sensual activities are different sides of the same coin. They constitute the dialectical unit of thinking. As Parmentier remarks, even an abstract symbol bears a kind of contextuality in that “a symbol necessarily embodies an index to specify the object being signified”; reciprocally, every contextual signifying act relies on a certain generality, for “an index necessarily embodies an icon to indicate what information is being signified about that object” (Parmentier, 1997, pp. 49).

In this context, there are a number of important research questions that must be addressed. One of them relates to our understanding of the relationship between body, actions carried out through artefacts (objects, technological tools, etc.), linguistic and symbolic activity. Research on the relationship between these three chief sources of knowledge formation is of vital importance for a better understanding of human cognition in general and of mathematical thinking in particular.

With regard to algebraic thinking –which has been the focus of my research program– the fundamental problem is to understand the way in which processes of symbolizing and meaning production relate to kinaesthetic activity and the artefacts employed therein. As our previous results suggest, highly complex algebraic symbolism cannot incorporate the students’ kinaesthetic experience in a direct manner. The severe limitations of a direct translation of actions into symbols require the students to undergo a dynamic process of imagining, interpreting and reinterpreting. The students have to pass through a dialectical process between (concrete or imagined) actions, signs and meanings. However, little is still known about this process. Further research needs to be conducted at the theoretical and experimental level.

In the next section I provide an overview of my previous results and of the research question that will lead our forthcoming research.

3. Some previous results

We were led to notice the role of kinaesthetic activity and artefacts in the students’ elaboration of mathematical conceptualizations in the course of observations that we began in a systematic way in 1998, in a longitudinal classroom based research program. We were studying the students’ processes of meaning-making in generalizing tasks, conducted within traditional technology (pencil and paper). Analyzing hours and hours of videotaped lessons, it became apparent that the students’ production of meaning was deeply sustained by natural language. A fine-grained analysis of the episodes made it possible to pinpoint two different key functions of language to which the students resort when they still cannot master the symbolic algebraic language and nonetheless must deal with mathematical generalizations. These were the deictic and the generative functions of language. The deictic function refers to the rich arsenal of linguistic terms (called deictics) with which the students can designate objects in their ongoing spatial-temporal mathematical experience (e.g. *this*, *that*, *here*, *there*, *top*, *bottom*, *before*, *after*). Deictics are ubiquitous in everyday communication. The generative action function refers to the linguistic terms that allow the students to convey the idea of generality. It is used by the students to express the idea of generality as a potential action that can be reiteratively accomplished. For instance, the students frequently use adverbs such as *always* to signify something that can be repeated forever (details in Radford, 2000). The generative action function can also appear in more subtle ways: instead of linguistic adverbs such as *always*, the students often use *rhythm*. They coordinate the flow of words with indexical or iconic gestures in order to produce rhythm and convey the idea that a pattern continues forever.

The deictic and the generative action functions of language empower the students with means for expressing the idea of generality – something that algebraists do using letters which stand for mathematical variables. The problem is that when the students are required to move into algebraic symbolism, they have to face the situation of expressing their mathematical experience through a semiotic system that does not possess deictics, adverbs, terms for generative actions or rhythm. The lack of such rich resources leads the students to a fundamental problem that struck me in a profound way. It is a problem of semiotic designation of objects: they have trouble designating, through algebraic symbolism, the number of e.g. circles or toothpicks in *Figure n*, that is, a non-specific figure identified only by its position in a sequence. I called this semiotic-cognitive problem the *positioning problem* (Radford, 2000, p. 250). *Figure n* cannot be seen, so reference to the number of circles or toothpicks that it contains can only be made *indirectly*, through signs.

However, this was not all. As our analyses progressed, we realized that the students were resorting to another semiotic system: gestures. Indeed, the students were continuously pointing to concrete figures in the sequence under study or imitating with some shapes of the figures with their hands. These gestures were not merely ancillary aids to communication. They appeared as *crucial* parts of their mathematical experience. Our students were more than cerebral thinking substances: their mental activity seemed, indeed, to be going beyond their internal cerebral processes and to be reaching the social world of body and artefacts. They were thinking *with*, and *through*, language, body and artefacts. This observation led us to search for ways to theoretically account for the role of body, tools and symbols in cognition.

At the end of his life, Vygotsky became more and more interested in the role played by the meaning of words in children's formations of cognitive functions, such as attention and perception. The problem about perception interested me the most. Our students were asked to deal with a general object which, because of its general nature, could not be perceived as one perceives a chair. In a way, their gestures and the whole semiotic activity that they were displaying were an attempt to supply the unperceivable general with something concrete. Following Vygotsky's work, I endeavoured to work out a theoretical account that could integrate the role of gestures, speech, symbols, and artefacts into the students' production of meaning. Certainly, Vygotsky's work is very rich, but the phenomenology of experience remained sketched in it only in very broad terms. I turned then to Edmund Husserl.

Husserl's phenomenology sets the basis for explaining how we become conscious of the things that we perceive. It seeks to explain the role of subjective intentions in the progressive apprehension of what is *there*. Husserl elaborated his account of how we become conscious of something in terms of noetic-noematic structures, but the problem of the conceptual object that we attend to in our phenomenological experience was subsumed into a rationalist idealism that was incompatible with the anthropological account that I wanted to offer. Meraleau-Ponty's work was instrumental in my research in order to elaborate the role of language and body in perception, and so was the work of the epistemologist Marx Wartofsky, which I discovered through my readings of Michael Cole's papers and the work carried out at the Laboratory of Comparative Human Cognition in California. I came across the papers of David Bakhurst, of Queen's University, a great specialist on a philosopher who in turn became an important influence on my work: Evald Vasilevich Ilyenkov.

These authors (as well as many others that I have not mentioned here) led me to suggest that students' acquisition of a mathematical concept is a process of becoming aware of something that is already there, in the culture, but that the students still find difficult to notice. The awareness of the object is not a passive process. The students have to actively engage in mathematical activities not to "construct" the object (for the object is already there, in the culture) but to *make sense* of it. This process of meaning-making is an active process based on understandings and interpretations where individual biographies and conceptual cultural categories encounter each other – a process that, resorting to the etymology of the word, I call *objectification*. To learn, then, is to objectify something (Radford, 2003). Now, to see the object, to become aware of it, teachers and students mobilize all sort of tools, symbols, words, gestures, etc. These are *semiotic means of objectification*. Knowledge acquisition requires one to become aware of abstract relations that cannot be fully indicated in the realm of the concrete but that, at the same time, cannot be noticed but through concrete objects, gestures, actions, and symbols.

I would like to end this short summary by mentioning one of the problems that we are currently investigating: the problem of *the disembodiment of meaning*. As a result of the contextual nature of actions and of the aspectual view deriving from language and signs, gesture and perceptual activity, a spatial-temporal relationship is created between the individual and the conceptual object leading to what we have termed an *embodied meaning*. This embodied meaning has to become somehow *disembodied* in order to endow the scientific conceptual object with its cultural, interpersonal value. This disembodiment is very difficult to accomplish for the students, as suggested by the following example (for a more detailed account see our research reports in the PME27 and 28 Proceedings). In a Grade 11 classroom activity, we wanted to start exploring the role of kinesthetic actions and semiotic activity.

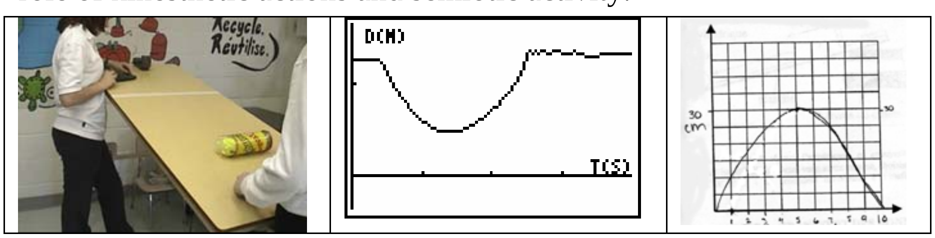

Figure 1. To the left, the students propelling the cylinder from the bottom of the ramp. The CBR can be seen at the top of the ramp. The second picture shows the graph produced by the calculator. The third picture shows the one drawn by the students.

The students were asked to make a graph of the relationship between the time spent and the distance traveled by a cylinder propelled from the bottom of a ramp (see Figure 1). Then the students carried out the experiment using a TI 83+ calculator connected to a Calculator Based Ranger (CBR) and were asked to compare their graph to the one produced by the CBR and calculator. Since the CBR was placed on top of the ramp, the calculator produced a convex parabola. The students drew a concave parabola and had difficulties understanding why the initial point of the graph was not at the point (0,0). As the transcript analysis reveals, for them, the bottom of the ramp is an important place.

The bottom of the ramp and the beginning of the cylinder motion orient the students' perceptual activity and become the centre of their mathematical experience. This point (that we have termed the *origo*, using the expression coined by K. Bühler, see Radford 2002), is confused with the mathematical origin of the Cartesian graph. The distinction between these two origins (the mathematical and the *origo*) is central for the disembodiment of meaning. In this example, to disembody meaning means to realize that the mathematical origin (defined by the position of the CBR) does not necessarily coincide with the place from where the experiment starts.

Let us summarize the general aim of the research program that we are conducting in light of the previous discussion. We are investigating the dialectics between the students' kinaesthetic and artefact-mediated activity and their processes of symbolizing and meaning production. One of the research goals is the following: To investigate the role of bodily and artefact-mediated (concrete or imagined) action, perception, and linguistic activity in algebraic symbolism and in the formation of meaning.

Acknowledgment: This paper is a result of a research program funded by the Social Sciences and Humanities Research Council of Canada (SSHRC/CRSH). I wish to thank Caroline Bardini and Cristina Sabena for their comments on a previous version of this paper.

References

Arzarello, F. & Robutti, O. (2001). From Body Motion to Algebra through Graphing. In H. Chick, K. Stacey, J. Vincent & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference*, The University of Melbourne, Australia, **1**, 33-40.

Beth, E., W. and Piaget, J. (1966). *Mathematical Epistemology and Psychology*. Netherlands: D.Reidel.

Edwards, L., Robutti, O. and Frant, J. (2004). Embodiment, Metaphor and Gesture in Mathematics. *Proceedings of the PME28 Conference,* Bergen, Norway, **1**, 270.

Goldin-Meadow, S. (2003)). *Hearing Gesture. How our hands help us think*. Cambdrige, Ma.: The Belknap Press of Harvard University Press.

Kita, S. (2003). *Pointing. Where Language, Culture, and Cognition Meet.* Mahwah, NJ.: Lawrence Erlbaum.

Nemirovsky, R. (2003). Three Conjectures Concerning the Relationship between body activity and Understanding Mathematics. *Proceedings of the 27 PME Conference*, University of Hawaii, **1**, 105-109.

Núñez, R. (2000). Mathematical Idea Analysis: What embodied cognitive science can say about the human nature of mathematics. *Proceedings of the 24 PME Conference,* Hiroshima University, Japan, **1**, 3-22.

Parmentier, Richard J. (1997). Semiotic approaches to meaning in material culture. *Semiotica*, **116** (1), 42 – 61.

Radford, L.: (2000). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis. *Educational Studies in Mathematics,* **42** (3), 237-268.

Radford, L. (2002). "The seen, the spoken and the written. A semiotic approach to the problem of objectification of mathematical knowledge. *For the Learning of Mathematics*, **22**(2), 14-23.

Radford, L. (2003). Gestures, speech and the sprouting of signs. *Mathematical Thinking and Learning*, **5**(1), 37-70.

Roth, M.-W. (2001). Gestures: Their Role in Teaching and Learning. *Review of Educational Research,* **71**(3), 365-392.

Sinclair, Hermine. (1971). Sensorimotor Action Patterns as a Condition for the Acquisition of Syntax. In Renira Huxley and Elisabeth Ingram (Eds.), *Language Acquisition: Models and Methods* (pp. 121-135). London & New York: Academic Press.