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THE SEMIOTICS OF THE SCHEMA[1]

KANT, PIAGET, AND THE CALCULATOR

ABSTRACT: What is the relationship between our mental activity and the empirical objects of the world? Kant raised this question in the *Critique of Pure Reason* and attempted to answer it by arguing that between the realm of concepts and that of sensuous phenomena lies the *schema*. Piaget re-elaborated the Kantian concept of schema and since then it has been extensively used in constructivist and psychological accounts of the mind. In this article, I discuss Kant's and Piaget's concept of schema from a semiotic-cultural perspective. Attention is paid to the epistemological premises on which the Kantian and Piagetian theoretical elaborations of the concept of schema were based and the role that signs played therein. I contend that the schema and its genesis can be better conceptualized if we take into account linguistic and non-linguistic mediated actions embedded in the social processes of meaning production and knowledge objectification. My discussion interweaves epistemological concerns with the semiotic analysis of a group of Grade 11 students dealing with the mathematical understanding and description of a natural phenomenon – the movement of a body along a ramp in a technological environment.

KEY WORDS: activity, cultural semiotics, gestures, Kantian and Piagetian epistemology, mediated action, phenomenology, schema.

INTRODUCTION

Kant believed – contrary to Hume, Locke and the empiricist tradition – that knowledge cannot be reduced to what impressions and senses give us. Ideas should certainly be more than the result of impressions that we receive from the contingent world. The guiding principles of experience should be more than customs if we are to avoid confining them to subjectivity. But Kant also believed – contrary to the rationalist tradition of Descartes, Leibniz and Wolff – that knowledge cannot be reduced to an inner mental activity governed by the a priori rules of Reason. Leibniz, for instance, had said that "our ideas, even those of sensible things, come from within our own soul" (Leibniz 1949, 15). If such were the case, Kant asked, how is it possible that the formal rules of Reason – removed from of all empirical content – can yield knowledge of the objects of the external world?

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Kant constructed a sophisticated system that tried to accommodate both the empiricist and the rationalist traditions. In this system, the senses were no longer considered as superfluous or as with merely heuristic value, as in Leibniz[2]. Kant provided the senses with an epistemological import. In an important passage of the *Critique of Pure Reason*, he says that knowledge is constituted of both sensual perceptions and concepts (A50/ B74, 92)[3].

But knowledge is more than a cocktail of conceptual and sensual ingredients. The sensual perceptions, Kant claimed, have to be linked to their corresponding concepts. To distinguish between the pen on the table and the book beside it, we need to be able to differentiate among the perceptions. To accomplish this we need to judge. Otherwise, Kant said, we would be led to a "rhapsody of perceptions" (A 156/ B195, 193). Judgment is a "peculiar talent" that distinguishes whether something (a perception) goes under a certain concept or not (A133/ B172, 177). For Kant, the *schema* is precisely a function of the faculty of judgment. A schema is something mediating between the mind's logical machinery and the phenomenal world. Its task is to ensure the link between concepts and senses, that is to say, between Form and Content.

THE ENCOUNTER OF *FORM* AND *CONTENT*

The schema is a kind of *analogical procedure* – a "monogram", as Kant said – that *unveils* the link between the intellectual and the sensual in the course of its empirical execution.

Like the concepts, the schema for Kant is itself void of empirical content. Yet it must contain something which is represented in the object that is to be subsumed under the concept (A137/ B176, 180). While the schema, in one respect, must be *intellectual*, said Kant, in another, it must be *sensible* (A 138/ B 177, 181). But the schema does not have to be confounded with an image:

> If five points be set alongside one another, thus**, ...** , I have an image of the number five. But if, on the other hand, I think only a number in general, whether it be five or a hundred, this thought is rather the representation of a method whereby a multiplicity, for instance a thousand, may be represented in an image in conformity with a certain concept, than the image itself. For with such a number as a thousand the image can hardly be surveyed and compared with the concept. This representation of a universal procedure of imagination in providing an image for a concept, I entitle the schema of this concept. (Kant, A140/ B179, 182)

In saying that the schema is a method or universal procedure Kant meant that its execution can be repeated again and again. The schema entails, in fact, a principle of iteration linking thereby *knowledge* and *action*. Kant's epistemology supersedes here the passive receptivity of impressions of the empiricist school and the reduction of knowledge to inner mental activity effectuated by the rationalist tradition. As a result, "there is knowledge only in the schematized experience." (Chiurazzi 1990, 155). This is also what Piaget meant when he said that we know an object only when we act upon it (Piaget 1970a, 85).

Now, since the schema is not only intellectual but is also sensual, we can ask: What is the *material* of which the schema is made? In addition to the schema of number

(quoted above), Kant mentioned other examples, among them the schema of a triangle and the schema of the concept of a dog. In the last two, the representation is made by drawing a figure that *during its execution reveals the method*; in the first one, the execution cannot reveal the method. There is no longer coincidence between execution and method. In the case of a number such as a thousand I can still draw point after point, except that, in this case, "the image can hardly be surveyed and compared with the concept." Judgments ("perceptual judgments", to use Peirce's term) do not work the same in geometry as in arithmetic. In the schema of arithmetic and algebraic objects highly cultural conventions underpin the very possibility of the execution of the method or universal procedure. It took Kant almost 10 years to disentangle the difference between these kinds of schemata. He came back to this difference in the third critique – *Critique of Judgment* – where, as Nichanian (1979) rightly observed, Kant met the symbol.

THE ROLE OF SYMBOLS IN KANT'S CONCEPT OF SCHEMA

It was, indeed, in the course of Kant's reflection on *Aesthetics* (by which he did not mean that which is related to art, but what in Greek is called "anaesthetic", i.e. "without sensation") that Kant encountered the symbol. How can we have or produce sensual presentations or re-representations of ideas (such as 'taste') "for which a commensurate intuition can never be given"? (Kant 1790, S57, 140)[4]. Kant wrote:

> All intuitions by which a priori concepts are given a foothold are … either schemata or symbols. Schemata contain direct [presentations of the concept], symbols [contain] indirect presentations of the concept. Schemata effect this presentation demonstratively, symbols by the aid of an analogy (Kant 1790, S59, 148).

The schema for the geometric concepts is hence based on a certain resemblance – it shows *ostensively* a certain commonality between the concept and its sensual presentation. As in the case of ideas of 'taste' or 'beautiful', the schema of arithmetic and algebraic concepts is only symbolic. They

> express concepts without employing a direct intuition [i.e. sensual presentations – LR] for the purpose, but only drawing upon an analogy with one, i.e., transferring the reflection upon an object of intuition to quite a new concept, and one with which perhaps no intuition could ever directly correspond. (Kant 1790, S59, 148)

The analogical process that allows us to move from an object of intuition to a new concept opens a window for a new kind of reflection – a reflection that will go from analogy to analogy. In contrast to the ostensive schema that functions as an "emblem", here the symbolic schema needs to enter into a new realm, a realm of possible experience. "The symbol is the analogy of an analogy, an analogy in abeyance". (Chiurazzi 1990, 158).

With his *Critique of Judgment* Kant provided room for semiotic considerations and went beyond the borders of the *Critique of Pure Reason*. His epistemology reached a new point of development but the possibilities of development were limited by his own ontological stance[5]. To understand this point, we need to note that, in its execution or materialization, the symbolic schema produces symbols, but the symbols thus produced designate something whose mode of existence is prior to all experience.

We may not know where the chain of analogies will lead us, but whatever the symbols are designating, their reference has always being *there*.

We have struck here one of the more fascinating and profound tensions in Kant's theory of knowledge. Although the symbol – as any intuition (presentation or representation) – has an *epistemological* import (as we saw in the previous section), the symbol cannot have an *ontological* constitutive role. Thus, it is unthinkable for Kant to conceive of a "pure symbolicity", i.e. a symbolicity without actual reference that, in its movement, could "participate" in the *constitution* of its own object. For Kant, the "symbol" can only be thought of in relation to a constituted reference: "the 'symbols' must always be 'symbolic' in the … sense that [their] pure reference must be constituted in the exterior of them." (Nichanian 1979, 287) The problem is that Kant adopted the rationalists' view on concepts and that, consequently, he considered concepts as independent of, and prior to, all experience.[6] Although considering himself a good Kantian, Piaget parted from Kant exactly at this point, as we shall see in the next section.

PIAGET

In 1924, Piaget published a review of Léon Brunschvicg's *L'expérience humaine et la causalité physique* [Human experience and physical causality][7]. He was seduced by the way Brunschvicg dealt with these two concepts that were vital in Kant's theory of Knowledge. The 28-year-old Piaget rephrased Brunschvicg's position saying that experience is not, as Kant assumed, something invariable, something given once and for all. On the contrary, experience has a historical context. The object of Raison, Kant was right, is to inform experience, but, in turn, Reason is constituted *in* experience. This claim was no longer Kant's. "Experience and reason are not two terms that we can isolate: Reason regulates experience and experience adapts reason." (Piaget 1924, 587). For Piaget, an account of human reason has to give up Kantian apriorism.

To better understand Piaget's solution to the problem between experience and apriorism let us return to Kant's schema of a dog. We recognize a dog because the empirical data (intuitions) that we collect in our experience are identified and filtered by the schema. The schema is not an *abstraction* drawn from experience. Experience is possible, and the empirical data become thinkable, *because* of the schema, and not the other way around. This is why Kant's theory of knowledge does not include a theory of abstraction. What Kant needed was a theory of *subsumption*, i.e. a theory indicating how representations and perceptions are subsumed under an a priori concept. In giving up apriorism Piaget found himself in need of a theory of abstraction. Central to it was the concept of schema – a revised one. He said: "Whatever is repeatable and generalizable in an action is what I have called a schema" (Piaget 1970b, 42).

As in Kant's case, a schema for Piaget is based on iteration. But the emphasis is now on the *actions*. However, in terms of human cognition, what is important in Piaget's version of the schema is not that we can iterate actions of one kind and then actions of another kind. This would lead us to a wonderful 'panoply of schemata' (similar

perhaps to Kant's "rhapsody of perceptions") that would remain in a chaotic situation in the absence of a higher organizing element. While Kant turned to the a priori concepts of the rationalist tradition, Piaget turned to structuralism:

> Any given scheme in itself does not have a logical component, but schemes can be coordinated with one another, thus implying the general coordination of actions. These coordinations form a logic of actions that are the point of departure for the logical mathematical structures. (Piaget 1970b, 42)

Piaget's anti-apriorism allowed him to conceive of symbols as playing a more decisive role in knowledge formation than they played in Kant's epistemology. Piaget's point of departure was the linking between action and representation. From the outset he insisted that it is a current mistake to reduce representation to language:

> Language is certainly not the exclusive means of representation. It is only one aspect of the very general function that Head has called the symbolic function. I prefer to use the linguists' term: the semiotic function. This function is the ability to represent something by a sign or a symbol or another object. (Piaget 1970b, 45)

In his book "*La formation du symbole chez l'enfant*" [The formation of symbol in children] – a particularly difficult book in its technical aspect because in it Piaget endeavored to show one of the central theses of his epistemology, namely that mental images are interiorized actions – Piaget argued that the symbol arises from non-symbolic schematism[8]. More specifically, Piaget was claiming that there is a *continuity* between the sensori-motor signifiers and the emergence of the first symbols in the children. In other words, that the sensori-motor intelligence prolongs itself into conceptual representation.[9]

The sensori-motor signifiers were seen by Piaget as 'indexes' or 'signals' but they still lack an independency vis-à-vis the signified object. The semiotic function begins precisely when there is a differentiation between signifiers and signifieds. This differentiation provides the signified with a spatial-temporal permanence and opens the possibility that a same signifier can be related to different signifieds.[10] For Piaget, the semiotic function includes differed imitations, symbolic play, mental images, gestures, and natural language. Following Saussure he distinguished between symbol and sign. A *symbol* is a « motivated » signifier, which means that the signifier bears a certain resemblance to the signified. A *sign*, in contrast, bears an arbitrary or non-motivated relationship to its signified. Thus, a letter that we use in an algebraic expression is a sign, while a figure standing for a triangle is a symbol.

If it is true that the constructive stance of his genetic epistemology led Piaget to pay careful attention to the way in which actions and gestures become conceptual representations, it is also true, however, that Piaget's attention to signs and symbols faded away in his analysis of older children's thinking. Reflective abstraction converts action into operations and signs come to symbolize the operations. Hence, in Piaget's epistemology, in opposition to Kant's, signs and symbol borne a constitutive ontological role, but because the primacy was given to the structure, signs and symbols were in the end merely the carriers and the expressions of a thinking measured by its structural features. Piaget wrote:

> reflective abstraction, which derives from the first concepts from the subject's actions, transforms the latter into operations, and these operations can sooner or later be carried

> out symbolically without any further attention being paid to the objects which were in any case "any whatever" from the start. (Beth and Piaget 1966, 237-238)

To sum up, Piaget elaborated a theoretical reformulation of the Kantian concept of schema. He emphasized the epistemological role of action and gesture. However, the emphasis on the operations' structure left little room for a thematization of the content of the operations and for a serious consideration of the semiotic systems and the cultural artifacts that the children use. Thus, for Piaget, the object that the hand holds in the schema is unimportant. It may be "any whatever" from the start, as he says in the last quotation. Verillon and Rabardel comment that

> the object submitted to the Piagetian subject is fundamentally non-historical and non-social: its main property is that it is structured by physical laws. … The introduction of artifacts in classic Piagetian experiments is mainly due to their convenience for highlighting the invariant properties of reality. (Verillon and Rabardel 1995, 80)

Piaget's recourse to structuralism (even if it was a dynamic one) introduced irresoluble tensions in his epistemology – tensions that were proportional, we may say, to the ones Kant introduced in his by having recourse to apriorism.[11] While in Kant the tension appears between Form and Content, between concept and sensual representation, in Piaget it appears as the tension between structure and object. In both epistemologies, nevertheless, the common denominator is that mind's activity is, in the end, reduced to abstract mental labour.[12].

In the next section I will claim that, from an epistemological and a psychological viewpoint, the concept of schema needs to be broadened so as to include not only the instruments that the individual uses (which has been Rabardel's recent claim[13]) but its cultural context and other semiotic means such as speech and gestures that, more than mere ephemeral descriptors of reality, prove to be fundamental in knowledge formation.

SCHEMA AND ACTIVITY

Let us come back to Kant's concept of schema. As previously seen, for Kant, the distinctive epistemological trait of a schema is to present or exhibit, through the execution of a procedure, the "intuition" of an object (the object of knowledge). I will take this idea as my starting point. However, as Peirce contended (Peirce 1966, p. 43), the way in which the object thus becomes intuited has a *volitional character* that Kant did not take into account. The volitional character underpinning the schema and its genesis, should be studied in the context of the individuals' activity.[14] As such, it is related to the activity's *goal*. But complex activities are often comprised of chains of actions. A chain is directed towards the attainment of an *aim*. An aim (in contrast to the goal of the activity) is not necessarily something that is set from the beginning: it is a reference point that hypothetically can lead us closer to the goal. The formation of an aim is part of the heuristic process underlying the activity.[15] Bearing these remarks in mind, the schema, I would like to suggest, is an organization of actions or a chain of actions related to the attainment of the goal and aims of an activity.

In this perspective the schema has a double nature: (1) a *functional* and (2) a *phenomenological* one.

(1) The phenomenological aspect of the schema:
In its *phenomenological aspect*, the schema is a mode of presentation – a mode of "exhibition" of the object, as Kant used to say –, an effort to render something (e.g. a conceptual object or a process) available, noticeable – even if, ontologically speaking, the object or the process (in short, what Husserl called *objectity*[16]) does not have necessarily precedence over the action. In this case, the schema *produces* the object and functions as a form of disclosure (in Heidegger's sense[17]). The schema *objectifies* the object (Radford 2003a).

(2) The functional aspect of the schema:
The *functional aspect* of the schema means that the schema is governed neither by the Kantian rationalist apriorism nor by the Piagetian's normative character of logico-mathematical structures. Indeed, its adequacy is not examined against a grid of truth but against its practical results.[18]

The schema, as I am formulating it, is still both a sensual and an intellectual action or a complex of actions. In its intellectual dimension it is embedded in the theoretical categories of the culture. In its sensual dimension, it is executed or carried out in accordance to the technology of *semiotic activity* (Radford 2002b). We still save some of the characteristics of the Kantian formulation – figurative synthesis in the heuristic process, the difference between the execution of the schema and its result, its reiteration – but I place it in the broader context of the individual's subjective awareness that, in its constructive and creative endeavor, grows sustained and framed by the theoretical categories of the culture, its technology of semiotic activity and the historically constituted mode of knowing (Radford 2003b). In the next section, I turn to a classroom episode that will help clarify the previous ideas.

THE TECHNOLOGICALLY MEDIATED SCHEMA: FILLING THE HOLES

In an artifact-mediated classroom activity, Grade 11 students were asked to investigate the relationship between time and distance of a cylinder moving up and down an inclined plane[19]. In one of the parts of the activity the students performed two experiments using a TI 83+ calculator and a Calculator Based Ranger (CBR) motion detector. In the first one, the students propelled the cylinder upwards, from the bottom of the inclined

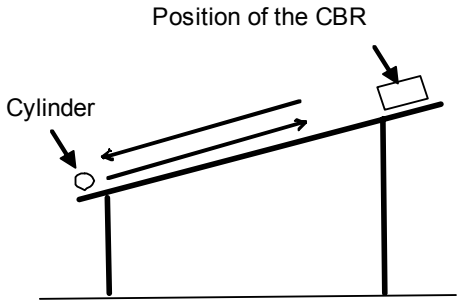

Figure 1 . Inclined Plane or Table

plane and activated the CBR as soon as the cylinder was put in motion. In the second one, the cylinder was propelled one second after the CBR was activated[20]. In both experiments the CBR was placed at the top of the inclined plane (see Figure 1).

Figure 2 shows the calculator graph for the second experiment. In this part of the activity the students had to explain the shape of the calculator's graph. Another part of the activity consisted in two 'thought experiments.' Here the students were asked to sketch two graphs: one for a cylinder moving on an imagined ramp that had a greater slope than the one of the experiment, and one for a cylinder moving on an imagined ramp having a lesser slope than the one of the experiment.

I will discuss first the students schema that resulted from the delayed motion (t=1; see Figure 2), and then I will comment on the use of this schema in the part concerning the 'thought experiments.'

In the first part, the schema, whose result is the graph shown on the calculator screen, consists of a sequence of actions, among them: (1) preparing the technological system calculator-CBR; (2) activating the CBR; (3) propelling the cylinder; (4) following the cylinder perceptually during its trajectory; (5) stopping the CBR when the cylinder comes back down and (6) making sense of the graph.

In order to better understand the schema we need to discuss the role of the technological system 'calculator-CBR' which was crucial in the experiment and in the generation of the graph. For sure, this technological system (TS) permitted a substantial economy in the carrying out of the experience. While Galileo went to great pains to figure out a way to measure the consumed time (a variable that, in contrast to distance, cannot be *seen*), the TS registered the measures of distance and time and, in the human-TS interaction, the calculator *produced* the graph.[21]

Now, the TS is more than a gadget to economize actions. It carries in itself, in a compressed way, socio-historical experiences of cognitive activity and scientific standards of investigation (Lektorsky 1984; Pea 1993). In addition to providing the students with economy and precision, the TS executes some of the human actions that it holds in a compressed way, and displays on its screen outputs of these actions. However, by taking over some of the human actions, certain aspects of the socio-historical experience that the TS holds remain "hidden" from the individuals using it. As a result, the schema loses an important aspect of the "sensuality" that it could have had for Galileo and the understanding that could have resulted from seeing, touching, and doing. The fact that the symbol-graph is not the result of the individuals' own actions but rather the result of the individuals' actions *and* those socio-historical ones that the TS executes, brings forward a very important element in the genesis of the schema: the resulting schema is a schema containing "gaps" or "holes". Indeed, while the *execution* of the symbol-figure of a triangle *reveals* the schema, in the technological experiment the *displaying* of the symbol-graph of the cylinder's motion on the calculator screen does not.

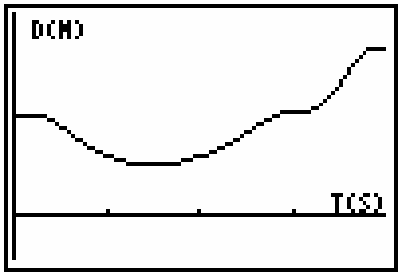

Figure 2. Calculator's graph.

There is no longer coincidence nor analogy between the execution of the procedure and the schema.

To obtain the schema, the holes have to be filled. However, the problem is not to repair the holes induced by the division of labor with their original substance (which would be impossible anyway). The problem is to *make sense* of the symbol-graph thus produced. In contrast to the schema of the triangle or of the dog, and the schemata discussed by Piaget, such as the baby hitting an object with a stick, the semiotic activity does not end with the production of the "image" (Kant) of the schema. The semiotic activity goes beyond the image (here the symbol-graph). The question is not primarily to judge but to interpret and to produce meaning. The students have to *make sense* of the image and to do so they will have recourse to other semiotic systems such as gesture and speech, as we will now see.

The students noticed that the graph was not perfectly curved in the part after its minimum value and that, in the graph, the value of the variable D (distance) in the ending points is not the same (i.e. $D_f > D_0$; see Figure 2). While the first difference was explained by a slight turn of the cylinder when it was rolling upwards on the inclined plane, the second difference was more difficult to understand. After discussing different ideas Judith said:

1. Judith: … (looking at the inclined plane) This thing there [the cylinder], does it go further? (the other two girls turn to see the inclined plane which was behind the students' desks) … like this … (she makes a gesture with her right arm; the gesture starts with her arm extended in front of her body and moves back, miming the cylinder motion in its coming back down trajectory) does it measure the …? Oh! (she thinks she understood something)
2. Vanessa: What?
3. Judith: You started on the table [i.e. the table that served as the inclined plane for the experiment], right? (Vanessa : Yes) And when it was rolling it fell off the table (with a similar gesture her arm is bent again and goes beyond her desk, as the falling cylinder did during the final part of its motion when it fell off the inclined plane and was caught by the student)… I don't know…
4. Vanessa: It has nothing to do with that.
5. Judith: It does have something to do with that […] That's the curve, right? Here (she points to the horizontal segment of the left part of the graph on the calculator screen) suppose this is when you started on the table and when you finished (she points now to the horizontal segment of the right part of the graph), you've finished further, that's further. […] Let's say that your distance here would be 30, and 45, that's the error! […]

In Lines 1 and 3 Judith makes an "iconic gesture", that is, a gesture that bears a resemblance with its referent. The iconic sign-gesture *enacts* the falling trajectory of the cylinder (see Figure 3). It allows Judith to call her group mates' attention to a specific part of the phenomenon. Like the Calculator-CBR system, the iconic gesture affords a segmentation of the phenomenon and operates a choice of what has to be taken into account. But in contrast to the Calculator-CBR system, the iconic gesture does not stress speed, time, accurate distance and other elements. What it stresses is the fact that the cylinder went off the table. The iconic gesture has made

an important fact *evident* (i.e. capable of being seen). The fact that, in its way back down, the cylinder went off the table and, consequently it traveled more distance, allows Judith a new interpretation of the graph. The new interpretation is elaborated on Line 5. Indeed, in Line 5, Judith has recourse to an "indexical gesture": pointing with her finger, she indicates two parts of the calculator graph on the screen (see Figure 4). In this case, numbers (30 cm and 45 cm) come to play the role of the iconic gesture that has previously shown the cylinder falling off the table. The first number represents the students' estimated distance from the cylinder's maximum point to the bottom of the table. However, the cylinder never went 15 cm off the table (i.e. 45-30), for it was caught immediately as it fell off the table. By exaggerating the numbers, the particular element of the phenomenon is highlighted.

Figure 3. Judith makes an iconic gesture that mimes the cylinder coming back down. In the genetic constitution of the schema, the students have to interpret the results of the artefact-mediated actions. To do so, they have recourse to gestures and speech.

I have discussed in some detail the previous students' dialogue because this dialogue shows aspects of the students' efforts to fill the schema and, overall, because I take these efforts as an important part of the genesis of the schema.

The students' dialogue suggests that to fill the holes in the schema the students produce a kind of simulation of the cylinder motion. The simulation was oriented towards understanding some 'remarkable points' on the graph. In the terminology of the previous section, these points are examples of *aims* and represent, as Arzarello and Robutti (2001, p. 37) indicate, strong connections between signs and experience. To attain the aims, the students had recourse to language. Through its rich arsenal of terms, in particular through some *objectifying deictics* (e.g. pronouns, locative words, time-related expressions), language allowed the students to "indexicate" and "iconize" essential features of their mathematical experience[22]. The stu-

Figure 4. Indexical Gesture.The students point to the right part of the calculator screen.

dents' dialogue also shows how language was coordinated with gestures in the production of meaning and understanding.

Once some understanding was reached and that the schema was apparently completed, the students could apply the schema to the proposed "thought experiments". To do so, the schema was significantly contracted. The technology of semiotic activity was not the same (now the students worked with pencil and paper). The key element that the students retained of the cylinder motion was the parabolic shape and the starting and ending points of the graph. They then produced the graphs shown in Figure 5.

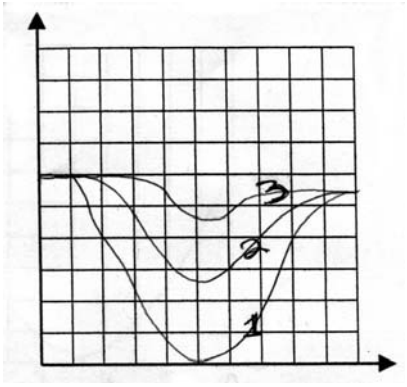

Figure 5. Graph for the cylinder motion on an inclined plane having a greater slope (graph 3) and a lesser slope (graph 1) than the original inclined plane (graph 2).

Graph 2 corresponds to the ramp of the original experiment. Graph 1 corresponds to the ramp having a greater slope and graph 3 to the ramp having a lesser slope. Of course, the results are not mathematically correct. The students focused on the kind of "effort" that it takes the cylinder to go up when the slope is greater and when the slope is lesser than the original one. All in all, the graphs show a partial understanding of the abstract mathematical spatio-temporal relationship of the cylinder motion.

SYNTHESIS AND CONCLUDING REMARKS

As we saw, Rationalists conceived of the mind as governed by a kind of abstract logical calculus ensuring deductions such as "$M \geq N$ and $N \geq P$, then $M \geq P$", regardless of the content of M, N, and P. Formal deduction removed from all empirical content, however, Kant argued, cannot yield knowledge. The question then was to explain how abstract concepts relate to their concrete content. In an important sense, the *Critique of Pure Reason* is an attempt to achieve this goal and the schema, in fact, was Kant's answer.

One of the distinctive theoretical features of Kant's concept of schema is that the individual is neither reduced to a passive receiver of impressions neither to a flesh box in whose interior logical calculations are effectuated. The schema entails the idea of an individual who, to acquire knowledge, has to become active. However, in Kant's theory of knowledge, the schema exhibits or unveils its concept – it does not produce it. Piaget retained the Kantian feature of an active individual, gave up apriorism and added a new ontological dimension: the schema was endowed with the power of producing concepts. Piaget's thesis, in fact, was stronger: concepts could not be produced in the absence of their correlated schemata[23]. The Piagetian elaboration of the schema opened a window for semiotic considerations. However, the concrete was rapidly evacuated and the relationship between Content and Form ended up being thematized against the rigid grid of logico-mathematical structures.

Since most of our actions are carried out using signs and artifacts, and since these actions are not performed in an arbitrary way but are framed by social goals and the cultural logic of meaning, I suggested that the schema can be reinterpreted as an organization of semiotical and artifactual actions or a chain of such actions related to the attainment of the goal and aims of an activity. The mediated nature of actions, nevertheless, leads to an important and difficult problem. Mediation means that, to accomplish something, we have recourse to an item of our environment (e.g. a word, an idea, a tool) that has already a social meaning. Carrying out a mediated action thus requires a lot of understanding. As a result of this intrinsic social nature of mediated action, the schema, generally speaking, cannot "exhibit" or show ostensively its object during its execution. The example of the graphic calculator and the motion sensor, I think, showed this point in a clear way. In the classroom episode, the students' schema was framed by a complex division of labor. The technological system calculator-CBR performed some key actions; as a result, even if the material product of the schema (i.e. the calculator graph) could be *seen*, the schema had "holes" that the students had to fill using creative imagination. The parabolic shape of the graph shown by the calculator underwent a process of interpretation. To do so, the sensual content of the cylinder motion had to be related to abstract aspects of the graph. Surely, language is a powerful means of objectification. However, in the genesis of knowledge, the relationship between conceptual descriptions and their referents cannot be reduced to linguistic terms (Otte 1998, 444). How then to account for the emerging schema and its encompassing description of the relationship between the concrete and the abstract? The interpretative process of the calculator graph (a crucial step in the formation of the schema), may shed some light on this problem.

In the course of this interpretative process, we saw the students displaying a range of semiotic forms of meaning production and knowledge objectification such as iconic and indexical reference (Figures 3 and 4) that were intermingled with language, intimating that the subsumption of a sensual content A into an abstract concept B by the schema may be much more complicated than perhaps Kant himself imagined. For one thing, both indexical and iconic reference involve types of "predication" different from those of the form "subject-copula-predicate", that is, of the form 'A *is* B' that Kant emphasized following the classical logic's view on judgments. It may very well be that 'Reality' is much less homogeneous than what we usually think and that the distinction between the concrete and the abstract might be placed on an "infinite

graduation of being, of perspective and of communication" (Otte 1998, 425) that language alone fails to capture.

If "the essential question of epistemology", as Otte suggests, is to understand that which "enables an A to stand for a B" (Otte 1998, 429) or that which makes an A to become subsumed into a B, a broader concept of predicative copula and relationship between A and B (between the concrete and the abstract or between Content and Form) would be required. In this line of thought, copular predication, I want to suggest, needs to be broadened so as to include other forms of semiotic reference capable of accounting for the dialectic ways of the constitution of subject and predicate, that is, of the semiotic processes through which the object of knowledge becomes noticed and socially thematized (in short, schematized) within a certain cultural discourse. It requires us paying attention to the technology of semiotic activity and its interaction with other semiotic systems in what Lotman (1990) calls the "semiotic space".

In the section titled Schema and Activity, I mentioned that I place the schema in the context of the individual's subjective awareness that grows sustained and framed by a historically constituted mode of knowing. I want to conclude these remarks by mentioning in what sense a schema relates to its cultural mode of knowing. I cannot find a better way to do so than to recall a phrase that Peirce wrote in a projected book that he never finished. Summarizing Kant's ideas, Peirce wrote: "Every cognition contains a sensual element."[24] In fact, every cognition (i.e. every phenomenon of our mental life) contains much more than a sensual element. It contains its cultural way of knowing. Thus, in the classroom activity, in addition to implicitly asserting, in a subtle way, the *existence* of a mathematical relationship between time and space that describes the cylinder motion, the design of the activity informs the students that such a relation becomes intelligible through *experimentation*. The question we asked the students and their conceptual procedures to answer it are framed and thus make sense within a particular, historically constituted mode of knowing. Had we asked a 17th century philosopher of nature this same question he would have certainly found it amusing – if not laughable. Vicenzo di Grazia (an Aristotelian philosopher and contemporary of Galileo), for instance, said:

> … those who want to demonstrate natural accidents through mathematical methods are delirious… the natural philosopher [scientifico naturale] studies natural phenomena whose essence entails movement, while, instead, the subject matter of mathematics does not comprehend movement. (Quoted in Biagioli 1993, 205).

When I said, in the application of the schema to the two thought experiments, that the student *only* retained the parabolic shape and the starting and ending points of the graph, I was forgetting the most important thing: the students' schema embodies a way of inquiring and of knowing about nature that only habit makes us now take for granted and to see as "natural".

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NOTES

[1] This article is a result of a research program funded by the Social Sciences and Humanities Research Council of Canada.

[2] In *New Essays Concerning Human Understanding*, Leibniz says: "necessary truths … must have principles whose proof does not depend upon examples, nor consequently upon the testimony of the senses, although without the senses it would never have occurred to us to think of them. This distinction must be carefully made, and was so well understood by Euclid, that he often proved by the reason, what is sufficiently seen through experience and by sensible images." (Leibniz 1949, 44)

[3] As usual in references to Kant's *Critique of Pure Reason*, A50 means page 50 of the 1781 edition; B74 means page 74 of the 1787 edition, etc.. Page 92 refers here to the English translation of Norman Kemp Smith. I will use this format throughout this article.

[4] In Kant's vocabulary "intuition" means an effected immediate relation that objects have on us (see A19/B33, p. 65). Examples of "intuitions" are impressions, perceptions, representations, etc.

[5] The problem, of course, is not that Kant had an ontology. We all need a theory of Being (even if it is only an implicit theory) in order to make assumptions or hypotheses. As Adorno pointed out, "If you refuse to make any assumptions, if you attempt to understand a thing purely on its own terms, then you will understand nothing." (Adorno 2001, p. 13).

[6] Daval (1957) deals with this topic in detail.

[7] Brunschvicg 1922.

[8] In the beginning of the book he says: "We will attempt to show how the [emergence of the] symbol is prepared by the non-symbolic schematism" (schématisme pré-représentatif). (Piaget 1968, p. 8).

[9] Piaget 1968, 68-69. See also Piaget 1972.

[10] Piaget in Piattelli-Palmarini 1982, p. 58.

[11] One of the tensions in Piaget's epistemology is its problematic concept of necessity, related to the growth of knowledge. It has been discussed in Otte (1998, in press). Another one is related to the problem of objectivity. It has been discussed in Radford 2002a.

[12] For a detailed elaboration of this point see Adorno 2001 and Buck-Morss 1975.

[13] Rabardel 1995, 1997.

[14] I use the term activity here in Leontiev's sense (Leontiev 1984).

[15] Leontiev 1984, p. 117.

[16] Husserl 1961, p. 44.

[17] Heidegger 1971.

[18] In his interesting work, Vergnaud (1985) was also confronted with the problem of the adequacy of the schema. In dealing with this problem in terms of *invariants*, he certainly succeeded in avoiding the Piagetian normative problem of logical structures. Among the invariants, Vergnaud included propositions (i.e. something that is true or false) and "propositional functions" – abstract functions having propositions as "variables". However, since "truth" as a conceptual category is adopted without critical stance, it is not clear how, epistemologically speaking, invariants are dependent and sensitive to the concrete cultural contexts of learning.

[19] The episode is described in detail in Radford *et al*. (2003).

[20] Thus, in the first experiment, the cylinder motion started at t=0 and, in the second experiment, motion started at t=1 sec.

[21] Commenting on the data collection in his experiment on an inclined plane, Galileo says: "As to the measure of time, we had a large pail filled with water and fastened from above, which had a slender tube affixed to its bottom through which a narrow thread of water ran; this was received in a little beaker during the entire time that the ball descended along the channel [carved on the inclined plane] or parts of it. The little amounts of water collected in this way were weighed from time to time on a delicate balance, the differences and ratios of the weights giving us the differences and ratios of the times, and which such precision that, as I have said, these operations repeated time and again never differed by any notable amount." (Galileo 1638, p. 170)

[22] A detail elaboration of the idea of "objectifying deictics" can be found in Radford 2002b.

[23] I will not dwell into this point here, limiting myself to mention that, to some extent, Radical Constructivism was elaborated as an effort to bring this point to its logical conclusions (for a critique see e.g. Lerman 1996; for a reply see Steffe and Thomson 2000).

[24] Peirce in Hooper (ed.) 1991, 17.

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