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THE ANTHROPOLOGY OF MEANING

Some have concluded that, if meaning is negotiable, then it is no longer of any use in explaining the way we understand one another. (Eco, 1999, p. 271)

ABSTRACT. Meaning is one of the recent terms which have gained great currency in mathematics education. It is generally used as a correlate of individuals' intentions and considered a central element in contemporary accounts of knowledge formation. One important question that arises in this context is the following: if, in one way or another, knowledge rests on the intrinsically subjective intentions and deeds of the individual, how can the objectivity of conceptual mathematical entities be guaranteed? In the first part of this paper, both Peirce's and Husserl's theories of meaning are discussed in light of the aforementioned question. I examine their attempts to reconcile the subjective dimension of knowing with the alleged transcendental nature of mathematical objects. I argue that transcendentalism, either in Peirce's or Husserl's theory of meaning, leads to an irresolvable tension between subject and object. In the final part of the article, I sketch a notion of meaning and conceptual objects based on a semiotic-cultural approach to cognition and knowledge which gives up transcendentalism and instead conveys the notion of contextual objectivity.

KEY WORDS: activity, cultural semiotics, epistemology, Husserl, mathematical objects, meaning, ontology, Peirce, subjectivity

1. INTRODUCTION

A time honoured tradition has it that meaning is the real and objective description of the intrinsic properties of objects or states of affairs. As a result, meaning cannot be subject to negotiation. To suppose that meaning is open to negotiation amounts to supposing that, at a given moment, we claim that things are a certain way and, a moment later, claim that they are a different way, thus making it impossible for us to communicate and understand each other.

In fact, this conception of meaning – of which contemporary dictionaries are perhaps the best example – goes back to Plato's time. Thus, Cratylus, in the dialogue that bears his name, maintains that there is a perfect match between signs and things. Within this context, the meaning of a sign (e.g. the meaning of the sign-drawing of a rectangle) discloses or uncovers the true nature of the thing (i.e. the referent). Meaning appears here as an objective act of *reference*.

Meaning, of course, can also be understood in a different way: it can be understood as the intentions that we want to convey. In this case, meaning appears as a subjective construct. Plato's contemporaries were aware of it. Nevertheless, they conceived of meaning above all as an act of reference because, for them, the true nature of things was supposed to be beyond human will.

Concepts of meaning indeed are based on presuppositions concerning the relationship between the cognizing subject and the object of knowledge. During the Renaissance, on the threshold of a world that was beginning to be oriented towards human production and exchange, when the epistemic contemplative subject of Greek thought was replaced by an active *Homo Faber*, to know something became equated with knowing the process of its production. By the end of the 19th century, when psychology conceptualized knowledge as a problem of consciousness, Edmund Husserl (one of Weierstrass' most brilliant students) endeavoured to understand how subjective intentions structure our awareness of what is being presented to us through our senses. Hence, meaning was elaborated as *that which we intend*.

However, the subjective stance surrounding the new concept of meaning clashed against the objective requirements of the sciences. On the one hand, meaning as an intentional act, i.e. as intention, necessarily appears as a subjective construct. On the other hand, because intentions are directed to an objective fact in the world, meaning carries objectivity. The link between these two facets of meaning proved to be more problematic than expected. If knowledge rests on the way we take notice of things, or on how they appear to us through our senses, or even on the actions that we carry out on them, how can the objectivity of conceptual entities be guaranteed? As Husserl put it, how do the objects of mathematics, sciences, aesthetics and other disciplines proceed from their primary intrapersonal origin to their ideal objectivity?

Behind this question of course lies the further question of the constitutive nature of human experience: in what sense can the subjective activity of individuals become constitutive of the ideal objects of all that is related, in one way or another, to the effective reality?¹

Mathematics educators have not been indifferent to the aforementioned questions. Thus, the importance of discussing ideas about mathematical objects in mathematics education has been clearly stated by Dörfler who, in a recent paper, wrote that

The issue for mathematics education (. . .) is what does it mean to know something about mathematical objects and how does the learner develop or construct that knowledge? The answer to this question will to some extent depend on the ontological and epistemological status that is ascribed to those mathematical objects. (Dörfler, 2002, p. 337)²

The objectivist referential approach to meaning (which locates mathematical objects outside semiotic activity) played an important role in the shaping of the pedagogical scene at the turn of the 20th century. It promoted a technological vision of mathematics, well tuned to a general view of the world which considered scientific progress to be the chief mark of civilization. Mathematics was no longer seen as a mere means to cultivate the faculties of the spirit and abstract reasoning: it was also seen as a useful modelling tool capable of unveiling the secrets of the world to us (see Nabonnand, 2004). However, the objectivist referential approach now appears to be challenged by the “discursive approach”, which “locates mathematical objects as discursive objects within the mathematical discourse” and claims that it is “the whole discourse of mathematics [which] lends meaning and existence to the mathematical objects” (Dörfler, 2002, p. 339).

It is my contention, nevertheless, that more discussion is needed in order to make explicit our assumptions concerning the subjective dimension of meaning and the sense in which mathematical knowledge may still claim some sort of objectivity. As one of the reviewers of this paper put it, these are “theoretical questions that mathematics educators meet when making choices concerning what mathematics to teach, and how.” Teachers holding opposing epistemologies will indeed convey different views of mathematics in the classroom. Questions about truth, mathematical proof and mathematical applications – to mention but a few – will certainly be addressed differently in the classroom depending on whether teachers adhere to e.g. Realism, Constructivism or Cultural Epistemology. Similar differences will become manifest in the way in which students’ learning is conceived (see Sierpinska, 1998).

Naturally, discussions here concerning meaning, subjectivity, and mathematical objects are not intended to necessarily lead us to overcome the different epistemological differences underpinning the various schools of thought in mathematics education. Rather, the relevance of such discussions is to make us aware of the ideas about mathematics that, directly or indirectly, we convey in the classroom and how these ideas influence our view of the teaching and learning of mathematics.

In this paper, I want to address the question of the way in which meaning and the intrapersonal nature of human experience can claim to be constitutive of conceptual mathematical objects. Since new insights may be obtained by examining the problem under different lines of inquiry, in the first two parts I discuss the phenomenological epistemologies of Peirce and Husserl. My interest in discussing these theories resides in the fact that, working from contrasting theoretical assumptions, these theories tackled the problem of meaning as an epistemological and ontological problem

interwoven with semiotic issues which are instructive for the problem at hand. In the last part of the article, I sketch what could be a theory of meaning and conceptual objects framed by a semiotic-cultural approach which emphasizes the central role of culture in the production of objects of knowledge and the way we come to know them.

In opposition to the objectivist referential approach, the semiotic-cultural approach that I am advocating (Radford, 2003a) does not claim a transcendental status for mathematical objects. Like the “discursive approach”, the semiotic-cultural one emphasizes the role of semiosis, but instead of locating the roots of mathematical knowledge in discourse, it conceives of knowledge in general and mathematical knowledge in particular as the result of a cognitive *praxis* (Wartofsky, 1979): ideas and mathematical objects, as I will argue, are conceptual forms of historically, socially, and culturally embodied reflective, mediated activity.

2. PEIRCE: TRUTH AS THE LIMIT OF UNLIMITED SEMIOSIS

In Peirce’s theory of knowledge, reality is not generated by the individual’s semiotic activity – nor are the attributes of mathematical objects (CP 4.156).³ Signs, indeed, are not empowered with a foundational status: they can neither create truth, nor can they produce facts or objects: “the sign”, said Peirce, “does not affect the object but is affected by it” (CP 1.538). Nonetheless, for Peirce, semiotic activity yields knowledge.

Indeed, although Peirce maintained that reality influences our thoughts but is not created by them (CP 8.12), he assumed that reality is cognizable by virtue of a harmonic correspondence between *ordo rerum* and *ordo idearum*, that is, between the order of things and the world of ideas. This harmonic correspondence that Peirce borrowed from the scholastic tradition, rests, in Peirce’s account, on an alleged commonality between the structures of the world and those of our semiotic-phenomenological experience.

In order to understand the sense in which semiotic activity yields knowledge, we have to bear in mind that Peirce conceived of reality as an evolving continuum encompassed by “indeterminacy” or pure *Firstness* and *hæcceity*, or pure *Secondness*, i.e. brute facts with no explanation (CP 1.405). Out of a universe displaying chaos in its beginning, uniformities and regularities were formed bit by bit and “habits” became possible, for “all things have a tendency to take habits” (CP 1.409). These habits took the form of laws: “Uniformities in the modes of action of things have come about by their taking habits. At present, the course of events is approximately determined by law. In the past that approximation was less perfect; in the future it will be more perfect. The tendency to obey laws has always been and always

will be growing.” (CP 1.409). This is why Peirce can say that “Reality consists in regularity. Real regularity is active law”, which is *Thirdness* (CP 5.121).

In Peirce’s work, reality forces itself upon the mind. “The reality of things consists in their persistent forcing themselves upon our recognition.” (CP 1.175). As he wrote,

The sense of hitting . . . is the sense of an actual occurrence, of actual action and reaction. . . . While I am seated calmly in the dark, the lights are suddenly turned on, and at that instant I am conscious, not of a process of change, but yet of something more than can be contained in an instant. I have a sense of a saltus, of there being two sides to that instant. (CP 1.380)

The contact with a hard or brute fact, (a Secondness) disturbs the individual’s thus far natural cognitive state of rest. “Consciousness”, as Peirce suggested, “is merely the sense of the shock of the non-ego upon us. Just as a calm sea sleeps except where its rollers dash upon the land.” (CP 8.266). Through thinking we repair the disturbed state in which we were put. Thinking, indeed, is the way to “the restoration of a homoeostatic, peaceful state of *ataraxia*” (Floridi, 1994, p. 562) – the state of which we have been deprived by the external objects that hit us. The function of thought is precisely to produce belief, that is, rules of actions which appease doubts by rendering things intelligible (CP 1.405). Belief “appeases the irritation of doubt; . . . it involves the establishment in our nature of a rule of action, or, say for short, a habit.” (CP 5.397).

Thus, thinking and semiotic activity are a *reaction* to our encounter with objects in the world. This reaction is led by a pragmatic motivation to understand the world that lies in front of us. In the “interest of intelligibility” we are “compelled” to experiment, to formulate hypotheses and carry out inferences:

the highest kind of synthesis is what the mind is compelled to make (. . .) in the interest of intelligibility (. . .) and this it does by introducing an idea not contained in the data, which gives connections which they would not otherwise have had. (CP 1.383)

Quantity, for instance, “is merely the mathematician’s idealization of meaningless vocables invented for the experimental testing of orders of sequence” (CP 4.154). This idealization, nevertheless, does not imply that quantity is something merely subjective (CP 4.156). Indeed, for Peirce, mathematics is objective, exact and universal (CP 4.237). If something has to be considered as subjective in mathematics, it is the idealization that the mathematician does in order to model a certain state of affairs (see CP 3.558; Otte, 2003). The mathematical idealization of a state of affairs serves to investigate the *objective* relations or laws governing it.

It is in this context that quantities and other mathematical objects play a central role. Mathematical objects are essential parts of the experimental enterprise carried out through abstractions, inductions and abductions. “An Abduction is Originary in respect to being the only kind of argument which starts a new idea.” (CP 2.96). In *Speculative Grammar*, Peirce wrote that “An Abduction is a method of forming a general prediction without any positive assurance that it will succeed either in the special case or usually, its justification being that it is the only possible hope of regulating our future conduct rationally, and that Induction from past experience gives us strong encouragement to hope that it will be successful in the future.” (CP 2.270).

Peirce, much as Cratylus, supposed that there was a kind of agreement between world and signs. The laws of the world are not human conventions. But they are intelligible:

Nature herself often supplies the place of the intention of a rational agent in making a Thirdness genuine and not merely accidental; (. . .) But how does nature do this? By virtue of an intelligible law according to which she acts. (CP 1.366)

Moreover, the laws of nature are cognizable:

every fact of a general or orderly nature calls for an explanation; and logic forbids us to assume in regard to any given fact of that sort that it is of its own nature absolutely inexplicable. (CP 1.405)

However, for Peirce, in opposition to Cratylus, the agreement between the world and signs is not a direct one. Even if, for Peirce, objects and laws are independent of, and occur prior to, the individual’s semiotic activity, he firmly believed that there are forms of transforming signs that would carry us to the ultimate truth of things. Success in the attainment of truth rests on the idea of “synechism”, or the idea of regarding everything in the ontological and in the epistemological planes as continuous: the progressive transformation of signs approximates our knowledge bit by bit to real truth. Notwithstanding Kant, we can accordingly be assured that, in our intellectual semiotic inquiries, (if they are “correctly” conducted) we are not running after mere appearances.⁴

How does meaning fit into Peirce’s semiotics? Meaning is at the heart of the grand semiotic process of understanding the world: meaning is precisely what allows one to go from sign to sign, from induction to induction, from abduction to abduction. “A sign”, wrote Peirce, “stands for something to the idea which it produces, or modifies. Or, it is a vehicle conveying into the mind something from without. That for which it stands is called its **object**; that which it conveys, its **meaning**; and the idea to which it gives rise, its **interpretant**.” (CP 1.339). In short, meaning is the relation that

links one sign to the next sign (i.e. its interpretant) in a semiotic chain (CP 4.536; see also Dewey, 1946, p. 88).

The way meaning allows us to go from sign to sign has to be understood in terms of Peirce's Pragmatic Maxim, which he stated as follows:

Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object. (CP 5.402)

As Almeder noted, "What the pragmatic maxim asserts is that the meaning of any proposition is nothing more than the conceivable practical effects which the assertion would imply – if the proposition were true." (Almeder, 1983, p. 333). This is why the meaning of something is a conditional statement, a law or a habit that governs the object of the proposition. "To say that a body is hard, or red, or heavy, is to say that it is subject to a law" said Peirce (CP 5.545).

In short, the pragmatic conception of meaning stresses the fact that meaning is related to a hypothetical or idealized state of affairs. In the end, meaning is a matter of heuristic semiotic investigation.

The heuristic nature of meaning was clearly stated by Brent, who wrote that, for Peirce

Meaning is of the nature of a **thought** experiment, in that it is what we find to be the *conceivable* consequence of an **abduction** we are considering. It is not those consequences, it is what we think them to be. Meaning is virtual, a matter of the transformation and interpretation of **signs**. (Brent, 1998, p. 354; emphasis as in the original)

Because Peirce postulated an adequacy between *ordo rerum* and *ordo idearum*, between what we know and what is to be known, knowing and being belong to the same grand scheme of things. This magnificent adequacy ensures that the contents of awareness constitute a unity of meaning and being. Commenting on Peirce's concept of meaning, Rosenthal wrote:

our lived perceptual experience is an intentional unity of knower and known which emerges through our modes of grasping the independently real continuum of qualitative events. . . The internal structure of meaning as habit both provides the tool for 'cutting the edges' of such a processive continuum and allows for a primordial experiential grasp of its continuities, real relations and real potentialities. (Rosenthal, 1983, p. 325)

Naturally, Peirce cannot escape the question of the legitimacy of the postulated adequacy between *ordo rerum* and *ordo idearum*. In considering Firstness, Secondness and Thirdness *both* as categories of being *and* as phenomenological categories, Peirce indeed cannot escape the reproach of having amalgamated the ontological and the epistemological in a way that

solves the problem of the objective and the subjective only in appearance. The Pragmatic Maxim was Peirce's attempt to get rid of a philosophical attitude entertaining propositions for which no precise answers could be given. To speculative metaphysics he opposed the verification procedures of science. However, in so doing, Peirce's choice commits him to a theory of knowledge that hangs upon the weight of a scientific rationalism that does not appear broad enough to anthropologically account for the diversity of the human mind. In characterizing the real through the method of theoretical western science, Peirce's epistemology leaves little room for the understanding of human activities of aesthetic, moral, religious, and political natures. Undoubtedly, Peirce was right in asserting that meaning is something belonging to the realm of the general, something mediating experience. But, in Peirce, that which is mediated by meaning is the rational validity of the hypothesized outcome of an experience where no attention is paid to the role of concrete social praxis and human interaction. It is true that Peirce talked about a community of thinkers and claimed that reality is inseparable from such a community. Nevertheless, his account of knowledge is definitely an account of knowledge carried out from a first-person perspective. Peirce's communism rests on the view that the community of thinkers is in fact defined by the *commonality of logic*. Peirce says: "So the social principle is rooted intrinsically in logic" (CP 5.354). If there is such a thing as a community of thinkers it is because there is an objective logic in the first place.

To sum up, Peirce advocated a view according to which we inhabit a world whose objects, laws and state of affairs are intelligible and semiotically knowable, even if to know them we have to go through an unlimited process of semiosis. Truth, indeed, is the ultimate point of this process which is governed by Meaning as the virtual engine that orients the chain of signs. As a result of Peirce's Realism and the epistemological status with which he endows semiotic activity, the agreement between knowledge and reality takes a singular form. While, for Piaget, the problem was to explain how the subjective process of abstractive actions can lead to objective knowledge of conceptual objects, for Peirce, the problem was to explain how semiotic activity can unveil the real nature of objects whose objective existence does not depend upon our system of signs and our interpretations.

For Piaget, truth is conceived of as simultaneously human-made *and* objective, i.e. human-independent.⁵ Peirce agrees with Piaget that truth is human-independent, but claims that it is not human-made. And for Peirce this is not a problem: even if truth is not human-made, it is humanly reachable – to the extent that he found it uninteresting that the angel Gabriel would descend from Heaven to reveal the answer to the riddle of reality to us (CP 5.553). For Peirce, we do not need such angelic gestures: the

riddle of reality could be solved by finding good tools: “if we can find out the right method of thinking and can follow it out – the right method of transforming signs – then truth can be nothing more nor less than the last result to which the following out of this method would ultimately carry us.” (CP 5.553). The success of the pragmatic enterprise lies in the fulfillment of an unlimited semiosis guided by Meaning and the methods of science.

Despite the emphasis on practice, Peirce’s pragmatism, as Smith correctly observed, “is quite rationalistic” (Smith, 1983, p. 49). And so is his concept of meaning. The individual remains an abstract construct and his subjectivity takes shape in his reaction to the non-ego. Man, for Peirce, is a natural entity carried out, as Nature itself, by the laws of evolution. Man is not a cultural historical product and neither is his knowledge of the world. Human reality appears as something abstract directed at most by a logicized community. As we shall see in the next section, Husserl gradually turned to an interest in historical praxis, something which placed his late work on the edge of one of the more vivid tensions between the objective and the subjective.

3. HUSSERL AND THE MEANING OF MEANINGS

The way in which Peirce theorized the contact between the cognizing subject and the object of knowledge shows the clear influence of empiricist thought in his work. His epistemology is an epistemology of the clash.⁶ Without discarding sensual activity, Husserl, in contrast, elaborated an *intentionalist* phenomenology, that is, a phenomenology based on the role played by intentions in our apprehension of things. Husserl was puzzled by a question previously raised by Frege: how can we claim to know something about e.g. big numbers, to whom we cannot have direct access? In his paper *The logic of signs* or *Semiotics*, Husserl rephrases the question as follows: “How is it that one can speak of ‘concepts’ which one, nevertheless, does not authentically possess, and how is it not absurd that the most certain of all the sciences, arithmetic, is to be based on such concepts?” (Husserl, ca. 1890–1908/1994, p. 20).

Following his master Brentano, Husserl distinguished between *proper representations* and *symbolic representations*. The former correspond to the representation of small numbers that can be grasped by normal perception. For this very reason they are, as Husserl said, authentic. The latter are indirect representations or representations by signs: they represent contents that we cannot access directly. They represent inauthentic concepts. Were we able to grasp in perception any multiplicity of things, we would then be able to decide, by merely seeing, whether equalities such as $7^6 + 9^6 = 18^6$ are true or not. If we had proper representations of all numbers, Husserl

argued, there would not be a science of arithmetic (Husserl, 1891/1972, p. 234).

According to Husserl, the solution to Frege's puzzle requires us to determine the foundations of symbolization (1891/1972, p. 241). He suggested that the only possible solution is to acknowledge that there are, among proper representations, *indexes* (*Anzeichen*, also translated as *indicative signs*) on the basis of which one can recognize the character of multiplicity (1891/1972, p. 246). In other words, Husserl believed that knowledge of numbers can in the end be traced back to concrete indexical representations of small numbers of which we can have an intuitive awareness via particular, concrete instances of them. Evidence is in the end validated by its intuitive and sensuous form, for evidence, as notes Bégout (2003, p. 161) is, for Husserl, "*verum index sui*".

As we can see, Husserl, like Peirce, conceived of indexicality as a key element in securing knowledge. However, Peirce thematized indexicality in terms of causation (I feel something in the realm of brute facts, and I feel it *because* something is there). Husserl elaborated it as an intuitive moment of intentional consciousness.

The requirement that arithmetic be based on indexicality or direct reference nevertheless raised a problem that ran through all of Husserl's work. Given that indexicality – this foundation of symbolization – presupposes an intention (hence a contextual and subjective view), how can objectivity be ensured?

Indeed, on the one hand, Husserl never doubted that the objects and state of affairs to which semiotic activity refers were objective in the strict scientific sense of the term.⁷

On the other hand, his theory of intentionality insists that our knowledge of objects is filtered by intentions, and every intention implies a subjective way of aiming at the object. Within this context, the question is: "how can subjectivity go out of itself in order to encounter or constitute the object?" (Derrida, in Husserl, 1989, p. 63).

In the *Origin of Geometry*, Husserl posed the problem of the passage from subjective sensual objects of experience to the objectivity of an ideal, omnitemporal object in the following terms:

Our problem now concerns precisely the ideal objects which are thematic in geometry: how does geometrical ideality (just like that of all sciences) proceed from its primary intrapersonal origin [. . .] to its ideal objectivity? (Husserl, 1989, p. 161)

Husserl tackled this problem again and again. At the center of the attempts to solve it was his concept of meaning. In 1900, he wrote: "an act of meaning is the determinate manner in which we refer to our object" (Husserl, 1900/1970, p. 289). In 1913, the problem was dealt with in Book 1 of *Ideas*,

through a refinement of the conditions of intentional meaning-giving experiences and of the concept of meaning itself. Pursuing his earlier reflections, Husserl remarked that we always experience objects from a certain angle, as having certain appearances, properties, or as having relations to other objects. This 'perspectival' or 'aspectual' intentional experience of attending to objects, Husserl called *noesis*. And the conceptual content of the experience he called noetic meaning or *noema*. Noema and noesis are at the centre of the phenomenological epistemology as developed by Husserl: conceptual contents (i.e. noetic meanings or noemata) always result from, and are correlated to, intentional experiences (*noeses*). In other words, no conceptual content can be attained without an accompanying or correlate experience. This is why Husserl claimed that there could be no noema without noesis.⁸

One of Husserl's own examples can help us to understand these concepts. We can think, he suggested, of an illuminating light. Our way of attending the illuminated object through the light is the *noesis*; it is an intentional experience without which we would not be able to see the object. "What is attended to, in the specific sense, subsists in the more or less bright cone of light, but can also shelve off into the half-shadow and into the full darkness." (Husserl, 1913/1931, p. 269). What we see is the conceptual content of our intentional seeing; it is the *noema* or the meaning. Now, we can also change the intensity of the lighting. "This alteration of the lighting does not alter that which appears in and through the meaning it conveys, but brightness and darkness modify its mode of appearing; they are to be found in the directing of the glance to the noematic object and there described." (Husserl, 1913/1931, p. 269).

In this example, each intensification of the lighting (each noesis) provides us with a certain content of the object. This content (noema) is not the object itself. In fact, it is one of its *meanings*.

The refinement of Husserl's concept of meaning resides in the attempt to provide a coherent view of it. Using a geological metaphor, he conceived of meaning as organized in *layers* where the "full noema" or full meaning is the ultimate coordination of all the noematic meanings that result from diverse phenomenological experiences: "the full noema consists in a nexus of noematic phases, and [...] the specific sense-phase supplies only a kind of necessary *nucleatic layer* in which further phases are essentially grounded" (Husserl, 1913/1931, p. 262).

In terms of our discussion of the relationship between meaning and conceptual objects, the important point here is the problem of the harmony or agreement between the full noema and the phenomenological object. How, for instance, can we ensure that it is the same mathematical object that is apprehended by different persons in their meaning-giving experiences?

For Husserl, the diverse meanings relate to their “object”, the “object simpliciter”, as he called it, and, in its transcendental, the object ensures the unity of the different noemata.⁹ The transcendental object is, in Husserl’s account, the content or the essence of meanings: it is the meaning of meanings.¹⁰

A Cartesian graph, an X/Y number table, an algebraic formula, each of them is a way of aiming at the *same* mathematical object, different intensities of a flash with which we light it. The graph, the number table and the formula are meaning-conferring intentional experiences (noeses) out of which a particular noema or meaning is attained. In Husserl’s account, we can be assured that these diverse noemata are lighting the *same* object, because its transcendental self-identity ensures the unity of individual meanings. The object attracts, so to speak, each of the lights, as a magnet attracts the metals around it. They are representations of the same mathematical object.

For Husserl, perception is only one of the three modes of intentionality – in fact, the simplest one. Along with perception, there are also imagination and signification. Signification, in Husserl’s account, is intimately related to language and the logical theory of judgements and predication – one of the central themes of Brentano’s seminar, which he attended in Vienna from 1884 to 1886. In harmony with classical logic, Husserl conceived of predication as an incidental means to bestow meaning on something: “through developmental and conceptual apprehension of our data we acquire a definite system of *predicates* . . . and these predicates . . . determine the ‘*content*’ of the object-nucleus of the noema in question” (Husserl, 1913/1931, p. 364–365), that is, this ideal and abstract object to which all the noemata point.

In other words, in carrying out intentional experiences we acquire a certain sequence of predicates $P_1, P_2, \dots P_n \dots$. Each one of these predicates P_j has a content or meaning N_j (its noema). The noemata N_1, N_2, N_3, \dots have a single object, the pure object of the predicates, the atemporal and self-identical “object simpliciter” X . This object X , the meaning of meanings, however, cannot be modified by the noemata. As Husserl said in the example about light, human experience cannot alter it.¹¹

Husserl’s concept of predication and its relation to meaning rests on a view of language according to which it becomes a kind of transparent tool that carries out intentions in a regular way. By decontextualizing language, judgements were reduced to an abstract and formal construct.¹² Nevertheless, as the anthropologist Sapir stated many years ago,

It is quite an illusion to imagine that . . . language is merely an incidental means of solving specific problems of communication or reflection. The fact of the matter

is that 'the real world' is to a large extent unconsciously built up on the language habits of the group. . . . We see and hear and otherwise experience very largely as we do because the language habits of our community predispose certain choices of interpretation. (Sapir, 1949, p. 162)

By removing the contextual and cultural factors surrounding intentionality, Husserl's account ends up portraying a theory of truth and meaning that is universal and beyond culture and time. As Derrida observed, Husserl comes back to language in order to allow the pure possibility of truth to emerge. Language is the sedimentation of reference and truth, not their constitution.¹³

4. MEANING AND CONCEPTUAL OBJECTS FROM A SEMIOTIC-CULTURAL PERSPECTIVE

It is worth noticing that, at the end of his life, Husserl became aware that human thinking needed to be understood against the background of the cultural context encompassing the phenomenological experience. In a letter dated March 11, 1935, sent to the anthropologist Lucien Lévy-Bruhl, whose book *The Primitive Mind* secured him a place in the sociology of knowledge, Husserl prized Lévy-Bruhl's work for its endeavour to understand the native's world as a world that was not mere "representation" but the real existent world. "From there", he wrote, "we come to apprehend, identify and think of their manners (*Arten*), hence [we apprehend] their logic as well as their ontology, those of the environing world with their corresponding categories." (Husserl, 1935).¹⁴

Certainly, one of the decisive contributions of his late work was the awareness that the sense-giving acts or noeses go beyond solely the relationship between individuals and their intentions. Husserl became aware of the fact that meaning can neither be reduced to the exteriorisation of thought (as in intending) nor to the indication of an object (as in referring). What Husserl was intimating was that our experiences of intending and referring are not only *object*-directed but also directed to *someone else*; in other words, that meaning is *other*-directed too. As a result, he ended up seeing meaning as much more than a subjective experience: meaning was a social experience through and through. But as his studies on intersubjectivity suggest, transcendental objectivity and the epistemological constitutive role of cultural categories did not merge in a coherent theory of meaning and conceptual objects. In a text written during the Christmas holidays of 1931/1932, a 72-year-old Husserl was still seeking to reconcile objectivity in its narrow scientific sense to the subjectivity of every distinct cultural world. He wrote: "We find ourselves here in front of the more profound problems of the objective world and of the apperception of the world, of

objectivity and of subjectivity.” (Husserl, 2001, p. 351). Husserl came to intuit that meaning and conceptual objects coexist with culture, but in no case could he conceive of the latter as consubstantial of the former. As Derrida notes in his comments on Husserl’s view of science,

Besides all the characteristics that it has in common with other cultural formations, [for Husserl] science claims an essential privilege: it does not permit itself to be enclosed in any historically determined culture as such, for it has the universal validity of *truth*. As a cultural form which is not proper to any de facto culture, the idea of science is the index of pure culture in general [. . .] Science is the idea of what, from the first moment of its production, must be true always and for everyone, beyond every given cultural area. (Derrida, in Husserl, 1989, p. 58)

In fact, Husserl’s later questioning ran along the lines of an understanding that seeks to comprehend how different cultural views of the world are possible, while leaving intact the question of the same and self-identical essence – the object *simpliciter*.

From an anthropological perspective, Husserl’s account fails to acknowledge the following central fact: sense-giving acts and all that makes them possible are essentially *cultural*. Thus, to continue with Husserl’s own example, the flash that we use to light an object, the intricate mechanism allowing us to control the intensity of the light and to *produce* different noises, have a cultural history which must be taken into account. The flash, like every artefact, is a cultural tool with a history behind it – a cultural history that, silently as it were, insinuates new lines of conceptual development (see Furinghetti and Radford, 2002). What appears in front of us in our intentional experience is consequently *ubiquitously framed by the cultural history of the means that we use to apprehend it*.

Husserl’s account hence fails to acknowledge the central fact that, in giving *meaning* to something, we have recourse to language, to gestures, signs or concrete objects through which we make our intentions apparent, and that language, signs, and objects are bearers of an embodied intelligence (Pea, 1993) and carry in themselves, in a compressed way, cultural-historical experiences of cognitive activity and artistic and scientific standards of inquiry (Lektorsky, 1984). If it is true that meaning is social, as claimed by interactionist and phenomenological studies inspired by Husserl’s thought (such as Berger and Luckmann, 1967) it is no less true that meaning is also cultural.¹⁵

In previous articles (Radford, 2000, 2002b, 2003b), I termed the whole arsenal of signs and objects that we use to make our intentions apparent *semiotic means of objectification*. Regardless of whether or not what we intend is personal or impersonal, what we convey in the experience of meaning can only be achieved in and through them. Merleau-Ponty’s reflections on language illustrate this point very well. He thought of language as a flexible system of possible significations. What we say is said using

a set of morphological instruments, syntactic and lexical systems, literary genres, figures of speech, forms of representation of events, etc. that are part of our cultural inheritance. We use all of these representational systems “to anchor the new signification (*la signification inédite*) in the already available significations” (Merleau-Ponty, 1960, p. 113).¹⁶

The intrinsically cultural nature of meaning lets us appreciate why, *contra* Cratylus, meaning cannot be reduced to the subject–object plane, where meaning appears as *referring*. In this case, meaning is reduced to those acts in which *someone* (the subject) refers to *something* (the object). Such a view leads us to a solipsistic account that overlooks the fact that ideal objects are already caught in a web of cultural conceptual categories (Luria, 1984, p. 62) that invisibly conspire against the direct experience of the object.

The intrinsically cultural nature of meaning also lets us appreciate why meaning cannot be reduced to the subject–subject plane. In this case, meaning is reduced to interaction and negotiation. Such a view of meaning removes the historicity of the tools, language, and sign systems used in the meaning-acts and forgets that noesis certainly is – to say it in a Vygotskian tone – a historical, semiotic mediated experience. It also leaves the question of the referent (i.e. the objects to which we refer) unproblematicized.

What, then, can a semiotic-cultural perspective that claims a prominent epistemological role for history and culture say about meaning?

I want to suggest that it is advantageous to think of meaning as a double-sided construct, as two sides of the same coin. On one side, meaning is a *subjective* construct: it is the subjective content as intended by the individual’s intentions. Meaning here is linked to the individual’s most intimate personal history and experience; it conveys that which makes the individual unique and singular.

On the other side and at the same time, meaning is also a *cultural* construct in that, prior to the subjective experience, the intended object of the individual’s intention (*l’object visé*) has been endowed with cultural values and theoretical content that are reflected and refracted in the semiotic means to attend to it.

“The route leading from the content of the individual psyche to the content of culture is a long and hard one”, noted Vološinov (1976, p. 87). Meaning, I want to suggest, is this route. However, precisely because “cultures make sense of the world in different ways, and what some see as a fact others do not” (Eagleton, 2003, p. 107), the twists of meaning are carved neither by the scientifically rationalized engine leading us from sign to sign, as in Peirce’s epistemology, nor by the intentional apparatus confined to remain captive in the subjective realm, as in Husserl’s account. The twists and turns of meaning are rather carved by the historically constituted modes

of rationality of the culture. It is in the realm of meaning that the essential union of person and culture, and of knowing and knowledge are realized.

It is time to turn to a concrete example. I want to refer to an excerpt from one of the hundreds of lessons that we videotaped in the course of a five-year longitudinal classroom based research program. It is a grade 12 three-day lesson dealing with the concept of rate of change and derivatives. In the first part of the activity, the teacher demonstrated the phenomenon to be mathematized in front of the class (see Figure 1). Following the teacher's demonstration, the students worked in small groups of three and were asked to solve a list of questions that the teacher and our research team had prepared for them.



Figure 1. The teacher shows a conical container with coloured water. The water flows out from the bottom of the container at a constant rate. The first day, the students were asked to make graphs expressing qualitative relationships between several variables (e.g. volume and height of the water in the container; volume and time; radius and time, etc.), as the water flows out.

In one of the small groups, after a sustained discussion, the students concluded that the graph of the relationship between the radius of the circle of the top surface of the water remaining in the cylinder and the elapsed time was linear (see Figure 2).

In the following excerpt, the teacher has just arrived to supervise the students' work. Noticing that, from the viewpoint of school mathematics standards, the graph still needs further conceptual refinement, the teacher tries to make the students reflect on the meaning of their graph. He says:

1. Teacher: Do you all agree on that (the graph)?
2. Diane: Yes!
3. Édouard: (*Answering with assurance*) Yes! We all agree. We discussed it!
4. Teacher: (*To prompt the students to reinterpret their graph, the teacher says*), What does this tell me about the change in the radius with respect to time? Just by looking at your graph (*he points to the graph. See Figure 3*).

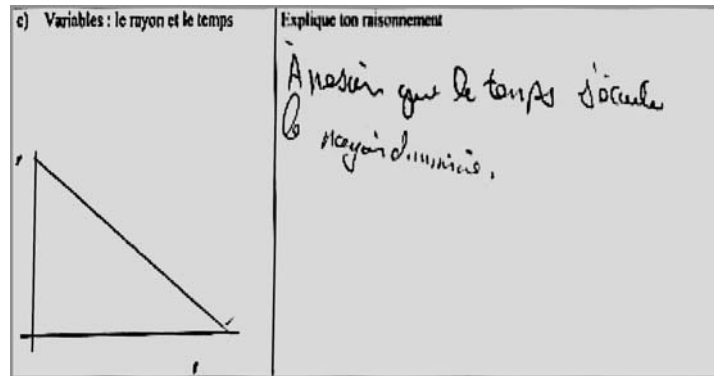


Figure 2. Students' graph of the relationship between the radius of the circle of the top surface of the water remaining in the container and the elapsed time. To the right, the students' explanation: "As time goes by, the radius diminishes."



Figure 3. The teacher prompting the students to reinterpret their graph (line 4 of the dialogue).

5. Diane: The radius diminishes as the time increases.
6. Sylvain: (*Completing Diane's remark*) As the time increases [...]
7. Teacher: Yeah, but (*emphasizing the words*) in which way does it diminish?
8. Diane: In a constant way!
9. Teacher: Is that what you believe?
10. Diane: I believe that it diminishes in a constant way.
11. Édouard: I think so.
12. Sylvain: Yeah! I don't see why (the radius) would diminish more quickly [...]. Why could it diminish more quickly? The opening (at the bottom of the cone) is always the same. The liquid always flows at the same speed.
13. Teacher: Yes, but there you're talking ... you're talking about the volume.
14. Diane: I think that it's in a constant way.
15. Édouard: Uh, humm?

16. Sylvain: Well!
17. Teacher: [...] the liquid that flows out is the volume.
18. Sylvain: Ah! That's true ...
19. Teacher: [...] You have to think in terms of the radius. There you're talking about the volume [...].
20. Édouard: Is it true that they (the radius and the time) change in an opposite way?
21. Diane: Yes, because if one of them increases the other diminishes.
22. Sylvain: But is it really in a constant way?
23. Édouard: Well, we don't know. I think that we ... (*interrupted by Sylvain*).
24. Sylvain: Like ... the volume (flows out) in a constant way... but the radius (*short period of reflection*) ... (*Talking to the teacher*) You confused us!
25. Teacher: Well... no! This is what you have to think about! It's exactly that ... [...]

In line 9, the teacher asks a crucial question. By using the verb “to believe”, he opens up a space for the students to revisit their interpretation. Although certainty is shaken (as witnessed in lines 10–12), the students still hold to their conclusion. In line 19, the teacher goes further and introduces a distinction between the volume of water flowing out from the cone and the radius of the water remaining in the cone. Line 24 expresses the students' awareness that the meaning of the graph has to be reconsidered. Since doubts have arisen, this is an opportune moment to repeat the experiment for the students.

26. Diane: Well... (*small period of reflection – they watch the teacher redo the experiment*) Well, that's not constant!
27. Teacher: So, every time that I have this (*he points to the cone*)... let's say ... at every regular (time) interval, the (radius of the) circle will diminish by a same amount. Is that what you predict will happen tomorrow?¹⁷
28. Diane: Yes!
29. Teacher: Yes?
30. Sylvain: No! [...]
31. Teacher: We could go further. Will the height (change) in a constant way?
32. Sylvain: Yeah, It will be the same thing as the radius [...]
33. Édouard: Well, the height at the end, it goes faster ... like ...
34. Teacher: Ah well, I haven't seen the graph showing that anywhere. You have all drawn straight lines. You have just used another concept and it doesn't go with your height-time there. There is something that

. . . you have to address there [. . .] Do you want to review [your graph] one more time?

35. Sylvain: (*Thinking aloud*) Because it's smaller at the bottom (of the cone), so even if it (the volume) flows out at the same speed . . . like . . . it (the height) descends more quickly.
36. Teacher: OK. I'll go see another group.

Since mathematical objects, like any other conceptual objects, are not particulars but generals, there is no direct way to reach them other than through semiotic actions. As Cassirer once remarked, "The object (of knowledge) is neither outside nor inside, neither on that side nor this; for our relationship to it is not ontic and real, but symbolic." (Cassirer, 1957, p. 318). And so, through words, artefacts, mathematical signs, and gestures – i.e. through semiotic means of objectification – the mathematical object '*relationship between the variables radius and time*' was made apparent to the students. In order to *see* it, the students underwent a process of objectification in the course of which their subjective meanings were refined. The students were right in noticing that, as the time increases, the radius decreases. But they still had to refine their understanding of the relationship. To do so, the teacher redid the experiment and interpreted it in terms of a constant rate of change (line 27). Looking at the experiment in this way, it became apparent that, in the beginning, the variation of the radius was lesser than it was at the end. The students then produced the graph shown in Figure 4.

The intended cultural conceptual object '*relationship between the variables radius and time*' has been formed in the course of a long historical process that goes back as far as Nicole Oresme's *De proportionibus proportionum* and other 14th century investigations on the motion of bodies. This cultural conceptual object has been endowed with a theoretical content prior to the students' mathematical experience. What the previous ordinary

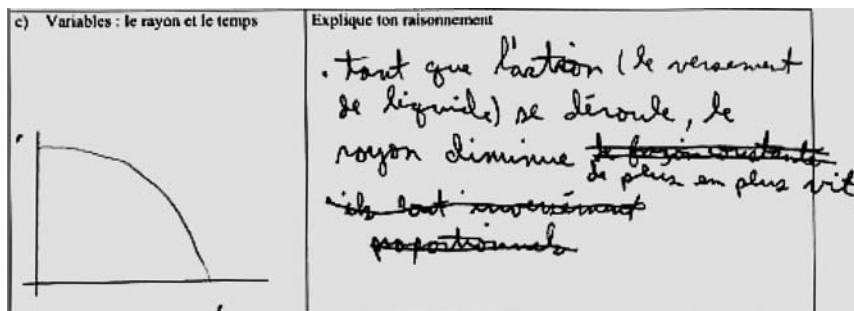


Figure 4. New graph drawn by the students to express the relationship between the radius and the elapsed time. The explanation now reads: "as the action (the pouring of the liquid) takes place, the radius diminishes more and more quickly."

classroom excerpt shows is the intermingled connection between the two sides of meaning: meaning as a subjective construct and as a cultural one. More specifically, what this ordinary classroom excerpt shows is how, encompassed by a social classroom process of objectification that unfolded along the route of meaning, the intended object of the students' mathematical experience sought to attain, with the decisive interventions of the teacher, the cultural conceptual object.

The previous classroom episode reminds us that the route towards culture does not have a random structure. The twists and turns of the route are carved, in this case, by the subjectivity of the students' intentions and the mathematical way in which the phenomenon has to be understood. This understanding entails the awareness that it does make sense to consider the water remaining in the cone as a genuine phenomenon and to busy oneself with questions such as the size of radius of the top surface of the water, as the water flows out of the cone. It also entails the capability of reflecting on this phenomenon through the conceptual categories of the culture (a cone, its radius, its height, the rate of change, a Cartesian graph, and so on).

Certainly, the students were actively engaged in what has been termed a "negotiation of meaning". But this term can be terribly misleading in that it may lead us to believe that the attainment of the concept is a mere consensual question of classroom interaction. As I mentioned previously, in addition to its social dimension, meaning also has a cultural-historical dimension which pulls the interaction up in a certain direction – more precisely, in the direction of the cultural conceptual object. It is in fact this cultural object that shapes and explains the teacher's intervention in the previous excerpt. Through the design of the lesson and the teacher's continuous interpretation of the students' learning, classroom interaction and the students' subjective meaning are pushed towards specific directions of conceptual development. Cultural conceptual objects are like lighthouses that orient navigators' sailing boats. They impress classroom interaction with a specific teleology.¹⁸

5. MATHEMATICAL OBJECTS

In the previous section, I suggested that cultural conceptual objects can be seen as lighthouses of classroom interaction. The question now is: are these cultural conceptual objects shadows of, and imperfect approximations to, transcendental, non-human made objects? I need now to try to provide a more precise explanation of the nature of these cultural conceptual objects without which the semiotic-cultural approach sketched here risks to fall into the respective traps of Platonism, Realism or Transcendental Idealism.

From the semiotic-cultural perspective advocated here, cultural conceptual objects are conceptualized in non-transcendental terms. As we saw, Husserl's noetic-noematic structure suffered from the dualistic immanence induced by the idealist transcendental stance. In a similar vein, in Peirce's approach, reality transcends the individual's semiotic activity. Knowledge of reality is obtained through processes of unlimited semiosis whose supposed convergence rests on the scholastic belief in an adequate correspondence between *ordo rerum* and *ordo idearum* and the optimist view of the scientific method.

To these idealist and realist views we here oppose the view according to which individuals are the producers of their conceptions and ideas. This view draws from Kant's theory of knowledge and the genetic epistemology developed by Piaget in that it conceives of conceptual objects as forms of interiorized reflected actions on the part of individuals. But, instead of endowing such actions with the teleology of logico-mathematical structures as Piaget did, the problem of knowledge in this semiotic-cultural approach is thematized as an interiorization of social practices, as reflected in consciousness by the cultural semiotic means of objectification. Hence, it is at this very point that the semiotic-cultural approach follows a different path from the various contemporary forms of Kant's constructivism. It instead adopts the view according to which the production of ideas and all cultural conceptual objects (mathematical objects included) are generated in activity – that is, in the dialectically interconnected sequence of mediated actions that individuals pursue in the attainment of a goal (see Leont'ev, 1978). In this context, mathematical objects are conceptual forms of historically, socially, and culturally embodied, reflective, mediated activity.¹⁹

The 'ideality' of mathematical objects – that is, that which makes them abstract or general – is consubstantial with, and is derived from human activities. As Ilyenkov wrote,

'Ideality' is rather like a stamp impressed on the substance of nature by social human life activity, a form of the functioning of the physical thing in the process of this activity. So all the things involved in the social process acquire a new 'form of existence' that is not included in their physical nature and differs from it completely – [this is] their ideal form. (Ilyenkov, 1977a, p. 86)

In more general terms, in the semiotic-cultural approach here proposed, knowledge is conceived of as the product of a *mediated cognitive reflexive praxis*.

Knowledge as cognitive praxis (*praxis cogitans*) emphasizes the fact that what we know and the way we come to know it is framed by ontological stances and by cultural meaning-making processes that shape a certain kind

of rationality out of which specific kinds of mathematical questions and problems are posed. The *reflexive* nature of knowledge is to be understood in Ilyenkov's sense, that is, as the distinctive component that makes cognition an intellectual reflection of the external world in the forms of the activity of the individuals (Ilyenkov, 1977b, p. 252). The mediated character of knowledge refers, naturally, to the role played by the semiotic means of objectification used in order to carry out the cognitive praxis.

Within this semiotic-cultural approach, an important distinction has to be made between learning and the production of new knowledge. While new cultural concepts arise from communal, reflective, mediated activities in the zone of proximal development of the culture, school learning is the process of actively and creatively transforming these cultural concepts embodied in texts, artefacts, language, and beliefs into objects of consciousness. This process, in which subject and object modify each other, is the process of meaning, the process where subjective knowing and objective knowledge merge.

We might ask ourselves whether or not the semiotic-cultural perspective here advocated entails a kind of relativism. If by relativism we mean the theory that asserts that 'anything goes', then a negative answer must follow. Knowledge is not the attainment of eternal essences but, as previously indicated, a sustained reflection of the world, a "'practical-critical', activity" where the question of truth "is not a question of theory but is a *practical* question." (Marx, in Marx and Engels, 1845/1968, p. 28).

Here, we abandon the idea of Truth in the essentialist metaphysical tradition, according to which Truth is that which remains once all that is ephemeral has been removed – an idea that goes back to Plato's aristocratic ontology (see Radford, 2004). We also abandon the idea of objectivity as an uncompromised access to transcendental entities. As Wartofsky noted,

The sheer externality of a state of affairs becomes 'objective' for us (. . .) only as it is mediated by our practice. What we can know is therefore always conditioned by the way that we come to know it. In a sense, our knowledge of the 'external' world is a knowledge of what this externality is amendable to, in our incursion upon it and intervention in it. (Wartofsky, 1979, p. 136)

Instead of the traditional category of objectivity, we suggest a less ambitious one – a contextual and cultural objectivity – where to know objectively means to insert yourself into a certain reflective praxis that allows you to see and to be seen, a praxis that provides you with a support or a position to engage yourself in the world, for "you can only know how the situation is if you are in a *position* to know." (Eagleton, 2003, p. 135).

From the semiotic-cultural perspective sketched here, the relationship between the observer and that which is observed is a culturally mediated

one: “The observer has no position *outside* the observed world, and his observation enters as a constituent part into the observed object.” (Bakhtin, 1986, p. 126). This is why, one way or the other, all objectivity is always subjective. Thus, instead of conveying the idea of a point in space that is nowhere from where you supposedly observe unobservable transcendental entities, a contextual objectivity means adopting a critical and reflexive position always open to change and discussion.

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NOTES

1. It was Kant, in the 18th century, who for the first time raised this problem in the clearest terms. He endeavoured to solve it through an aprioristic stance that Piaget attempted to correct by emphasizing the constructive role of the individual’s actions. Objects, Piaget suggested, become endowed with meaning as they are inserted into systems of schemata: “To assimilate a sensorial image or an object . . . is to insert it in a system of schemata, in other words, to give it a ‘meaning’” (Piaget, 1953, p. 189).
2. Some other papers dealing with mathematical objects are: Duval (1998), Godino and Batanero (1999), Sfard (2000), D’Amore (2001) and Otte (forthcoming).
3. To conform to tradition, mentions to Peirce’s work are given in the format ‘CP N.M’, where ‘CP’ refers to Peirce’s *Collected Papers* (details in the Bibliography), ‘N’ to the book number and ‘M’ to the number entry.
4. See Nesher (1997) and Parker (1994).
5. The fact that Piaget conceives of truth as human-made is part of his Constructivism. There is nothing new here. The objective nature of truth, in contrast, cannot be guaranteed by Constructivism alone. Like Peirce, for Piaget the objective nature of truth can only be guaranteed by the methods of science – supposedly the only methods capable of ensuring us of certainty. He espouses them as a way to avoid the “illusions of philosophy”, which, in his view, instead of *certainty*, can at most give us *wisdom*. In so doing, Piaget ends up adopting a kind of mild Realism (I discuss this point in Radford, 2002a), which leads his epistemology into similar problems as Peirce’s when faced with the question of the agreement between the subjective and the objective dimensions of knowing and knowledge.
6. The picture that comes out of Peirce’s work is that “in the development of knowledge the mind does not ‘go towards’ external reality, but it is rather *vice versa* . . . it is the mind that attempts a cognitive defence against the intrusion of external reality into its internal world, not nature that has to defend itself against the scientific aggression of the human mind.” (Floridi, 1994, p. 564).

7. As he said in a posthumous work, *The Origin of Geometry*, written in 1936: “The Pythagorean theorem, [indeed] all of geometry, exists only once, no matter how often or even in what language it may be expressed. It is identically the same in the ‘original language’ of Euclid and in all ‘translations’” (Husserl, 1989, p. 160). The objective nature of the ideal objects to which reference is made in semiotic activity is atemporal and it cannot arise from thinking. In 1900, in *Logical Investigations* (a book that sought to avoid the psychologism of Husserl’s early work) he said: “Truth, however, is ‘eternal’ [. . .] and so beyond time.” (Husserl, 1900/1970, p. 148). “If truth were essentially related to thinking intelligences, their mental functions and modes of change, it would arise and perish with them, with the species at least, if not with the individual.” (Husserl, 1900/1970, p. 151).
8. See (Husserl, 1913/1931, p. 271). The experiential nature of noesis is reflected in the fact that noetic acts are usually expressed in the gerund: symbolizing, knowing, seeing, etc. (the adjective noetic comes from the Greek *noetikos* and *noetos*, ‘perceiving’). Noema is usually expressed in a substantive or objectual language. It is about what is symbolized, known, seen, etc.
9. The “object simpliciter, namely, the identical element which is at one time perceived, a second time directly represented, and a third time exhibited in figured form in a picture, and so forth, indicates only *one* central concept.” (Husserl, 1913/1931, p. 266; emphasis as in the original).
10. “The essence of meaning is seen by us . . . in its ‘content’, the single, self-identical intentional unity set over against the dispersed multiplicity of actual and possible experiences of speakers and thinkers” (Husserl, 1900/1970, p. 327). For a detailed account of the identification of meaning and noema see McIntyre and Smith (1976) and Føllesdal (1969).
11. In his comments on Husserl’s *Ideas I*, Ricoeur noted that according to Husserl “we do not construct the essence but rather the consciousness of the essence.” (Ricoeur, 1996, p. 82).
12. As Husserl says in *Experience and Judgment* (1973) – a book made up from a number of manuscripts compiled by Landgrebe – “The whole layer of expression . . . is certainly inseparably linked to predicative operations” (Husserl, 1973, p. 199). Husserl’s leading question was to understand the basic judgement “S is p” and others connected to it.
13. Derrida in (Husserl, 1989, p. 76).
14. I am most grateful to Professor Sebastian Luft from the *Husserl Archives* in Leuven, for helping me in my search for Husserl’s letter.
15. To be as clear as possible about the difference between the social and the cultural dimensions of meaning and human experience in general, I want to suggest that the social refers essentially to invisible structures that organize the pattern of social relationships and interactions between people. The cultural, in contrast, refers to cultural attitudes, beliefs, and values, as they are embodied in texts, artefacts, language, paintings and so on. From an educational viewpoint, this difference amounts to saying that the cultural cannot be reduced to the social. Indeed, the classroom is not a closed, self-regulating system. Even if still not apparent for the students, the objects of discourse in a classroom appear framed, from the outset, by the historical-cultural values of scientific inquiry. These values are imported into the classroom through the curriculum, the teachers’ general knowledge, textbooks, tools, and so on (I shall come back to this point later).
16. The opening lecture of the 27th Conference of the International Group for the Psychology of Mathematics Education provided the participants with the extraordinary opportunity to attend the lecture of the celebrated Hawaiian navigator Nainoa Thomson. In coordinating a succession of significant cultural deeds involving ancestors, masters, and family, recounted in a first-person narrative, Thomson was using forms of

- representation of events, figures of speech and other elements of discourse production proper to the scientific Pacific tradition. Expression (to use one key Husserlian term) was achieved through a texturized discourse reflecting the navigator's knowledge of the sky, the winds and the sea. It would be vain to look for Thomson's lecture in the Proceedings. Thomson belongs to a different scientific tradition from ours – one privileging orality.
17. According to the three-day activity, it was planned that the next day the students would spend some time in the laboratory performing the experiment, measuring the variables and using a TI-83+® graphic calculator to obtain the graphs.
 18. I do not have space here to discuss the impact that the teacher's awareness of the historical-cultural dimension of the mathematical objects has on the formats of classroom interaction. See e.g. Furinghetti, 1997.
 19. In Radford (2003c) a concrete example of the social formation of the abstract object of "value" is discussed against the background of Renaissance culture.

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