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Introduction
The topos of meaning or the encounter between past and present

A concrete understanding of reality cannot be attained without a historical approach to it. Ilyenkov, 1982, p. 212.

Luis Radford · Fulvia Furinghetti · Victor Katz

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Can teachers and educators take advantage of the history of mathematics to enhance the students' understanding of mathematics? How? These and other related questions still remain without a definite answer. One of the reasons relates to the theoretical presuppositions adopted by contemporary theories of knowing and learning.

For educational theories adopting a pragmatic, empiricist or rationalist stance, knowledge formation is limited to actual experience – to the experience that we make of the world as we engage in it. Within this context, the epistemic import of the historicity of knowledge is something merely irrelevant. However, as the philosopher Immanuel Kant remarked in the eighteenth century, all attempts to derive our concepts from experience and to attribute to them a merely empirical origin are "entirely vain and useless." (Kant 1929, p. 139). Hans-Georg Gadamer, the father of Hermeneutics, remarked that the naïve empirical posture which leads one to believe that the reality in front of us can be grasped in a direct manner, fails to take into account the decisive *historical dimension* embedding all human experience. As he put the matter, "What we call experience and acquire through experience is a living historical process" (Gadamer 1989, p. 221). For Gadamer there is a fundamental continuity between past and present knowledge.

However, Gadamer's notorious insight may still leave the skeptical reader indifferent inasmuch as it does not tell us *how* the continuity between past and present affects and can be used to enhance the manner in which we acquire mathematical knowledge today. The theoretical elaboration and the pedagogical implications of the link between this past and

L. Radford (✉)
Ecole des Sciences de, Universite Laurentienne, l'Education, Sudbury, ON P3E 2C6, Canada
e-mail: lradford@laurentian.ca

F. Furinghetti
Dipartimento di Matematica, Università di Genova, via Dodecaneso 35, 16146 Genova, Italy

V. Katz
Department of Mathematics, University of the District of Columbia, 4200 Connecticut Ave. N.W., Washington, DC 20008, USA
e-mail: vkatz@udc.edu

present living historical process has indeed proven to be of great difficulty. To conciliate the historical with the subjective-individual dimensions of experience, Piaget elaborated an epistemology in which the individual is supposed to *reconstruct*, in its general aspects, the path followed by history. As we know, this is the old idea of recapitulation. Although of biological origin, the idea of recapitulation could only be imported into psychology thanks to the durable influence of the idea of the self forged by Kant and other philosophers of the Enlightenment. Indeed, within this grandiose cultural movement of the eighteenth century that sought to emancipate men and women from tradition and political power, the self was conceived of as a kind of quasi-hero – an autonomous, rational, auto-sufficient, culturally-detached being capable of reconstructing everything (Radford 2006). This Kantian idea of the self provided psychology in general and Piaget in particular with the perfect ground to import the biological concept of recapitulation into the study of the human psyche.

Piaget, of course, did not endorse a crude recapitulation theory (Furinghetti and Radford 2002). His description of human intelligence in terms of logical-mathematical structures made possible nonetheless an apparent conciliation between past and present. Recent developments in anthropology, epistemology, ethnomathematics, and cognitive sciences have, however, shown the limitations of casting the study of the human mind as Piaget did. The human mind, it turns out, is much more sensible to its cultural context. There is more to the mind than the logical structures Piaget was talking about. The way we think and come to know are subsumed in cultural practices that guide our contact and become consubstantial with the historically formed objects of knowledge.

How, then, can mathematics educators and teachers benefit from the history of mathematics? Hieronymus Georg Zeuthen answered this question more than 100 years ago by saying that a certain acquaintance with the history of mathematics would help us to get a better general sense of our discipline (Zeuthen 1902). However fruitful this humanistic stance may be, the history of mathematics has much more to offer to mathematics education. To make our point, let us refer to a basic statement in which A. N. Leontiev stressed the social-historical nature of knowledge. In an essay titled *Man and Culture*, Leontiev said,

> No one's personal experience, no matter how rich it might be, can result in thinking logically, abstractly and mathematically, and in individually establishing a system of ideas. To do this, one would need not just one lifetime, but thousands. (Leontiev 1968, p.18)

Translated into the classroom, what Leontiev's statement tells us is that the students' actions and deeds are necessary conditions for knowledge attainment, but they are not sufficient conditions. The very possibility of learning rests on our capability of immersing ourselves − in idiosyncratic, critical and reflective ways − in the conceptual historical riches deposited in, and continuously modified by, social practices. This is why the students' deeds and actions are just *one part* of the mathematical knowledge that is acquired in a classroom. Classroom emergent knowledge is rather something encompassed by the Gadamerian link between past and present. And it is precisely here, in the unraveling and understanding of this link, which is the *topos* or place of Meaning, that the history of mathematics has much to offer to mathematics education.

Indeed, the particular meanings that we form arise within the limits of our own finite experience − a limit that, as Bakhtin argued, can only be overcome by the encounter with foreign meanings. Bakhtin said:

> A meaning only reveals its depths once it has encountered and come into contact with another, foreign meaning: they engage in a kind of dialogue, which surmounts the closedness and one-sidedness of these particular meanings (Bakhtin 1986, p. 7)

History in general, and the history of mathematics in particular, is a reminder of the finiteness of human existence, of the limits of our individual cognitive capacities. At the same time, history erects itself as the place where we can surmount the one-sidedness of our particular meanings; it is a place to enter into a dialogue with others, and with the historical conceptual products produced by the cognitive activity of those who have preceded us in the always changing life of cultures. History provides us with a unique experience, one that completes the ephemeral moment in which we live and that reveals to us the depth of the conceptualizations that we share now with our contemporaries. History allows us to overcome Narcissus' position, one in which he could only see himself as someone reflected in the mirror of his own ego.

The aforementioned cultural-epistemic conception of the history of mathematics leads to a series of important theoretical and practical problems. As Bakhtin remarked,

> We raise new questions for a foreign culture, ones that it did not raise itself; we seek answers to our own questions in it; and the foreign culture responds to us by revealing to us its new aspects and new semantic depths. (Bakhtin 1986, p. 7)

This dialogue with past cultures through which we not only deepen our functional understanding of mathematics but contribute to our growing into whole human beings (Fried 2007), requires acquaintance with suitable forms of "dialogism." What questions to pose to history? How to seek the answers to our own questions? What kind of knowledge do we want the teachers to have in this respect? How to use the history in the design of a specific mathematics lesson?

Many of the papers of this Special Issue address these and other questions. The papers seek to deepen our understanding of the pedagogical role that the history of mathematics may play in contemporary mathematics education. Some of the papers provide examples of the use of the history of mathematics in school practice and in teacher education. Other papers address theoretical questions that have become crucial to understanding the profound intertwining of past and present conceptual developments from spreading new epistemologies and theories of learning.

In designing this Special Issue our intention was to bring together scholars with different expertise. At the bottom line lies the idea of the complexity of contemporary mathematical concepts, the historicity embedded in them, and the belief that epistemology and anthropology are two key pivots between history and mathematics education. The papers have been grouped accordingly into a few categories. The first set of texts – authored by Abraham Arcavi and Masami Isoda, Fulvia Furinghetti, Luis Radford and Luis Puig, Yannis Thomaidis and Constantinos Tzanakis − deals with some aspects of the links between the history and the teaching of mathematics. History is presented as a subtle artifact for teaching and for reflecting on teaching. The paper by Victor Katz and Bill Barton stresses the role of history in providing a background against which to design teaching sequences and to gauge the conceptual complexity of contemporary algebraic concepts.

The second set of papers starts with Michael Fried's article. Fried's main concern is related to the fact that the history of mathematics and mathematics education have their own goals and methodologies and that their link cannot be taken for granted. Fried's paper discusses some key elements that make possible a mathematics education in which the history of mathematics has an essential place. Evelyne Barbin's article sheds light on the decisive contribution of the history and epistemology of mathematics to the contemporary teaching of mathematics. She shows how the apparently transparent notion of "simplicity" rests on often unnoticed epistemological presuppositions.

The final two papers are by Michael Otte and Jens Høyrup. The first one deals with the transformation of mathematics during the nineteenth and twentieth centuries and shows how mathematical thinking of the time grew out of a dialectic between operative and functional or relational views about mathematical objects. Otte's paper shows us the conflicts and reluctance of many great mathematicians in the course of becoming aware of the transformation from objects as processes to objects as products and sheds light thereby on the historical complexity embedded in our contemporary mathematical concepts. The second paper is written by an eminent historian whose contribution to our contemporary understanding of Babylonian Mathematics is crucial. Working within a historiographical approach that relates mathematics to its cultural setting, Høyrup has shown in an impressive number of articles how Babylonian mathematical thinking was imbedded in its political and economical contexts. We are very proud to make Høyrup's work accessible to our community of mathematics educators.

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