

DIAGRAMMATIC THINKING: NOTES ON PEIRCE'S SEMIOTICS AND EPISTEMOLOGY

Luis Radford

In this paper, I discuss the role of diagrammatic thinking within the larger context of cognitive activity as framed by Peirce's semiotic theory of and its underpinning realistic ontology. After a short overview of Kant's scepticism in its historical context, I examine Peirce's attempt to rescue perception as a way to reconceptualize the Kantian "manifold of senses". I argue that Peirce's redemption of perception led him to a series of problems that are as fundamental as those that Kant encountered. I contend that the understanding of the difficulties of Peirce's epistemology allows us to better grasp the limits and possibilities of diagrammatic thinking.

Keywords: Culture; Diagrammatic thinking; Kant; Peirce; Perception; Semiotics

Pensamiento Diagramático: Notas sobre la Semiótica y la Epistemología de Peirce

En este artículo se discute el papel que desempeña el concepto de pensamiento diagramático en el contexto de la actividad cognitiva, tal y como es concebida dentro del marco de la teoría semiótica de Peirce y su subyacente ontología realista. Luego de presentar una visión general del escepticismo kantiano en su contexto histórico, se examina el esfuerzo de Peirce por rescatar la percepción, esfuerzo que lo lleva a indagar de manera innovadora el "multiespacio de los sentidos" del que hablaba Kant. Se mantiene que este esfuerzo lleva a Peirce a una serie de problemas que son tan fundamentales como los que Kant encontró en su propio itinerario epistemológico. Se sostiene que la comprensión de las dificultades intrínsecas a la epistemología de Peirce nos permite cernir mejor los límites y posibilidades de su pensamiento diagramático.

Términos clave: Cultura; Kant; Peirce; Percepción; Pensamiento diagramático; Semiótica

Commenting on Pearson's *Grammar of Science*, Peirce says: "Professor Pearson... tells us that each of us is like the operator at a central telephone office, shut out from the external world, of which he is informed only by sense-impressions." (Peirce, CP 8.144) Peirce found such a position untenable. However, this was neither because he believed that senses do not have a role to play in cognition, nor because he conceived of concepts as cut off from all empirical content. Like Kant, he endeavoured to elaborate an epistemology that, while running against empiricist and rationalist ones, combines elements of both of them. Thus, in a projected book on which he was working in 1861, but never finished, Peirce noted what he considered to be two of Kant's basic ideas: first, that all cognitions contain a sensual element, and second, that every cognition rests on inferences (Hoopes, 1991, p. 17). These two Kantian ideas clearly run throughout Peirce's work, and yet he took a different path from Kant. According to Peirce, Kant "drew too hard a line between the operations of observation and of ratiocination" (Peirce, CP 1.35). In an attempt to overcome Kant's scepticism, he restored to perception the epistemological role that it had lost since Plato. Not only did he theorize the relationship between the sensual and the conceptual differently, but also made perception the central tenet of the theoretical formulation of his three categories of being. Perception also became a central feature of Peirce's conception of cognitive activity. Indeed, in an important sense, in his account, diagrams appear as the semiotic artefacts of an epistemic subject engaged in the search for the truth that, with Kant, had become beyond human reach. Diagrams are at the heart of Peirce's remarkable endeavour in the restoration of truth—an endeavour that, nevertheless, remains incessantly haunted by Kant's ghost.

In this paper, I discuss the role of diagrammatic thinking within the larger context of cognitive activity as framed by Peirce's semiotic theory of and its underpinning realistic ontology. More specifically, after a short overview of Kant's scepticism in its historical context, I examine Peirce's attempt to rescue perception as a way to reconceptualize the Kantian "manifold of senses". Then, I argue that Peirce's redemption of perception led him to a series of problems that are as fundamental as those that Kant encountered. I contend that the understanding of the difficulties of Peirce's epistemology allows us to better grasp the limits and possibilities of diagrammatic thinking.

KANT'S SCEPTICISM

As Floridi reminds us, in Greek classical antiquity, as well as in medieval philosophy, the intrinsic nature of reality was considered to be intelligible. Thus, for Aristotle, there was a human desire that moved humans to know, and this knowledge "corresponded, on the ontological side, [to] the intrinsically knowable nature of the world itself" (Floridi, 1994, p. 547). In the scholastic tradition, the possibility of knowing external reality resided in knowing a world that God had

created without the intention to deceive us (p. 549). In short, until the end of the Middle Ages, there was an assumed correspondence between *ordo rerum* and *ordo idearum*.

The invention of the telescope in the sixteenth century, however, made men realize that the earth revolved around the sun and that neither everyday direct sense experience, nor reason could secure knowledge. The perplexed scepticism that arose from this fact led to a subjectivism that claimed that that which is knowable is that which is man-made. Arendt (1958) argues that the beginning of the modern era was marked by a shift from “what” to “how”, that is, from objects themselves to processes. In this context, Kant’s work is based on the “how-question”. Kant’s epistemic subject is a constructor and epitomizes the idea of man as *homo faber*. Indeed, in actively synthesizing intuitions and impressions, the Kantian epistemic subject becomes the producer of knowledge. This knowledge nevertheless carries its own limits. Kantian Doubt is in fact the doubt that the universe is not as intelligible as once supposed. In claiming that what we come to know is not Reality itself, but only what is given to us in sensations, Kant questioned the belief that we can ascend from the sensible to the non-sensible by abstraction. Kant insisted that the difference between the sensual and the conceptual is not just a matter of degree but a transcendental difference¹. Between the thing-in-itself and the phenomenon, Kant argued, there is a gap, and it is one that we cannot cross.

However, the Kantian epistemic subject was entangled with the conception of a world conceived of as constructed from a first-person perspective and, at the same time, with an unavoidable process of *reification*, that is, a process which brought forward the objective dimension of the world as something contrasting the subject. The Kantian epistemic subject was thus led to a view in which the world appeared as *subjectified* and yet required, for the sake of objectivity, the effacement of the self.

Adorno (2001) links the emergence of Kant’s philosophy and the phenomena of subjectivization and reification to the bourgeois element that asserts that the world of experience is the product of the individual’s labour. Taking the word “bourgeois” as a sociological category whose etymology means “those who live in the burgs” as opposed to “those who live working in the fields”², Adorno says that the

growth of subjectivism and reification expresses... the essential antinomy of bourgeois society in general... human beings have increasingly made the world in their own image, and the world has become progressively theirs. At the same time, however, the world has increasingly become a world that dominates them. (p. 115)

¹ For a detailed discussion see Allison (1973, p. 75).

² I am indebted to Yvan Morin for calling to my attention this remark.

To sum up, knowledge in Kant appears as the product of a synthesizing consciousness. Since, within this framework, it becomes impossible to achieve objectivity (in the sense of the rational program), truth becomes synonymous with coherence. Yet, Kant's work has to be placed within the grandiose Platonic paradigm that opposed the world of truth—that is, the world of unchanging things—to the world of senses and phenomena, that is, the world of delusion and deception. Following Descartes, Leibniz and other rationalists, Kant assumed a theory of truth according to which “truth is whatever remains once everything sensory, everything ephemeral and hence deceptive has been subtracted” (p. 25). Of course, Kant's work here reaches a tremendous tension. This tension arises from a programme that wants to restore to sensations their epistemological import, while at the same time remaining committed to the logical apparatus of the rationalists. It arises from a Reason that seeks to reach a truth that escapes its own possibilities.

Peirce's philosophy can better be understood against this sketch of Kantian background. Indeed, Peirce's attempt to rescue perception is in fact part of a larger effort aimed at rescuing truth from where Kant left it. To do so, Peirce gave up Kant's apriorism and proceeded to a reconceptualization of the Kantian “manifold of senses”. His account relies on a phenomenological gradation of our awareness that something is *there*, in our field of attention. But the observation that perception has intrinsic limits and that a predication about general events is beyond visible facts, led him to develop an epistemology deeply rooted in a theory of signs. In the next section, I briefly outline Peirce's phenomenological enterprise in light of my previous remarks. I will leave a discussion of Peirce's theory of cognitive activity and of diagrammatic thinking for the subsequent sections.

PEIRCE'S RESCUE OF PERCEPTION

In “On a New List of Categories”, Peirce says, in a Kantian tone, that “the function of conceptions is to reduce the manifold of sensuous impressions to unity” (Peirce, CP 1.545). But to this he also adds a gradation of knowledge. In so doing, he both draws from Kant and departs from him. As he says

This theory gives rise to a conception of gradation among those conceptions which are universal. For one such conception may unite the manifold of sense and yet another may be required to unite the conception and the manifold to which it is applied; and so on. (CP 1.546)

Like Kant, Peirce started his account of knowledge with *intuition*. Kant defined intuition as a singular presentation depending immediately on the presence of the object (Kant, 1783/1959, A19/B33) and registered by the faculty of sensibility—a faculty which is merely receptive, i.e. a faculty involving no thought. In a simi-

lar way, for Peirce, intuition “occurs in the immediate present, [and] it is not accompanied by thought” (Hoopes, 1991, p. 49).

Now, for Kant, intuitions are synthesized by the faculty of understanding and subsumed to the a priori concepts of Reason. For Peirce, things function differently. The path that intuitions travel in order to end up in cognitions is explained by him on the bases of a theory of perception.

We become aware of things because we are able to recognize their own *quale*. A quale is the distinctive mark of something, regardless of something else (it is its *suchness*). “Each quale is in itself what it is for itself, without reference to any other” (Peirce, CP 6.224). Thus, what allows us to perceive a red rose is the quality of redness. Were we to be left without qualia, we would not be able to perceive anything.

However, quale is not perception yet. It is its mere *possibility*: it is *firstness*—the first category of being in Peirce’s account. “The mode of being a redness, before anything in the universe was yet red, was nevertheless a positive qualitative possibility” (CP 1.25).

Qualia—such as bitter, tedious, hard, heartrending, noble (CP. 1.418)—account hence for the possibility of experience, making it possible to note that something is *there*, positioned, as it were, in the boundaries of consciousness.

As Rosenthal (2001) remarks,

... qualia, as the recognizable but ineffable element in experience, are the most abstract of the phenomena of experience and of the nature of firstness, for while pure abstraction can be reached by the agreement of things in some respect, Peirce denies the relativity of firstness to anything else by which it is grasped. Thus, in its phenomenological sense, a First is an immediately recognized quale.

To an important extent, the conditions of possibility of knowledge rest on firstness and their qualia. And since a quale is considered as something non-subjective, Peirce here paves the road for a rational conception of objectivity and truth that avoids Kantian apriorism. In Peirce’s phenomenological road to knowledge, objectivity is thus far ensured by the unanalyzed encounter of subject and object, an encounter where consciousness remains with no change, no comparison (Peirce, CP 1.306). The object gives itself to the subject’s consciousness as it is, in its immaculate suchness.

Now, the very eruption of the object into our field of perception marks the indexical moment of consciousness. It is a moment of actuality or occurrence. Here, we enter *secondness*. Peirce (CP. 1. 358) says:

We find secondness in occurrence, because an occurrence is something whose existence consists in our knocking up against it. A hard fact is of the same sort; that is to say, it is something which is there, and which I cannot think away, but am forced to acknowledge as an object or second

beside myself, the subject or number one, and which forms material for the exercise of my will.

Because we have reached awareness, the object now becomes an object of knowledge. But, knowledge is not an array of isolated facts or events. Rather, it results from a linkage between facts, and this link, Peirce argues, requires us to enter into a level that goes beyond quality (firstness) and factuality (secondness). This new level (*thirdness*) requires the use of symbols:

I see a man on Monday. On Tuesday I see a man, and I exclaim, "Why, that is the very man I saw on Monday." We may say, with sufficient accuracy, that I directly experienced the identity. On Wednesday I see a man and I say, "That is the same man I saw on Tuesday, and consequently is the same I saw on Monday." There is a recognition of triadic identity; but it is only brought about as a conclusion from two premises, which is itself a triadic relation. If I see two men at once, I cannot by any such direct experience identify both of them with a man I saw before. I can only identify them if I regard them, not as the very same, but as two different manifestations of the same man. But the idea of manifestation is the idea of a sign. Now a sign is something, A, which denotes some fact or object, B, to some interpretant thought. (CP 1.346)

In other words, because it is humanly impossible to *perceive* the events of Monday, Tuesday, and Wednesday *at the same time*, the identification of the three sightings of the same man can only be made possible if we go beyond the facts themselves. To do so, we need to see the facts, not as facts, but as representations, that is, as *signs*.

The semiotic nature of thirdness allows us to gain an understanding of facts and to introduce new ideas (CP 1.537). Thus, "the highest kind of synthesis is what the mind is compelled to make... in the interest of intelligibility... and this it does by introducing an idea not contained in the data, which gives connections which they would not otherwise have had" (CP 1.383).

In more general terms, the reason for Peirce having recourse to an epistemology deeply rooted in a theory of signs is to overcome the limits of perception: in order to be able to recognize, to assert and to know something about a certain state of affairs, we need to uncover connections among data and this we do by entering the world of representations. Without signs, Peirce argued, we cannot have generality: we cannot even have thought, for "A thought... is not a quality. No more is it a fact. For a thought is general." (CP 1.420) More precisely, "every thought is a sign" (CP 1.538, 2.253, 5.314, 5.470).

Yet, we still need a kind of adequacy between the represented state of affairs and the signs representing it. This adequacy is provided, in Peirce's account, by an ontological stance that he conveniently marries to a theory of cognitive activity.

The next section clarifies this matter and prepares the terrain for the discussion of diagrammatic thinking.

PEIRCE'S THEORY OF COGNITIVE ACTIVITY

Peirce upheld an ontology according to which the course of events of nature had a tendency to obey laws (CP 1.409). These laws, he claimed, are not human conventions: "Nature herself often supplies the place of the intention of a rational agent in making a thirdness genuine and not merely accidental... But how does nature do this? By virtue of an intelligible law according to which she acts." (CP 1.366)

As the quoted passage shows, in addition to asserting that the rules of nature are not conventions, Peirce also claimed that Nature is *intelligible*. This point is mentioned over and over again. Thus: "every fact of a general or orderly nature calls for an explanation; and logic forbids us to assume in regard to any given fact of that sort that it is of its own nature absolutely inexplicable." (CP 1.405) We can accordingly be assured that in our intellectual inquiries (if they are correctly conducted), we are not running after mere appearances. In fact, in CP 1.366 Peirce is asserting more: He is saying that Nature's intentions can genuinely be read through signs. "Had there been any process intervening between the causal act and the effect" he said "this would have been a medial, or third, element", that is, a thirdness (CP 1.328), for "so far as the idea of any law or reason comes in, thirdness comes in" (CP 8.330)³.

The assumption that Nature is semiotically describable has a repercussion in the pragmatic side of human conduct: our knowledge of the laws of nature can guide our actions in the world. Thus, in answering the question "What is thinking?" Peirce says: "Pragmatism makes thinking to consist in the living inferential metaboly of symbols whose purport lies in conditional general resolutions to act." (CP 5.402, n.3)

As we can see, within Peirce's pragmatism, the teleology of the individual's actions is seen against the background of a postulated nature governed by laws.

In Peirce's account, this claim makes sense within a conception of cognitive activity that finds an unconditional ally in nature: There is an unbreakable agreement between thinking and being that manifests itself in a nature that collaborates with us, a nature that will not play tricks on us, and that has somehow become inserted into our semiotic processes of thinking. Nature's signs and hu-

³ The idea reappears, for example, in CP 1.536 and CP 8.268. In CP 8.268, for instance, Peirce says: "The third element of the phenomenon is that we perceive it to be intelligible, that is, to be subject to law, or capable of being represented by a general sign or Symbol." Of course, the idea that Nature can be read in the book of signs is part of Galileo's influence on Peirce (see Eisler, 1979).

man thinking go hand in hand. So Peirce could confidently say that “The sole immediate purpose of thinking is to render things intelligible.” (CP 1.405)

The aforementioned ontological point frames an interesting concept of cognitive activity that, nevertheless, relates thinking and being in a way that soon runs into difficulties. Had Kant had the opportunity to read Peirce’s account of knowledge, he most probably would have asked whether or not the objects of the successive sign-thoughts of cognitive activity were the exact objects of reality, and if so, how we could know it. Peirce, who used to say that he spent three years reading Kant’s *Critique of Pure Reason* (De Tienne, 1993, p. 640), once said, “The present writer was a pure Kantist until he was forced by successive steps into Pragmaticism” (CP 5.452). He refuted Kant’s refutation of idealism and ended up adopting what is called Peirce’s “extreme scholastic realism” (Parker, 1994, p. 67).

In Peirce’s scholastic realism, the problem is solved by having recourse to the regulative idea of *truth* as the end of semiosis (Parker, 1994; Nesher, 1997). Briefly stated, this idea claims that if we were to conduct the process of reasoning unlimitedly, discarding the wrong hypotheses as we detect them in the course of our actions, reality would become disclosed. Peirce said:

The cognitions which thus reach us by this infinite series of inductions and hypotheses (which though infinite a parte ante logice, is yet as one continuous process not without a beginning in time) are of two kinds, the true and the untrue, or cognitions whose objects are real and those whose objects are unreal. (CP 5.311)

In another passage (CP 8.12), Peirce says:

But human opinion universally tends in the long run to a definite form, which is the truth. Let any human being have enough information and exert enough thought upon any question, and the result will be that he will arrive at a certain definite conclusion, which is the same that any other mind will reach under sufficiently favorable circumstances.

Reality for Peirce becomes entangled with *truth*, and *truth* is what we get in the end of the process of unlimited semiosis. In a comment on Kant’s *Transcendental Aesthetics* he said: “Apply induction not to any limited experience but to all human experience and you have the Kantian philosophy, so far as it is correctly developed” (CP 5.223).

However, a problem arises precisely at this point. Let us concede to Peirce that a multitude of individuals can carry out experiments indefinitely. Can we be certain that the limit obtained through the cognitive process coincides with the real object?

There is a letter written on March 22, 1801 by Heinrich von Kleist, one of Kant’s contemporaries. Commenting on Kantian philosophy, von Kleist (quoted in Adorno, 2001, p. 252) says:

If men had green glasses instead of eyes, they would have to believe that the objects they saw were green —and they would never be able to decide whether their eyes showed them things as they were. Or whether they did not add something to them that belonged not to them, but to the eye. The same is true of the mind. We cannot decide whether what we call truth is truly true, or whether it only appears so to us.

It may very well be the case that, after exhausting the endless cognitive march, our crowd of individuals, who have been wearing green glasses without knowing it, have attained an object *different* from the “real” one. Granted that the cognitive process is of the order of representation, it may even be the case that the found object does not have a relation with the “true” aspects of the universe.

Of course, the problem of the adequacy between knowledge and reality is not specific to Peirce’s account. In his genetic epistemology, Piaget had to face it and, in order to explain it, had recourse to the biological roots of the individual’s actions (see Radford, 2002a). In Peirce, however, it takes a singular shape as a result of his perceptual, representational theory. While for Piaget the problem was to explain how the subjective process of abstractive actions can lead to the objective knowledge of conceptual objects, for Peirce the problem was to explain how a semiotic activity that identifies signs and thoughts can nevertheless unveil the real nature of objects whose objective existence does not depend upon our system of signs and our interpretations.

For Piaget, truth is conceived of as simultaneously human-made *and* objective, i.e. human-independent. Peirce agrees with Piaget that truth is human-independent, but claims that it is not human-made. In fact, Peirce adds that even if truth is not human-made, it is humanly reachable—to the extent that he found it uninteresting that the angel Gabriel would descend from Heaven to reveal to us the answer to the riddle of reality. For Peirce, we do not need such angelical gestures: the riddle of reality could be solved by finding good tools (Peirce, CP 5.553):

if we can find out the right method of thinking and can follow it out—the right method of transforming signs— then truth can be nothing more nor less than the last result to which the following out of this method would ultimately carry us.

The success of the pragmatic enterprise lies on the *entelechy* of semiosis and the methods of science.

DIAGRAMMATIC THINKING

The methods of science nevertheless require a great deal of creativity. Inferences, for instance, involve the elaboration of hypotheses and abductions. “An Abduc-

tion is Original in respect to being the only kind of argument which starts a new idea.” (CP 2.96) In *Speculative Grammar*, Peirce wrote that

An Abduction is a method of forming a general prediction without any positive assurance that it will succeed either in the special case or usually, its justification being that it is the only possible hope of regulating our future conduct rationally, and that Induction from past experience gives us strong encouragement to hope that it will be successful in the future. (CP 2.270)

His concept of abduction is closely related to the concept of diagrams, which he derived from Kant’s concept of construction (CP 3.560):

Kant is entirely right in saying that, in drawing those consequences, the mathematician uses what, in geometry, is called a “construction”, or in general a diagram, or visual array of characters or lines. Such a construction is formed according to a precept furnished by the hypotheses. Being formed, the construction is submitted to the scrutiny of observation, and new relations are discovered among its parts, not stated in the precept by which it was formed, and are found, by a little experimentation, to be such that they will always be present in such a construction.

Diagrammatic thinking is a central piece in Peirce’s endeavour to rescue the epistemological import of perception. It is strongly linked to a heuristic process that exhibits, via intuition (i.e., in a sensual manner), some aspects of the object under scrutiny, thereby making these aspects available for observation, in order to help us discover new conceptual relations. The epistemological potential of diagrammatic thinking rests then in making apparent some relations that have thus far remained hidden from perception or beyond the realm of our attention. This is why, etymologically speaking, diagrammatic thinking relates to actions of *objectification* and a diagram, considered as a semiotic artefact, is a *semiotic means of objectification* (Radford, 2002b, 2003a).

We should note that, in light of the epistemological and ontological options framing Peirce’s theory of cognitive activity, the “new relations to be discovered” as well as the way “that they will always be present in such a construction” (see Peirce, CP 3.560, quoted above) acquire a particular meaning.

Indeed, on the one hand, for Peirce’s realism, the relations to be discovered are conceived of as *preceding* the semiotic experience. This is why Peirce’s realism cannot be aligned with a constructivist view of knowledge.

On the other hand, the idea that the new relations thus discovered “will *always* be present in such a construction” supposes: (a) the ontological invariability of a state of affairs to which the diagram refers, and (b) our capability of grasping it. The first point is well tuned to Peirce’s realism and its insertion in the more general ontological framework that locates being beyond human will. The second point is an attempt to transform Kantian Synthetic apriorism into relations born

by objects now open to perceptual inspection, relations that can be subjected to the scrutiny of scientific experimentation. As Stjernfelt notes, with this move, Peirce effects a pragmatization of Kant's apriorism and attempts "to escape the threatening subjectivism", concerning the ontological status of the relations perceived in the object, since now it is implied that "the rational relations are inherent in the universe and are not our inventions..." (Stjernfelt, 2000, p. 365)

Thus, to the Kantian epistemic subject who, in his various processes, can create everything except being, Peirce opposes an epistemic subject who cannot produce being either, but who, armed with the powers of perception and the methods of science, can attain the truth that scepticism placed beyond human reach. But again, Kant would have said that the diagram and the experimentations that we perform upon it are our own, and as such, diagrammatic thinking unveils for us, at most, nothing but the ontology of appearances. How—he might have said—on account of the "transcendental gap", can we ensure the validity of the generalizing act underling diagrammatic thinking?

MAN AS DIAGRAM

For Peirce, semiotic activity "is the mirror of being" (Peirce, CP 1.487); "there is in the being of things something which corresponds to the process of reasoning" (NEM 4:343-44, quoted in Parker, p. 60), so that the communion between the living universe and human thinking provides us with much more than a mere casual confidence. Since Reality is regularity (Peirce, CP 1.175), what the adverb "always" (CP 3.560, quoted above) conveys in the generalizing act is precisely *this* regularity. In their quiet and immobile lives, the qualia (perturbed perhaps only by the slow progress of the laws of evolution) pick up our very first impressions anyway, and put them on the right track. Of course, the actions of the demon of deception that threaten all the epistemologies based on a theory of perception cannot be discarded here, but—Peirce might have argued—a scientific attitude is well fitted to help us to correct whatever has to be corrected.

In fact, how can the generality that the diagram exposes be accomplished, if we give up regularity? I think that the problem is not regularity itself. Rather, one must become aware of what makes regularity possible. It is one thing to believe that this regularity is an intrinsic part of reality, and quite another to believe that we endow reality with regularity.

To illustrate this point, I would now like to discuss the case of generalization as it appears in the geometric proofs of classical Greek mathematics. It is well known that Greek mathematics had recourse to diagrams. Now, recent historical analysis has ruled out the once accepted idea that the diagram was the representation or the surrogate of an ideal mathematical object⁴. In actual fact, it turns out that the object of the proof is literally *the particular diagram* mentioned in the

⁴ See Netz (1999, p. 51).

proof. The question that I wish to address is: How then was Greek diagrammatic generality ensured?

The typical organization of a Greek mathematical proof starts with the statement of what is going to be proven. This is a general conditional statement called *protasis* that, following Netz (1999, p. 252), we can abbreviate as $C(x) \rightarrow P(x)$.

Next a *particular* geometric object “ a ” is considered (e.g., Let ABC a triangle...), which allows one to express the premise C as $C(a)$. The predication about the particular object “ a ” is called *ekthesis*, for example, “the (particular, eventually drawn) triangle ABC verifies such and such”. Following this, the mathematician routinely affirms “I say that...” and he asserts the conclusion P of the *protasis* as applied to the object “ a ”, that is, $P(a)$, which is called the *diorismos* (e.g., “the bisectors of the angles CAB , and ABC satisfy this and this”). So far, then, the structure is:

1. $C(x) \rightarrow P(x)$ (protasis)
2. $C(a)$ (*ekthesis*)
3. I say that $P(a)$ (*diorismos*)

This chain is followed by a sequence of constructions $C(b), \dots, C(n)$, $P(b), \dots, P(a)$, eventually accompanied by a drawing.

The proof ends with the *diorismos* $P(a)$ and the mention once more of the *protasis*, $C(x) \rightarrow P(x)$, preceded this time by the expression “therefore”. In other terms, the previous chain is followed by:

4. $C(b), \dots, C(n)$, $P(b), \dots, P(a)$
5. Therefore $C(x) \rightarrow P(x)$.

As Netz (1999) remarks, the sequence $C(b), \dots, C(n)$, $P(b), \dots, P(a)$ is *not* a proof of $P(x)$. This sequence is a proof that $P(a)$ results from $C(a)$. What then is that which ensures the validity of the general proposition $C(x) \rightarrow P(x)$? Netz says (p. 256):

The fact that the diorismos is provable on the assumption of the ekthesis—the fact that $P(a)$ may be necessarily inferred from $C(a)$ —is the proof for the general result, that $C(x) \rightarrow P(x)$. The crucial thing is that, assuming the ekthesis and nothing else, the diorismos is thereby necessarily true. The necessary nexus between $C(a)$ and $P(a)$ forms the ground for $C(x) \rightarrow P(x)$.

The deepest sense of this nexus rests on the mathematician’s conceptualization of the object a *not as a particular as such, but as a general object*⁵.

⁵ I differ with Netz on this point. He suggests that the nexus ensuring the passage to the *protasis* rests on the *possibility* of going through the same sequence again and again, starting from another singular object a' , then a'' , then a''' , and so on, so that, for Netz, in its deepest sense, *gen-*

From a Peircean perspective, the drawing accompanying the proof and the whole sequence of formulas are diagrams. From a Kantian perspective, both of them are schemata. Nevertheless, because in both perspectives the diagram/schema and the regularity shown by a mathematical proof are conceived of differently, the passage from the particular to the general is justified in a different manner. This is one of the important points here. Now, in so far as the reality of mathematical relations (Peirce) or the transcendental ontological status of mathematical objects (Kant) precedes all semiotic experience, Kant and Peirce coincide in leaving the actions of the subject out of the ontological realm. This is the second important point.

Yet, as Netz's historical reconstruction intimates, there is another possible way to conceptualize the diagram: the latter can be conceived of as a kind of *cascading schema*. In fact, as we saw, the semiotic activity unfolds as a sequence of formulas embedding a drawing, which in turn embeds formulas, which in turn embed other general objects (a', a'', \dots) thereby giving rise to other formulas that may not be written, but that are intimated by the subject's actions. We may then see this cascading diagram as produced in an ampler semiotic text that henceforth includes not only the different aforementioned steps and the perceptual drawing accompanying the proof, but also the actions of man as an epistemic acting subject whose actions *ensure* the passage from $C(a) \rightarrow P(a)$ to $C(x) \rightarrow P(x)$. Within this context, the diagram can be seen as a hyper-cascade diagram written in a multi-dimensional semiotic text that also includes man as a diagram.

ZERONESS: THE CATEGORY THAT PEIRCE FORGOT

It should be noted that the epistemic acting subject to which I am referring is neither the reified subject of Kant's philosophy nor the logicized individual of Peirce's epistemology. By saying that man is himself a diagram embedded in a multi-dimensional semiotic text, I want to convey the idea that an explanation of human knowledge has to take into account something that enters Peirce's philosophy only at too late a stage, and that does not enter the Kantian epistemology of the *Critique of Pure Reason* at all. Namely, I want to stress the contextual and cultural character of knowledge and knower. In what follows, I will explain this point further.

In Peirce's account, culture enters the edifice of knowledge in thirdness (see e.g. Thellefsen, n/d). The gate of knowledge is firstness. Indeed, firstness is the foundation of cognitive activity (Eco, 1999) and hence the route to knowledge; it

eralization, for the Greek mathematicians, signifies *repeatability* (Netz, 1999, p. 246). The problem with this line of thought is that the object "a" predicated in the *ekthesis* (say a triangle ABC) is not a particular, but something considered as a general.

is the place where, we are told, “The idea of the absolutely first must be entirely separated from all conception of or reference to anything else.... It must be initiative, original, spontaneous, and free.” (CP 1.357) The First is, in short, the place where intuitions and sensations are thematized within an epistemology where the subject is seen as being continuously hit by a torrent of objects — making Peirce’s epistemology an “epistemology of the clash” that bears clearly precipitated sediments and old vestiges of empiricism, as rationalized by Kant.

However, the object that hits us in the realm of firstness, the object that in hitting us makes possible (in Peirce epistemology) the “secretion of new knowledge” (Floridi, 1994), already comes into existence with an orientation toward a cultural system of ideas that we have absorbed earlier. It is not a meteorite falling out of the blue. Rather, it is a historically constituted object, keeping in itself an embodied intelligence (Pea, 1993) and carrying in itself, in a compressed way, socio-historical experiences of cognitive activity and scientific standards of investigation (Lektorsky, 1984).

In actual fact, secondness (the realm of what exists, the realm of indexicality), and firstness (i.e. the realm of suchness, the realm of “potential being”, of what “may-be” or of uninterpreted iconicity) are already cultural through and through. Cognitive activity does not become cultural only when thirdness mediates between firstness and secondness. Rosenthal (2001) is right when she says that, in addition to being a mere repository of sensory element of experience, firstness is also (and I would say overall) “quality determining a class”. In this second aspect, as a class and quality of possible experience, qualia should neither be understood in a Platonic sense, nor solely as something subject to the laws of natural evolution⁶. The conditions of possibility of firstness should to be understood as related to, and framed by, the concrete world of individuals. “Whiteness”, for instance, as quale, presupposes not only a sensorial capacity, but also a linguistic activity (Hjelmslev, 1969) subsumed in social practices (Radford, 2003b, 2008). In this context, we might need to pay more careful attention to the signifying social practices that encompass the processes of semiosis, interpretations and understanding referred to by Peirce.

As mentioned previously, culture does enter into account in Peirce’s epistemology. It appears in the third category of being, but it does so as a mere human convention —the convention which makes symbols different from indexes and icons. However, to reduce culture to convention is to adhere to a *nominalist theory of culture*. From an epistemological viewpoint, culture is certainly much

⁶ Perhaps because Peirce’s first two categories of being are considered as *given*, as antihistorical and a-cultural, some readers of Peirce have been led to detect a certain kind of Platonism in Peircean ontology. But this is not the case, at least in so far as he subjected the Platonic forms to the laws of evolution: “The evolutionary process is, therefore, not a mere evolution of the existing universe, but rather a process by which the very Platonic forms themselves have become or are becoming developed.” (CP 6.194) For a more detailed discussion see Hoffmann (2002).

more than conventions. It is consubstantial to knowledge through and through. The phaneron, that is, “the collective total of all that is in any way or in any sense present to the mind” (CP 1.284), is ubiquitously cultural. So, when Peirce tells us that “It is therefore not surprising to find that beyond the three elements of firstness, secondness, and thirdness, there is nothing else to be found in the phenomenon.” (CP 1.347), I believe that he forgot Zeroness —that is, culture as a category of being made by concrete human praxis.

CONCLUDING REMARKS

The understanding of a sign is, after all, an act of reference between the sign apprehended and other, already known signs; in other words, understanding is a response to a sign with signs. And this chain of... understanding, moving from sign to sign and then to a new sign, is perfectly consistent and continuous: from one link of a semiotic nature... we proceed uninterruptedly to another link of exactly the same nature. And nowhere is there a break in the chain...

These words, couched in the idea of semiosis as a continuous flow of signs, could have been Peirce’s. In fact, they are Voloshinov’s (1884/5-1936) —a collaborator of Bakhtin (see Voloshinov, 1973, p. 11). There is nevertheless an important theoretical difference that may help us to make sense of the previous remarks —it is a difference concerning the role that other persons play in the experience that we make of the world. In the confines of firstness, Peirce’s epistemic subject is as lonely and solitary as was Adam when he first woke up in paradise (e.g., Peirce, CP 1.357, 4.77). And, although Peirce talks about a community of thinkers, and claims that reality is inseparable from such a community, his account of knowledge is, like Kant’s, definitely an account of knowledge carried out in the first-person perspective⁷. Peirce’s communism rests on the view that the community of thinkers is in fact defined by the commonality of logic. Peirce says: “So the social principle is rooted intrinsically in logic.” (CP 5.354) For Voloshinov (1973), the social principle is rooted intrinsically in the interaction between individuals and their understanding of signs as conveyers of systems of ideas resulting from the individual’s historically and culturally situated social activities (p. 22):

The sign is a creation between individuals, a creation within a social milieu. Therefore the item in question [the item to which a sign will refer] must first acquire interindividual significance, and only then can it become an object for sign formation.

As a result, consciousness —a key Peircean epistemological concept— becomes, in Voloshinov account, that which “takes shape and being in the material of signs

⁷ For a similar critique see Colapietro, Midtgarden and Strand (2005).

created by an organized group in the process of its social intercourse” (p. 13). Thus, while talking about language, but it is true of all indexes and icons, Voloshinov says: “What is important about the word in this regard is not so much its sign purity as its *social ubiquity*.” (p. 19)

Within this context, diagrammatic thinking may be understood in a larger context than the one-person perspective epistemology (where it appears as a self-controlled management device of one’s own thoughts). Diagrammatic thinking may be understood as unfolding in a semiotic text embedded in the historicity and culturality in which it is produced. Its historicity and culturality account of course for the kind of sign systems that are available in a certain historical period (Hoffmann, 2002). But they also account for the cultural forms of generalization at the heart of abductive processes, as I attempted to show in the example of the mathematical proofs of the Euclidean tradition.

The relevance of diagrammatic thinking resides not in seeing it as a means of experimentation in the search for a truth that, in its objectivity, requires the extinction of the self (as in Kant), or the asymptotic displacement of a logicized individual (as in Peirce). Diagrammatic thinking is relevant in as much as it is seen as a mode of expression of human subjectivity that includes, on the epistemic side, the subject that is producing it in his or her endeavour to understand human reality—a reality, let me insist once more, that is not posited as the metaphysical end of an endless semiosis. It is a reality that individuals socially construct as they act and interact, as they “experiment” (to use Peirce’s term) in accordance with dynamic cultural semiotic systems of signification (Radford, 2003c) which frame scientific, aesthetic, artistic and poetic creation.

REFERENCES

- Adorno, T. W. (2001). *Kant’s critique of pure reason*. Stanford: Stanford University Press.
- Allison, H. (1973). *The Kant-Eberhard controversy*. Baltimore: Johns Hopkins University Press.
- Arendt, H. (1958). *The human condition*. Chicago: The University of Chicago Press.
- Colapietro, V., Midtgarden, T., & Strand, T. (2005). Introduction: Peirce and education: The conflicting processes of learning and discovery. *Studies in Philosophy and Education*, 24(3-4), 167-177.
- De Tienne, A. (1993). [Hypotheses of space and time: A response to Kant] Charles S. Peirce, Appendix No. 2. *Transactions of the Charles S. Peirce Society*, 29(4), 637-673.
- Eco, U. (1999). *Kant and the Platypus. Essays on language and cognition*. San Diego: Harcourt, Inc.

- Eisle, C. (1979). *Studies in the scientific and mathematical philosophy of Charles S. Peirce* (Edited by R. M. Martin). The Hague: Mouton.
- Floridi, L. (1994). Scepticism and the search for knowledge: a Peirceish answer to a Kantian doubt. *Transactions of the Charles S. Peirce Society*, 30(3), 543-573.
- Hjelmlev, L. (1969). *Prolegomena to a theory of language*. Wisconsin: The University of Wisconsin Press.
- Hoffmann, M. H. G. (2002). Peirce's "Diagrammatic Reasoning" as a solution of the learning paradox. In G. Debrock (Ed.), *The quiet revolution: Essays on process pragmatism* (pp. 147-174). Amsterdam: Rodopi Press.
- Hoopes, J. (Ed.). (1991). *Peirce on signs*. Chapel Hill and London: The University of North Carolina Press.
- Kant, I. (1929). *Critique of pure reason*. (Norman K. Smith, Trans.). New York: St. Marin's Press. (Original work published 1781 and 1787)
- Kant, I. (1959). *Prolegomena to any future metaphysics that will be able to present itself as a science*. (Peter G. Lucas, Trans.). Manchester: Manchester University Press. (Original work published 1783)
- Lektorsky, V. A. (1984). *Subject, object, cognition*. Moscow: Progress Publisher.
- Nesher, D. (1997). Peircean realism: Truth as the meaning of cognitive signs representing external reality. *Transactions of the Charles S. Peirce Society*, 33(1), 201-257.
- Netz, R. (1999). *The shaping of deduction in Greek mathematics*. Cambridge: Cambridge University Press.
- Parker, K. (1994). Peirce's semiotic and ontology. *Transactions of the Charles S. Peirce Society*, 30(1), 51-75.
- Pea, R. D. (1993). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), *Distributed cognitions* (pp. 47-87). Cambridge: Cambridge University Press.
- Peirce, C. S. (1931-1958). *Collected Papers* (Vols. 1-8). Cambridge: Harvard University Press.
- Peirce, C. S. (1976). *The New Elements of Mathematics (NEM)*. (C. Eisele, Ed.). The Hague: Mouton Publishers.
- Radford, L. (2002a). The object of representations: Between wisdom and certainty. In F. Hitt (Ed.), *Representations and mathematics visualization* (219-240). Mexico: Departamento de Matemática Educativa CINVESTAV-IPN.
- Radford, L. (2002b). The seen, the spoken and the written. A semiotic approach to the problem of objectification of mathematical knowledge. *For the Learning of Mathematics*, 22(2), 14-23.
- Radford, L. (2003a). Gestures, speech and the sprouting of signs. *Mathematical Thinking and Learning*, 5(1), 37-70.
- Radford, L. (2003b). On the epistemological limits of language. Mathematical knowledge and social practice during the Renaissance. *Educational Studies in Mathematics*, 52(2), 123-150.

- Radford, L. (2003c). On Culture and Mind. A post-Vygotskian semiotic perspective, with an example from Greek mathematical thought. In M. Anderson, A. Sáenz-Ludlow, S. Zellweger, & V. V. Cifarely (Eds.), *Educational perspectives on mathematics as semiosis: From thinking to interpreting to knowing* (pp. 49-79). Ottawa: Legas Publishing.
- Radford, L. (2008). Culture and cognition: Towards an anthropology of mathematical thinking. In L. English (Ed.), *Handbook of international research in mathematics education* (2nd Ed.). Mahwah: Erlbaum.
- Rosenthal, S. (2001). Firstness and the collapse of universals. In *Digital Encyclopedia of Charles S. Peirce*. Retrieved on June 2003, from <http://www.digitalpeirce.org/>
- Stjernfelt, F. (2000). Diagrams as centerpiece of a Peircean epistemology. *Transactions of the Charles S. Peirce Society*, 36(3), 357-384.
- Thellefsen, T. L. (n/d). Firstness and thirdness displacement - The epistemology of Peirce's three sign trichotomies. In *Digital Encyclopedia of Charles S. Peirce*. Retrieved on June 2003, from <http://www.digitalpeirce.org/>
- Voloshinov, V. N. (1973). *Marxism and the philosophy of language*. Cambridge: Harvard University Press.

Luis Radford
Université Laurentienne
lradford@laurentian.ca