18 Culture and cognition

Towards an anthropology of mathematical thinking[1]

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> Like food taboos, mathematics is not free of other, very significant aspects of culture.
>
> Owens, 2001, p. 157

INTRODUCTION

It seems that there is no cliché more popular in this day and age than affirming that cognition is related to culture. To explain how exactly the former relates to the latter remains, however, an open problem. Indeed, even if in the past few years there has been an increasing awareness concerning the role played by the social, political and cultural contexts in the way in which we think about the world, it is still unclear in which exact sense our concepts — mathematical as well as others — are informed by culture.

In all likelihood, the mitigated success of our current understanding of the link between culture and cognition finds its explanation in the longstanding rationalist epistemological tradition which adopted the logical characteristics of mathematics in order to build its paradigmatic models. Thus, in the first half of the 20th century, Lucien Lévy-Bruhl suggested in his book, *The Primitive Mind*, that the overcoming of the so-called pre-logical thinking that appeared to afflict the tribal individuals described by missionaries and travellers in their visits to "exotic" places, was only possible through the abandonment of cultural "collective representations" (Lévy-Bruhl, 1922). In his account, culture was seen as an obstacle to objective, logical thinking. Adherence to the same rationalist epistemological tradition — a tradition that goes back to Plato and which was substantially refined by Leibniz, Kant and other 17th- and 18th-century philosophers — led Piaget to conceive of thinking in terms of logical-mathematical structures. A few years after Piaget's genetic epistemology reached its summit and started a slow decline, cognitive sciences, inspired by the same epistemological tradition, followed a similar path and reduced the mind to a kind of mental computational device (Dupuy, 2002).

By and large, within the rationalist epistemological tradition, the role of culture was relegated to something that plays only an "external" role in mathematical knowledge formation (Lakatos, 1978). Armed with the rhetoric of an evolutionist discourse, culture was considered by the rationalist epistemological tradition as something that obstructs the right course of knowledge. True knowledge, in fact, was the reward for the individual's emancipation from his or her culture.

However, the traditional view of knowledge and the mind offered by the rationalist paradigm has been the target of contemporary criticism (Berman, 1990; Eagleton, 2003; Foucault, 1980; Geurts, 2002; Tyler, 1987).

On the one hand, there is an increasing dissatisfaction with the idea of cognition as a series of disembodied and unhistorical logical calculations. A few years ago, Francisco Varela summarized this matter in a clear way. As he noted, "A purely procedural account of cognition, independent of its embodiments and history, is [now] seriously questioned" (Varela, 1992, p. 250).

On the other hand, there is a growing interest in understanding the role of tools and artifacts in cognition (see, e.g., Bartolini Bussi & Mariotti's chapter 28, this volume; see also Bartolini Bussi & Mariotti, 1999; Kieran & Saldanha, 2005; Verillon & Rabardel, 1995) — an interest which no doubt has been motivated by the important place of technology in our contemporary societies (Franklin, 1990; Heath & Luff, 2000; McLuhan, 1962, 1966). As the anthropologist Clifford Geertz suggested, the human brain is thoroughly dependent upon cultural resources for its very operation; and those resources are, consequently, not adjuncts to, but constituents of, mental activity (1973, p. 76).

We have now been led to a point in which culture as well as its artifacts are said to play a cognitive role. But what role is it? If thinking cannot be reduced to a set of mental procedural mechanisms, what is it then?

Mathematics educators have shown great sensitivity to these theoretical problems and their practical consequences, as witnessed by the various plenary sessions, research fora, and working groups held at recent national and international conferences where the question of the nature of mathematical thinking, new theories of learning and conceptual development have been discussed (English & Sriraman, 2005; Nemirovsky, 2003; Radford, Bardini, Sabena, Diallo, & Simbagoye, 2005; Sfard & Prusak, 2005).

Drawing on anthropological and cultural schools of thought, my goal in this chapter is to present a conception of mathematical thinking that attends to its historical and cultural situatedness. I will suggest that thinking in general, and mathematical thinking in particular, are forms of reflective, mediated social praxis where the organization of individuals' sensuous cognitive processes are related to the meaning of things as they become objectified in practical and theoretical activity.

But to avoid the ubiquitous temptation of seeing practical and theoretical activity through Western rationalist prisms, an effort has to be made to try to understand — in Gadamer's sense (1989) — the activity of individuals and the goals that they pursue in terms of their own cultural, "rational" contexts. The understanding of such cultural, mathematical and scientific, rationalities is, indeed, the primary business of any anthropological approach and, as far as I can see, a way to investigate the link between culture and cognition.

In the following section, I examine some results from ethnomathematics and cross-cultural investigations that clearly show different cultural modes of mathematical thinking. Next, I contrast two types of mathematical thinking stressing the epistemic and ontological dimensions that shape each of them. Since classical historical and anthropological discourse have usually presented Western mathematics as the peak of an evolutionary process and have pictured other kinds of mathematics as "primitive" versions of it, I cannot avoid discussing in some detail the historical roots of such views: I return to Lévy-Bruhl and the myth of progress. Then, drawing on the results of previous sections I formulate the concept of thinking as cognitive praxis. One of the crucial points here is to show that all cultures are subsumed in super-symbolic structures which lend meaning to artifacts and actions and that these dynamic super-structures orient the forms of mathematical and scientific behaviour and investigation. They frame, so to speak, the imaginary dimension of cultures (Castoriadis, 1987) and, at an ontogenetic level, the organization of cognitive processes. In the last section, echoing social theorists who stress the inherently heterogeneous and conflicting nature of cultures, I deal with the issue of knowledge and power. In the conclusions, some implications for mathematics education are mentioned.

THE PLURALITY OF MATHEMATICS

Research conducted in ethnomathematics, cross-cultural psychology, ethnography, history, sociology, etc., shows an impressive diversity of *types* of mathematics (e.g., Barton, 1999;

Bishop, 1991; Cole, 1990, 1995; Cole, Gay, Glick, & Sharp, 1971; Contreras, Morales, & Ramírez, 1999; D'Ambrosio, 2006; Gay & Cole, 1967; Gerdes, 1996; J. Harris, 1982, 1987; Lave, 1988; Lizcano, 1993; Powell & Frankenstein, 1997; Saxe, 1991; Strathern, 1977; Zaslavsky, 1973). My purpose here is not to give an overview of this diverse field. I will limit myself to mentioning a small number of examples that will help me to reformulate the problem of mathematical thinking from an anthropological viewpoint.

Number systems

Drawing on data collected at the University of Papua New Guinea (PNG) and the University of Technology of Lae, Lancy (1983) suggested a four-fold classification of number systems. The classification is based on the kind of signs used to count and represent numbers. Type 1 includes those systems in which parts of the body above the waist serve to count objects. They were common in Western cultures before the invention of printing and the relative spread of writing as a social phenomenon during the Renaissance (see Figure 18.1). Different body parts can be used to count in cyclic form, leading to what we may call, in our own terminology, systems with different "bases." Figure 18.2 shows two of these systems from two communities in Papua New Guinea. The first one is used by the Yupno; it is an "extended" Type 1 system in that it includes body-signs below the waist to count up to 33. The second one is a typical Type 1 system used by the Oksapmin; the body-signs go up to 29.[2]

Type 1 systems are also used in Africa. The extended Type 1 is not common.[3]

Type 2 counting systems use signs of different kind. Here objects are "tallied" using sticks that represent collections or certain kinds of objects as commodities. They are grouped by twos, threes, fours or fives.[4] To designate basic numbers their users employ lexemes which do not name parts of the body.

Type 3 systems have a mixed base of 5 and 20. The number for 58, for example, would be 20 by 2 and 5 by 3 and 3. Generally speaking, only one, two, three, and four are designated by number words. Other numbers are built up from them.

Type 4 number systems have a base of 10. These systems do not use body parts and have several number words. That is, there are terms for numbers 1–6, 100 and 1,000 that have no other meaning in the current language.

Although, in general, these systems are distributed in different geographical regions, some overlaps occur. People may use a number system to count some objects and another system to count other objects. For instance the Buin use a Type 4 system, but they use a Type 1 system to count shell money (Lancy, 1983, p. 105).

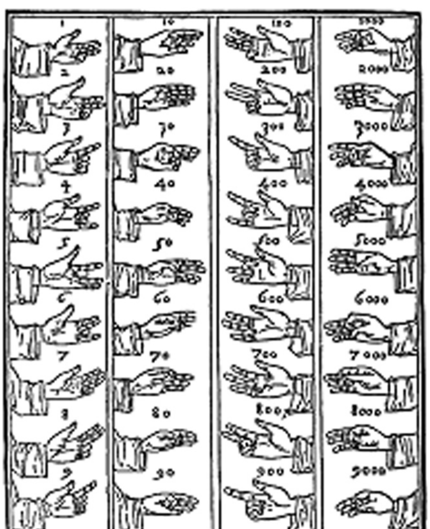

Figure 18.1 The use of fingers to represent numbers in Pacioli's *Summa Artmetica* (1494). The first two columns show the left hand; the two last columns the right hand (D. E. Smith, 1958, p. 199).

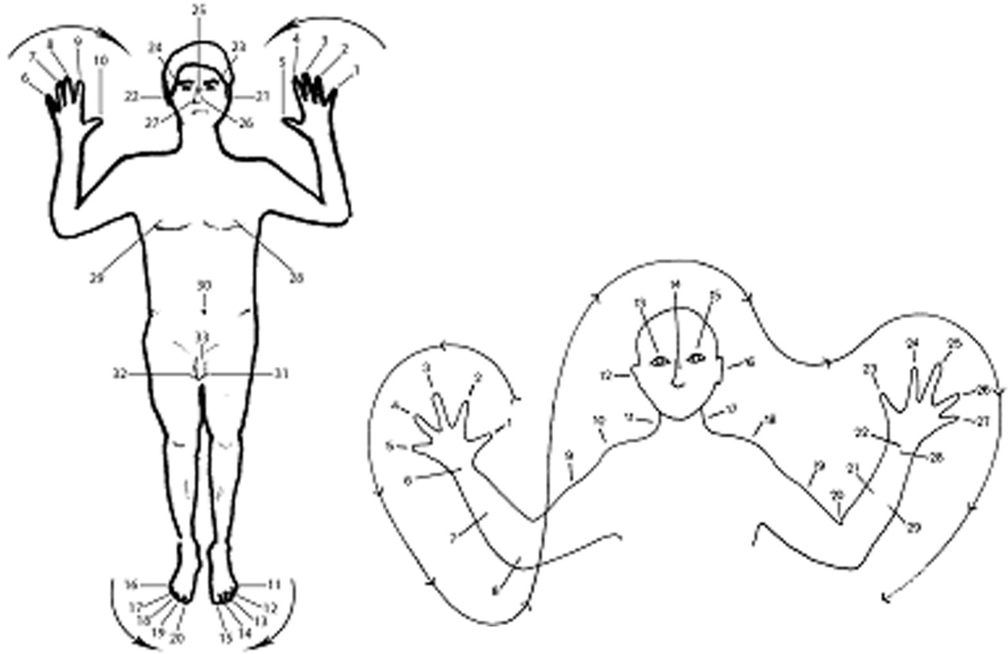

Figure 18.2 Illustrations of two arithmetical sign systems: the Yupno (left) (from Wassmann and Dasen, 1994, p. 84) and the Oksapmin (right) (from Saxe, 1982a, p. 585).

The Yupno inhabit an isolated region of the Eastern part of the Finisterre Range, Madang Province of Papua New Guinea. They are semi-nomadic: they leave their village for several months a year and live in dispersed settlements to collect screw-pine (Pandanus) nuts and hunt marsupials, feral pigs, cassowaries and possums. Around their village they cultivate sweet potatoes, and more recently, they introduced vegetables. Wassmann and Dasen (1994) describe the Yupno numeric system as follows. To count, the Yupno use different parts of the body, as shown in Figure 18.2. They start counting on the left hand. They have number words for 1, 2, and 3. Number 4 is expressed as "2 and 2." The thumb is associated with Number 5 and is called the "finger with which one peals bamboo shoots." Similarly, numbers 6 to 9 are counted on the right hand. Ten is two hands, and also called "mother." Number 15 is called two hands and one foot, or stepfather. Number 20 is called two hands and two feet, also "a man is dead," which means "one complete man." As Wassmann and Dasen (1994, pp. 82–83) point out, the Yupno do not name the testicles and the penis; oblique reference to these parts of the body is made through phrases like "the bow strings" and "the man thing."

While the Yupno numeric system has specific number words for 1, 2, and 3, the Oksapmin use body part names to refer to numbers. Thus, the expression for the term "14th" in a series is "the nose." Cardinal numbers are expressed by adding a suffix to the corresponding body name.

It would be a mistake, however, to think of number systems as defined by their numerical base only. The conceptual foundations of number systems are much more complex. For instance, in their work on the Kakoli-speaking people of the Upper Kaugel Valley of New Guinea's Highlands districts, Bowers and Lepi report the use of a kind of 24 (tokapu) numeric base and an embedded sub-base of 4 (kise) that emphasizes a counting by 2 (talu) (Bowers & Lepi, 1975). As it will become clear in the example below, in the counting process, the counted objects are not reduced to merely appending them to the previous ones to form a "bunch," but are seen as *belonging* to the next group.

Furthermore, the counting process is generally carried out in front of the objects (e.g., pigs, game animals), placed in rows, and is accompanied by gestures and utterances. The person taps the stakes or the objects while saying:

i talu	(here 2!)
i talu kise	(here 2 4)
i talu	(here 2)
i talu kise engaki	(here 2 4 8)
rureponga talu	(2 of 12)
rurepo	(12)
malapunga talu	(2 of 16)

What we call "10" appears here as 2 of the key group of 12 (rurepo). The expression "rureponga talu" means "2 of my 12" (the suffix "–nga" is a possessive suffix). The same idea leads to count 14 as "2 of my 16."

As Bowers and Lepi indicate, "If the objects to be counted total less than a named set of four, the total is expressed as part of the next named set, for example, supunga yepoko "3 of 20" i.e., 19 (Bowers and Lepi, 1975, p. 313).

Since counting large sets of objects may be cognitively demanding a stick may be placed after each set of 24.

Generally speaking, the terms for arithmetical operations unveil the embodied nature of counting as reflected in language. Yupno use the word "timit timit" (timit = to hold, to group) to refer to "addition." Subtraction is referred to as "urok mavi kit" (urok = to throw out; mavi = to throw away; kit = movement –thus to take away). However, even if the Yupno language does have a term for subtraction related to the action of throwing away, arithmetically speaking, subtraction is conceptualized as the inverse of addition. For instance, 17–9 is interpreted as if one already has 9. Then, the problem is to determine how many is needed to reach 17 (details in Wassmann & Dasen, 1994, p. 82).

The cultural-based conceptualization of elementary arithmetical operations becomes even more apparent when we turn to multiplication and division. As suggested by Wassmann and Dasen (1994, p. 82), multiplication cannot be translated exactly into the Yupno language. Thus, 8 × 3 can at best be expressed as "8" yan ong kabi "3" (ya n = thus, on g = this; kabi = many, group), whose translation would roughly be "here are 8, make many with 3." Furthermore, division cannot be translated exactly.

Counting and problem solving

The Yupno's additive strategies

Wassmann and Dasen were interested in finding a suitable translation in the Yupno language for our arithmetical operations, for they were studying the Yupno's strategies for solving elementary problems.

Here is how an old man solved the problem 12 + 13, using sticks:

> He counts both sets (2, 2, 1) breaking them up in groups of five. He shows two hands and two toes (12), then he shows the three other toes of the same foot, and groups the units (3 + 2 = 5) saying, "One foot is finished." Looking again at the sticks that now form five groups of five, he says, "One man is dead [i.e., complete]. I start on another man, only one hand… I am starting with Sivik (another informant present), from him only one hand." (Wassmann & Dasen, 1994, p. 89)

The Oksapmin's additive strategies

The Oksapmin, "a recently contacted cultural group" (Saxe, 1982a, p. 583), who live in a mountainous region of contrasting climates between the headwaters of the Fly and Sepik Rivers, "one of the most isolated spots on the globe" (Lancy, 1983, p. 46), cultivate sweet potato and taro, and obtain animal food by hunting, gathering and the raising of domestic pigs (Perey, 1973).

In his ethnographic studies on the Oksapmin, Saxe classified several strategies used to solve arithmetic problems (Saxe, 1981, 1982a, 1982b; see also Saxe & Esmonde, 2005). In one of the problems, the participants were asked to calculate 6 + 8 without using coins or other tangible objects.[5] In one of the strategies (that Saxe calls *Global Enumeration*), the participants start counting from the thumb (the sign of number 1; see Figure 18.2) to the wrist (number 6); then they continue counting from the forearm (7) upwards. The limitation of this strategy, as Saxe remarks, is that the participants cannot necessarily keep track of when the second number has been completed; as a result, this strategy usually ends up giving only an estimate of the answer and often leads to error.

To cope with this problem, some participants used other strategies. One of them is what Saxe calls *Body-part substitution*. This strategy is based on a coordination of body-signs, uttered signs and pointing gestures. The subject counts up to the wrist (6). Then, he or she *says* "thumb" (1) and *points* to the body-sign "forearm," he or she says "first finger" (2) and points to the body-sign "elbow" (8), and so forth, until he or she says the number corresponding to 8 (elbow), and points to the result ("nose"; see Figure 18.3).

The previous short examples give us a hint of the rich diversity (Owens, 2001) of ways of dealing with counting processes and thinking about numbers. Concrete objects (like sticks) and body parts become mathematical signs through which the counting tasks can be carried out. As a few additions and subtractions using, e.g., the Oksapmin Body-part-substitution strategy would convince anyone trying them, rather than a mental process, mathematical thinking unfolds here at the crossroads of perception, spoken language, actions (e.g., moving sticks) and gestures.

Naturally, there are clear limitations in terms of what is mathematically thinkable and possible using the aforementioned number systems and arithmetic strategies. But for this remark

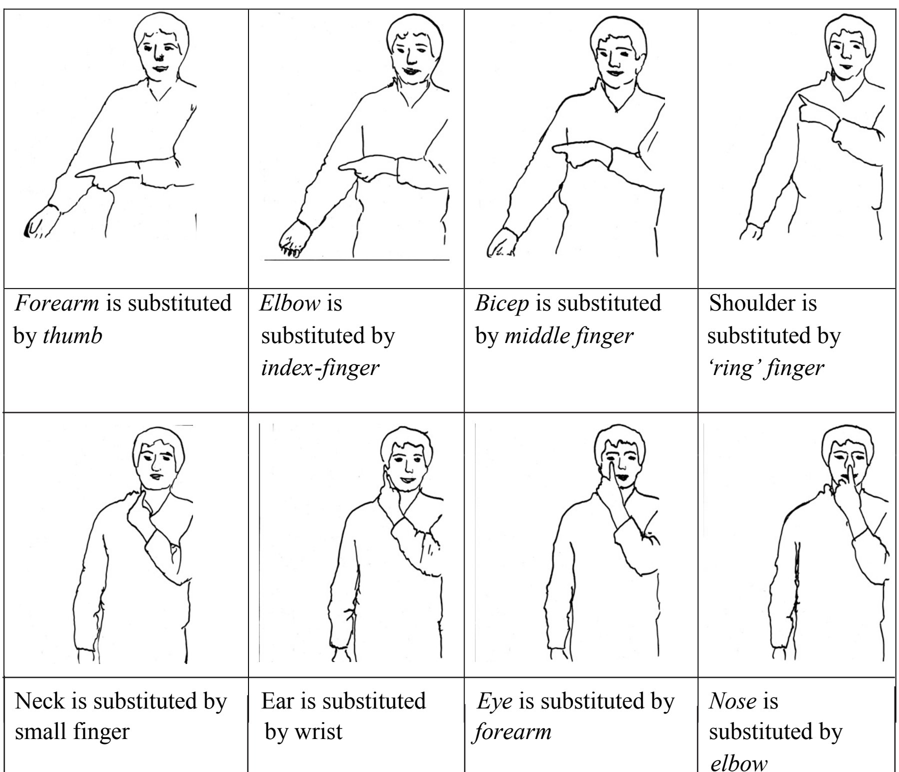

Figure 18.3 The Oksapmin's Body-part substitution strategy to calculate "6 + 8". The drawing at top left shows the first added number to wrist (i.e. 6), then, the second number added to wrist, etc. up to the eighth number (elbow) added to wrist. The result: nose.

to be meaningful, these limitations have to be measured against their own context. As Tulviste (1991) suggests, "if we have to construct hypotheses on the nature of thinking in one epoch or another or in one culture or another, we must first of all ask how the people there were occupied [and] what kind of problems they had to solve" (p. 111).

In general terms, the question can be formulated as follows. Given a culture, C, with a certain mode of mathematical thinking, MT_C, what are the kinds of social, economic, aesthetic and other activities that gave shape to MT_C? To answer this question the broad cultural context needs to be taken into account.

Mathematical thinking in its cultural context

To try to cast one of the subtractions that they gave to their Yupno subjects in meaningful terms, Wassmann and Dasen used the following "Bride price story": "You want to marry P's daughter. The bride price was set at 19 pigs. You have already paid 8 pigs. How many will you have to pay later?" The answer was: "Friend, I am not rich enough to buy a new wife. Where would I find 8 pigs? Besides, I am an old man and have no more strength." As the interviewers remark, "After this he could not be moved to tackle the problem again, it having been rejected as preposterous" (Wassmann & Dasen, 1994, pp. 88–89).

The Yupno people do not seem to be in need of developing more complex systems and techniques for "the Yupno count neither days nor people, nor sweet potatoes nor betel nuts" (Wassmann & Dasen, 1994, p. 83). And this is so "*because counting has no practical meaning*" (Wassmann & Dasen, 1994, p. 83; emphasis added). As Wassmann and Dasen note, at the market, individual objects are put into heaps of a value of 10 toea. If you are interested in the product, you pick it up and leave a 10 toea coin in its place. This way of buying and selling makes unnecessary the numerical calculations. In this context, no exact and precise numeric answer to the Yupno's everyday problems is necessary.

Summing up a general reflection on the various NPG arithmetic systems, Owens (2001) says:

> In a few different language groups, the larger amount of objects is compared by the amount of space taken up rather than by counting objects precisely. This is not an area or volume per se but a recognition that approximation and spatial abundance can be sufficient for a transaction. (p. 160)

However, these remarks do not amount to saying that mathematics is absent. From an anthropological point of view, the challenging problem is to try to elicit other forms of mathematical thinking different from the ones we are used to. In the next section, I contrast two types of mathematical thinking stressing the epistemic and ontological dimensions that shape each of them.

NON-NUMERICAL AND NUMERICAL-ORIENTED CULTURES

Number systems (be they written or body-based) are certainly cultural artifacts that allow their users to deal with everyday life and to reflect about the world. However, as Thune (1978) remarks, there are other means different from counting that may prove to be successful in dealing with the world.

In his work on numbers and counting in a village of Normanby Island, in Papua New Guinea, Thune analyses the case of a non-numerical-oriented culture: the people of the Loboda village. Lobodan people do use mathematics, but it differs from numeric-oriented mathematics in striking ways. It is not that the Lobodans are unable to count. In fact, they may count if needed. They even have a counting system with words for a few numbers (e.g., "Kebwehu" for 1, "labui" or "luwa" for 2, "toi" for 3, "hata" for 4, "nima" for 5). Numbers are, in general, combinations of lower numbers such as "nima kaigeda gumorai" for 6 ("one hand and one finger of the other hand"). The point is that, for the most part, counting is unnecessary in Loboda activities. In fact, Loboda activities revolve around qualitative organizing systems and principles.

To describe lengths, quantities, years, time, etc., the Loboda people use qualitative comparative measures. The measured object is compared to another familiar object. The length of a necklace may be compared to the length of one's arm. In a story, Thune tells us, a man was sent back to his village, which according to our distance system was about 40 miles away. In the story, this distance is referred to as far away as "from here (Loboda) to Sanaroa Island" (Thune, 1978, p. 72). Following this same contextual comparative pattern of thought, mothers do not express the age of their children in years, but in terms of crucial stages of aging. They have terms for infant (memeyo), child (gwama), adolescent boys and girls (tubuhau, gomwagwehine), etc. As Thune observes, "It is not so much that one couldn't develop means for keeping track of age using the Loboda numerical terminology, or for that matter the introduced English terminology, as there is no interest in doing so" (p. 74).

One of the important social activities of the Loboda is giving and receiving. And according to the Loboda's epistemology, this activity is organized through qualitative principles. Thus, the items distributed at a feast have to be repaid at another feast, and the repayment has to be of an "equivalent" (as opposed to "equal") amount. It would not make sense to say that the receiving side has to repay the *same number* of, say, yams, for yams are not counted. They are heaped together and considered as a collective gift. Repaying the collective gift means that a heap of yams of the same (approximate) size (Thune, 1978, p. 75) must be given. Thus, when a pile of yams is divided equally among some recipients, the latter do not think that they received six or seven yams; rather they think in terms of having received a basket or half a basket of yams, the basket serving here as the qualitative comparative measure.

Items from different categories cannot be mixed up. It is not possible to replace tobacco with yams. Because of the Loboda's epistemological principles, it is impossible to add all the received goods and number or quantify them in a grand total. In contrast, the giving side meticulously records the goods given in separate categories (tobacco, yams, necklaces, etc.) by size and form. Even money cannot measure all things, for it is considered to be a good like any other. Instead of measuring all things, it may be part of the list of goods offered by the giving part.

We see hence that the Loboda's mathematical thinking is interested in stressing quantities in terms of practical comparisons to other situated elements rather than to absolute standards. As Thune suggests, numbers may be included in the Loboda's speech, but the meaning is not quantitative. Thune talks about *rhetoric meaning*:

> In many respects it is perhaps best to think of numbers in Loboda as being primarily rhetorical figures, that is as being figures of speech rather than as truly numerical in the English usage of the term. (1978, p. 77)

Expressions to refer to a historical event such as "in the time of our ancestors"; or, "many years ago," or, "200 or 100 years ago," Thune argues, are used to emphasize the extraordinary length of time between the occurrence of the event and now.

The Loboda's mathematical thinking is rooted in general cultural conceptual categories encompassing other non-mathematical activities. These cultural conceptual categories reappear in, for instance, kinship terminology. Thune (1978) notes that

> A frequent theme in Loboda stories concerns the marriage of a person to a being alien to himself, for example a fish, or to a woman who is a witch … The moral of these stories, then is that it is best not to break down the barriers between separate categories … In feasting we see this theme represented in the unwillingness of people to treat yams and tobacco or pigs and dishes as comparable and hence exchangeable. (p. 78)

Even the Loboda grammar reflects this epistemological stance through the use of "mass nouns" which are neither counted nor pluralized: "For example, for the most part the word yam, *bebai*, is treated as if it were a mass noun for grammatical purposes thus precluding its being modified by a number or by an indefinite pronoun" (Thune, 1978, p. 79).

Thune summarizes the Loboda people's lack of interest in using numbers as resulting from the fact that their world is not conceived of as organized by numbers. Unlike European and other modes of thinking,

> Loboda people measure analogically using a wide variety of unnamed scales which may be divided into any fraction necessary to describe the quantity to be indicated. (1978, p. 72)

In opposition to the Loboda, numbers and measures play a fundamental role in Western mathematical thinking — even if throughout its historical development this numerical orientation was not always one of its chief characteristics. As a matter of fact, Indo-Arabic numerals were only introduced in the early 13th century, progressively replacing Roman numerals, which were not practical for calculations. In the pre- and classical Greek periods, numbers had a pronounced mystical meaning and tended to be a source of metaphysical speculations (Robbins, 1921a, 1921b; Roochnik, 1994). Still, in the Middle Ages, the Schoolmen practiced mathematics without attending to numerical or measuring concerns. Crosby (1997) says:

> As with Aristotle, the Schoolmen considered things as more and less than each other, but not in terms of multiples of a definite quantity such as inches, degrees of arc, degrees of heat, and kilometers per hour. The Schoolmen, paradoxically, were mathematicians without being quantifiers. (p. 67)

It was only in the late Middle Ages and early Renaissance that numbers started to become omnipresent. As Crosby notes, one of the bases of the quantifying sources in the Renaissance was *money*. In contrast to other cultural traditions, money became the common denominator of everything, even of time, and "in the dizzy vortex of a cash economy the West learned the habits of quantification" (1997, p. 73).

Among other things, the shift that occurred at the end of the Middle Ages and that gave rise to Western capitalism was related to the emergence of free labor, systematic manufacture and other new forms of production. Renaissance merchants faced new problems and numbers were a way to deal with them. Describing the Renaissance merchants' daily worries, Bec says:

> … one worry for merchants is how to be able to keep fair and precise accounts. Thanks to numbers, the *mercatores* can measure the universe and bring it back to human scale. In their account books, they carefully specify the weight, length, volume, surface and price of the merchandise and goods that they buy or sell. (1967, p. 316)

It would be misleading to say that numbers made the emergence of Western capitalism and its encompassing epistemology possible. But it is not an exaggeration to say that without numbers and careful calculations modern Western rationality would not be able to emphasize the systematization of empirical knowledge, experimentation and prediction — in short those elements in which Max Weber uncovered the characteristics of modern rationality, that is, this peculiar *instrumental rationality* that "has been … strongly influenced by the development of technical possibilities… especially the natural sciences based on mathematics and exact and rational experiment" (Weber, 1992, p. 24).

Even geometry, the science of shape and form par excellence, fell under the empire of numbers and calculations and — with Descartes and Fermat — a new branch of mathematics arose: analytic geometry.

It is worthwhile to point out that, like in the case of the Loboda, Renaissance mathematical thinking was subsumed in a general cultural episteme. And, as in the case of the Loboda, the Renaissance episteme influenced other spheres of everyday life, such as law, art, music, and architecture (see Weber, 1992). In the Renaissance, mathematics ceased being an intellectual ascetic exercise as encouraged by Plato and the Athenian Academy and instead became a method ensuring *certitude*. In the Renaissance the world was supposed to be governed by indefectible laws and — more importantly — these laws were supposed to be knowable. As Da Vinci said before Galileo, the route that leads to these laws is the route of mathematics. What makes the laws reachable is not some algebraic models, but the world of numbers — more precisely, the *proportions*. For the episteme of the Renaissance and its mathematical thinking "the proportion inhabits in numbers and measures, it resides in sounds, time, and space and in every existing force" (Cassirer, 1983, p. 206).

Talking about the Westerners of the 15th century, Crosby says, "no people on earth [were] more obsessed with counting and counting and counting" (Crosby, 1997, p. 74). In order to seize the difference between numerical and non-numerical-oriented cultures, the Western obsession with counting can be contrasted with Lancy's enlightening observation, derived from the vast Papua New Guinea Mathematical Indigenous Project:

> The available evidence would suggest … that counting has little practical merit in the traditional cultures of Papua New Guinea, nor do extraordinary exchange ceremonies inevitably call forth an orgy of counting. (1983, p. 109)

"PRIMITIVE" THOUGHT AND THE MYTH OF PROGRESS

The Yupno answer to the "Bride price story" mentioned previously is strikingly similar to the ones Alexander Luria received again and again from his subjects in Uzbekistan, during the famous "psychological expeditions" to Central Asia. In these expeditions, conducted in the early 1930s under Vygotsky's initiative (Luria, 1931, 1934, 1979), Luria and his team presented Uzbek peasants with problems about logical thinking (syllogisms), categorization, generalization, etc. In the following excerpt, the interviewer (I) has a discussion with an almost illiterate 36-six year-old lady (L):

I: It takes 20 hours to go on foot to Dzhizak, or five times faster on a bicycle. How long will it take on a bicycle?

L: Twenty hours on foot to Dzhizak, and five times faster on a bicycle… I can't reckon at all. Ten hours, maybe? I know that bicycles go faster than bullock carts. Probably it would get there in about 10 hours (…)

I: How do you know?

L: I guessed myself (…)

I: (giving 20 buttons to the lady) If it's 20 hours on foot, you may not get there in 10 hours on a bicycle.

L: [She sorts through the buttons, but doesn't use them as a means for solving the problem] Probably much faster … I don't know, I never rode. (Luria, 1976, p. 121–122; editing slightly modified)

A few years before Luria conducted the psychological expeditions, Lucien Lévy-Bruhl — one of the founders of the French school of sociology — introduced the term "pre-logical thinking" to refer to one of the principles that, along with others, forms the basis of "traditional" or "primitive" thought. According to Lévy-Bruhl, the main difference between "pre-logical" and Western "logical" thinking resides in the fact that while the latter finds in causal laws the essence of nature and the foundation of reality, the former is governed by *col-*

lective representations, i.e., ideas in which the world of humans is thought of as ruled by mystic powers (Lévy-Bruhl, 1922, p. 19).

In opposition to the evolutionist stance of his time, Lévy-Bruhl did not consider "pre-logical" thinking as a rudimentary form of thought or as a kind of inferior thinking comparable only to the thought of children in the "civilized" world, as, e.g., Edward Burnett Tylor did (Tylor, 1891). Lévy-Bruhl adopted a different position. If "pre-logical" thinking was a question of evolutionary slowness, Lévy-Bruhl reasoned, how to understand the fact that when the missionary teaches something to both the "primitive" child and the white child, the former learns as well as the latter?

He suggested that if we see the "primitive" mind in its own context and institutions, it appears as normal, according to the conditions in which it works: it appears as complex and developed according to its own requirements (Lévy-Bruhl, 1922, p. 17).

In a letter written to the anthropologist E. E. Evans-Pritchard in 1934, after reading Evans-Pritchard's critique of his theory of "primitive" mind (Evans-Pritchard, 1934), Lévy-Bruhl says that the fact that the mental habits of the "primitives" are different from the habits of "civilized" individuals does not mean that the "primitive" thinks differently from the "civilized": "his thought", he said, "is neither more nor less logical than ours" (Lévy-Bruhl, 1952, p. 121). In this letter, written at the end of his life, Lévy-Bruhl admitted that the term "pre-logical thinking" was an unfortunate choice, for, he says, "according to me 'primitive thought' is eminently coherent, maybe even over-coherent" (Lévy-Bruhl, 1952, p. 120).

For Lévy-Bruhl, the lack of logical thinking on the part of the "primitives" results from the fact that "pre-logical" thinking remains a prisoner of collective representations. It was culture that was responsible for keeping pre-logical thinking in its own sphere.

Drawing on the ethnographic research of their time, Luria and Vygotsky also posed the problem of "primitive man" in terms of the opposition between "pre-logical" and "logical thinking." For them — like for Lévy-Bruhl — "pre-logical" or "mystic thinking" (a synonymous term) was the distinctive nature of the "uncivilized" world — i.e., the world populated by people "at the lower level of cultural development" (Luria & Vygostky, 1998, p. 40). "Logical" thinking, in contrast, was the distinctive feature of the "civilized" world. Summarizing Lévy-Bruhl's ideas they approvingly said: "By 'prelogical' he [Lévy-Bruhl] simply meant a type of thinking that had not yet developed as far as the form of logical thinking" (Luria & Vygotski, 1998, p. 45).

Although Luria and Vygotsky credited Lévy-Bruhl for having been the first to point out that logical processes are not merely a by-product of natural selection, they criticized Lévy-Bruhl and the French school of sociology for erring in describing the formation of the individual mind as a purely spiritual event occurring in isolation from concrete practice, the particular social systems of the individuals and their histories (Luria, 1976, p. 7). Luria pointed out that the omission of social practice led Lévy-Bruhl to describe the formation of the mind within the confines of the sphere of beliefs, and that this omission also impeded Lévy-Bruhl from seeing that the gap between "pre-" to "logical" thinking could only be accomplished through sociohistorical shifts.

The differences in their views can be gauged by reference to the following passages concerning human perception. Lévy-Bruhl wrote:

> When I said that "primitives" never perceive anything exactly as we do, I never meant to assert a truly psychological difference between them and us; on the contrary I admit that individual physio-psychological conditions of sensory perception cannot be other among them as among us. (Lévy-Bruhl, 1952, p. 121)

Luria, in contrast, claimed that perception changes with the acquisition of theoretical concepts: "The perception of colors and shapes changes" for linguistic theoretical categories introduce verbal and abstract meanings in such a way that “direct impressions [become] related to complex abstract categories” (1976, p. 162).

As a result of the radical changes that the peasants of Uzbekistan underwent under the Russian reform, it became apparent for Luria that changes in social practices entailed much more than an expansion of experience (Antsyferova, 1976): it entailed the *alteration* of cognitive processes and the formation of new psychological systems. Furthermore: “some mental processes cannot develop apart from the appropriate forms of social life” (Luria, 1976, p. 10).

In short, for Vygotsky and Luria, the essential point missing in Lévy-Bruhl’s account is that

> sociohistorical shifts not only introduce new content into the mental world of human beings; they also create new forms of activity and new structures of cognitive functioning. They advance human consciousness to new levels. (Luria, 1976, p. 163)

But how exactly is the shift from “pre-” to “logical” thinking accomplished? It is clear that sociohistorical shifts can be of different kinds. Will all of them lead to this distinctive feature of the “civilized world” which is “logical” thinking? Luria and Vygotsky argued that two ingredients are necessary: changes in the psychology of “primitive” man, which “are to be found in the development of technique, and the corresponding development of social structure” (Luria & Vygotsky, 1998, p. 84). To overcome “pre-logical thinking” and fill in this gap that keeps it apart from “logical” thinking it is necessary that a separation be made between man and his surroundings, between thinking and nature. “In actual fact, the complete separation of the objective and the subjective becomes possible only on the basis of a highly developed technique whereby man, while influencing nature, comes to know it as something outside himself and subject to its own special laws” (Luria & Vygotsky, 1998, p. 81). This is why “More advanced technical development eventually separates the laws of nature from the laws of thinking, and magical action begins to fade away” (Luria & Vygotsky, 1998, p. 85).

In short, along with social changes, the cure for pre-logical thinking is technology and the mastery of nature.

This perspective is at odds with claims made by contemporary aboriginal and other epistemologies, which posit the relationship between humans and nature in different terms. Instead of seeing the purpose of human activity as the mastery of nature, these epistemologies see it as a form of living with it.

For example, Cajete, explaining the concept of Native science, says:

> Native science is a metaphor for a wide range of tribal processes of perceiving, thinking, acting, and “coming to know” that have evolved through human experience with the natural world. Native science is born of a lived and storied participation with the natural landscape. To gain a sense of Native science one must participate with the natural world. (2000, p. 2)

To sum up, Luria and Vygotsky’s claim according to which cognition and consciousness are products of cultural development placed them on the opposite side of Lévy-Bruhl. However, they were closer to Lévy-Bruhl than they thought. Indeed, like Lévy-Bruhl, they also adopted the idea that there is a kind of “primitive,” “uncivilized” culture, and that Western culture and mind are modern, rational, and logical. Even more, like Lévy-Bruhl, they thought that it makes sense to talk about progress and that Western science, its technology and its logic were the highest point of development. While Piaget, seduced by Western mathematics, described the development of the mind in terms of logical-mathematical structures, Luria and Vygotsky anchored the development of the mind in cultural developments, the latter being understood in the sense of the grand epic narrative of modernism and its enlightened notions of progress, civilization, reason, and the mastery of nature.

THINKING AS COGNITIVE PRAXIS

In light of the diverse cultural ways to deal with numbers, space, time, money (Crump, 1990; Gell, 1992; P. Harris, 1991), what, hence, can we say about mathematical thinking? What can we say if evolutionism seems inadequate to explain cultural and conceptual developments and if "logical" thinking, instead of being the *telos* (i.e., the end) of its alleged "primitive" predecessors, seems rather to be *one* among the multitude of possible conceptual forms of human thinking?[6]

Drawing on the epistemologist Marx Wartofsky (1979) and in light of the previous discussion, I want to suggest that thinking is a cognitive praxis — a *praxis cogitans.* More specifically, thinking is *a mediated reflection of the world in the form of the individuals' activities.*

In the rest of this section I elaborate upon this idea.

Mediated Reflection

It is well known that Vygotsky was the first to outline the role that artifacts (objects, instruments, semiotic systems, etc.) play in cognition (Vygotsky, 1981). In saying that thinking is a *mediated* reflection, what I mean is that instead of conceiving of artifacts as mere aids to thinking and acting or as simple amplifiers (as cognitive psychology does), I conceive of artifacts as co-extensive of thinking: we act and think *with* and *through* artifacts.

Now, since artifacts are bearers of the historical cognitive activity deposited in them by previous generations (e.g., the Oksapmin digging stick or the hunting arrow or the school calculator), in using them in the course of our activities the subjective domain and the cultural-objective one become imbricated into each other. Artifacts (that Wartofksy also called "models") are indeed historical "embodiments of purpose and, at the same time, instruments for carrying out such purposes" (Wartofsky, 1979, p. 142). This is why in each culture artifacts define a "region" (Voloshinov, 1973) which is both subjective and objective, where thinking finds its space for unfolding and the mind goes "beyond the skin" (Wertsch, 1991).

But I want to broaden Vygotsky's idea of mediation here, so as to also include gestures and other kinesthetic actions — in fact all those embodied aspects of tactile, visual, and other sensuous experiences through which we get acquainted with, and position ourselves in, the world (Merleau-Ponty, 1945). These embodied cognitive elements have been excluded from the realm of knowledge by the rationalist epistemologies of the West since Plato's time. In the *Phaedo* (65a–65b, 1961, p. 47), Simmias is asked to determine who, among all sorts of men, would be able to attain true knowledge. Is it not him — Plato has Socrates ask — who

> pursues the truth by applying his pure and unadulterated thought to the pure and unadulterated object, cutting himself off as much as possible from his eyes and ears and virtually all the rest of his body, as an impediment which by its presence prevents the soul from attaining to truth and clear thinking? Is not this the person, Simmias, who will reach the goal of reality, if anybody can? (*Phaedo,* 65e–66a, p. 48)

He then continues: "we are in fact convinced that if we are ever to have pure knowledge of anything, we must get rid of the body and contemplate things by themselves with the soul by itself" (66b–67b, p. 49).

However, as Kathryn Geurts suggests, *sensing* — understood as a form of being in touch with our surrounding and of gathering knowledge — "is profoundly involved with a society's epistemology, the development of its cultural identity, and its forms of being-in-the-world"

(Geurts, 2002, p. 3). She argues for the importance of what she calls *sensory order* or *sensorium*, that is,

> a pattern of relative importance and differential elaboration of the various senses, through which children learn to perceive and to experience the world and in which pattern they develop their abilities. (p. 5)

In her ethnographic work, conducted in the 1990s with Anlo people who speak a dialect of Ewe — a West African language used in southern Togo and the southeastern corner of Ghana — Geurts was exploring a culture with an extremely developed *sensorium*. In the same way that Artic cultures seem to have a richer perceptual category than Western cultures for distinguishing tones of white, so the Anlo-Ewe-speaking people have a broad set of words for different tones of tactility. Geurts says:

> While gathering information on sensory experiences in Anlo-land, the phenomenon I think of as "touch" was probably the most problematic. There seemed to be a profusion of expressions for what all seemed to be "tactility." (p. 55)

The translation of the various words that the Anlo-Ewe-speaking people use to describe the different aspects of tactility proved to be particularly difficult:

> Translation into my own experience and cognitive framework proved to be extremely confusing [...] Five root words appeared over and over again in my observations and discussions about touch, and they are arranged phenomenologically into a kind of continuum of intensity. In its barest simplicity, the continuum consisted of *li* (caress), *ka* (contact), *le* (seize), *to* (push), and *fo* (strike). Initially I was reluctant to accept as *tactility* the last three categories of seize, push, and strike (*le*, *to*, and *fo*), but people consistently offered them as aspects of *contact* or *touch* and argued for the correspondence with a kind of haptic experience. (Geurts, 2002, p. 55)

Seselelame (literally "perceive-perceive-at-flesh-inside," Geurts, 2002, p. 41) was in the end an expression that seemed to best capture tactility, balance, embodied intuition as well as other sensorial aspects that can be translated as *sense*. But, in opposition to our common cultural idea of the latter, *seselelame* houses the cognitive function of perception as well as other subtle somatic sensorial phenomena that we do not distinguish or attend to. *Seselelame* is an indigenous epistemological category that keeps physical sensation and the cognitive processes of thinking together.

Thus, in saying that thinking is a mediated reflection, I am using the term "mediation" in a sense that encompasses not only the technological dimension of culture but also its s*eselelame* or cultural epistemic mode of embodiment.

The reflective nature of thinking

For rationalism, reflection "is nothing else than attention to what is in us" (Leibniz (or Leibnitz), 1705/1949). This is what contemporary cognitive psychology calls *meta-cognition*. The idea of reflection that I want to convey here is different.

The reflective nature of thinking means neither a dialogue with what we already have *in* us, nor merely an assimilation of an external reality (as empiricism and behaviorism have suggested) or an *ex nihilo* individual conceptual construction.

Not only does reality not unfold itself in a direct and immediate fashion, as the empiricists thought, it can hardly be reconstructed on the basis of personal experience alone. Indeed, no personal experience, regardless of how rich it might be, can manage to establish on its own a cultural system of ideas, such as the qualitative mathematical thinking of the Loboda, the Roman legal system, formal logic or set theory. An undertaking of this magnitude requires not one lifetime, but thousands (Leontiev, 1968), and perhaps even more.

As Durkheim (1968) noted, the concepts through which we think are those recorded in the vocabulary of a culture and express the product of elaborations of collective experiences that go beyond the experiences of the individual. He said:

> there are scarcely any words among those which we usually employ whose meaning does not surpass, to a greater and less extent, the limits of our personal experience. Very frequently a term expresses things which we have never perceived or experiences which we have never had or to which we have never born witnesses… Thus in the word there is a great deal of condensed knowledge in whose formation I have not participated, a knowledge which is more than individual [...]. (p. 483)[7]

Thinking as re-flection means rather a dialectical process between a historically and culturally constituted reality and an individual who reflects and modifies it according to his or her interpretations and personal meanings. One of the roles of culture is to suggest to its individuals forms of attending to (Merleau-Ponty, 1945), or intuiting (Husserl, 1931), reality and its phenomena.

In short, in more general terms, the reflectivity of thinking resides in the fact that, from a phylogenetic viewpoint, individuals produce the objects of knowledge. At the same time, from an ontogenetic viewpoint, the thinking of any concrete individual is oriented by the general concepts recorded in his or her culture. "Social being," says Eagleton (1997, p. 12), "gives rise to thought, but is itself caught up in it."

The form of the activity: Semiotic systems of cultural significations

In previous sections we saw that, to count, the Yupno use their body. They start counting on the left hand; but the choice of this hand is not arbitrary. The Yupno's choice results from a symbolic dimension that ascribes different values to the two sides of the body: the left side of the body is associated with the passive and female side of man, a part whose function is to help. The right side is the active one; it is associated with the hot male side. "This symbolism applies especially to the two hands: The right hand is the one that tightens the bow; the left one helps in holding it" (Wassmann & Dasen, 1994, p. 91).

We also saw that the Kakoli-speaking people as well as the people from the Lombada village deal with problems involving numbers, but they do it in very different ways. For the Kakoli-speaking people, "counting does not exist in isolation. It quantifies and qualifies relations between people, objects and other entities" (Bowers & Lepi, 1975, p. 322). For the people in the Lombada village objects are qualitatively counted and objects of different sorts (tobacco, yams, etc.) cannot be mixed up.

These choices in dealing with numbers rest on some basic *beliefs*. These beliefs orient the *form* of the individuals' activities and their corresponding mathematical thinking.

Beliefs of this kind are not specific to the mathematics of the Yupno, the Kakoli-speaking people or the Lombada villagers. Pythagoreans, for instance, believed that the universe is governed by numbers, and Plato claimed that numbers and mathematical objects in general are unchangeable forms. All of these beliefs reflect accepted principles of an *ontological* nature, i.e., principles about the way the world is.

These principles are part of a symbolic superstructure that may be termed "Semiotic Systems of Cultural Significations" (SSCS; Radford, 2006).[8] In addition to beliefs about mathematical objects (their nature, their relationship with the concrete world, etc.), they also include ideas about truth, the methods to inquire about it, what counts as a fact and evidence, etc., and the legitimate forms of knowledge representation.

As to the legitimate forms of knowledge representation, it is worthwhile to pause here for a moment and recall an often quoted passage from Plato's *Republic*. In this passage, talking about the geometers, Plato says, "You know too that they make use of and argue about visible figures, though they are not really thinking about them, but about the originals which they resemble" (*Republic,* 510d). Concerning the methods for inquiring about truth, Plato clearly condemned those relying on the use of artifacts. For instance, he criticized the use of mechanical instruments — as used by Eudoxus and Architas among others — in the study of the two mean lines in a proportion. As Plutarch tells us,

> Plato took offense and contended with them that they were destroying and corrupting the good of geometry, so that it was slipping away from incorporeal and intelligible things towards perceptible ones and beyond this was using bodies requiring much wearisome manufacture. (Plutarch, *Lives*: Marcellus, xiv; quoted by Knorr, 1986, p. 3)

These cultural significations — e.g., to start counting with the left hand (Yupno), the avoidance of mixing objects from different categories (Lomboda), and the restraining and suppression of embodiment and artifacts (Plato) — function as links between the individual consciousness and his or her cultural objective reality. They are prerequisites and conditions of the cognitive activity of the individual (Ilyenkov, 1977). They orient the individual's mathematical activities and give a particular shape to them. If it is true — as we stated previously — that the sensual practical activity mediated by artifacts enters in the process of thought and its actual content, the way this happens depends on the cultural significations that underpin and give *form* to the activity.

The understanding of the SSCS is an important task in the investigation of the type of mathematical thinking of a culture. SSCSs interact with activities — goals, actions, distribution of labor, etc. (Leont'ev, 1978) — and with the technology of semiotic mediation (i.e. the territory of the artifact). In so doing, the SSCSs give rise, on the one hand, to *forms of activity*; and, on the other hand, to specific modes of knowing or *epistemes* (Foucault, 1966). While the first interaction gives rise to particular manners in which activities are carried out at a given historical moment, the second interaction gives rise to modes of knowing that identify the "interesting" problems and situations, and highlight the methods, arguments, facts, etc. that are considered valid.[9]

The triangle shown in Figure 18.4 illustrates the complexity of human activities and their diversity.

POWER IN CULTURE: THE POLITICS OF PLATO'S FORMS

Social theorists have always stressed the dynamic, heterogeneous, and diverging nature of cultures. Thus Quantz and O'Connor (1088) note that culture is not "a superorganic entity demanding obedience; rather, it is a world full of unique individuals, each expressing personal views within their cultural interactions" (p. 96). Fay (1996) remarks that

> Cultures are neither coherent nor homogeneous nor univocal nor peaceful. They are inherently polyglot, conflictual, changeable, and open. Cultures involve constant processes of reinscription and of transformation in which their diverse and often opposing repertoires are re-affirmed, transmuted, exported, challenged, resisted, and re-defined. This process is inevitable because it is inherent in what it means for active beings to learn and apply cultural meanings, and in the ideational nature of culture itself. (p. 61)

Cultures, as sites of sameness and difference, of convergence and opposition also become sites of power and control. Power is a producer of knowledge (Foucault, 1966) and vice-versa:

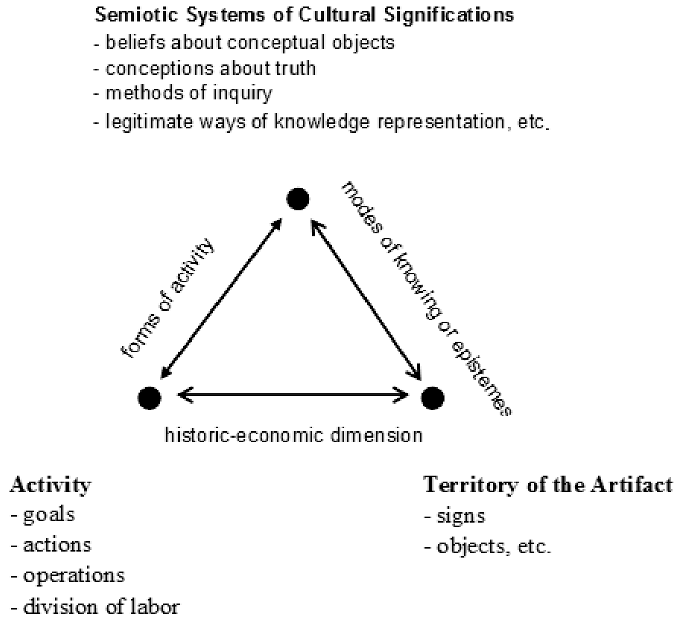

Figure 18.4 The arrows show the interaction between Semiotic Systems of Cultural Significations, Activity, and the Territory of the Artifact. The interaction generates the forms of activity and the modes of knowing on the base of the specific historic-economic dimension. In a dialectic process, forms of activity, modes of knowing, and the historic-economic dimension alter the triangle's vertices.

knowledge also produces power. And mathematical knowledge is not an exception. As any anthropological inquiry about mathematical thinking cannot leave this point aside, I want to illustrate it by referring to Platonism. Although I could have chosen another example, my choice is justified by the influence of Plato's views on Western mathematics.

From an anthropological viewpoint, the problem, as formulated by non-traditional epistemologies, is to unveil the processes of knowledge production within determinate traditions and to disclose "the presuppositions that circumscribe what is believed to exist and identify the mechanisms by which facticity is accredited and rendered unproblematic" (Hawkesworth, 1989, pp. 551–552).

Bearing this idea in mind, let me start by recalling some basic well know facts about the Greek philosophical and political context, the more obvious being that Plato equated the knowable with "what is" (N. D. Smith, 2000, p. 165), and that for Plato something *is*, if it is without change. As Plato says, "knowledge … has to do with being and reality, and sameness and unchangeableness" (*Philebus,* 58a–58a, 1953, vol. 3, p. 618).

All sensible things are always changing; as a result, the actual triangle on the blackboard, which is in a continuous process of degradation, remains unknowable and indefinable. The only objects that can be defined are those that are real, unchangeable, and these are the *forms.* What can be known is not the actual and visible triangle on the blackboard or the paper but the *form* of the triangle. The very possibility of knowledge rests, for Plato, on this principle.

Plato's epistemological exigency is so peculiar that it even surprised his disciple Aristotle, who, without denying the existence of general objects, found nonetheless this manner of formulating the problem a bit extravagant or at any rate "an ontological complication that can be avoided by a proper analysis of the empirical basis for knowledge" (Modrak, 2001, p. 7; see also Fine, 1993).

Why did Plato pose the question of knowledge in those terms? Our mathematics is so deeply immersed in Plato's ideas that we cannot leave the question under the carpet. To answer this question, we have to go back to the historical and cultural context in which Plato's theory of *Forms* was developed.

What was this context? It was in the aftermath of the defeat of a prosperous Athens in the Peloponnesian War. Before the war, under Pericles' government, Athens experienced a population growth, the rise of commerce, and the emergence of new social classes, leading to a social restructuring where the old values of the oligarchic elite were shaken. The concept of the "good," related to manliness and good birth progressively elaborated since Homer's times was challenged by the new context shaped by the arrival of "[r]ootless foreigners in their origins; skeptical, nominalistic, subjectivistic, and relativistic thinkers" (Levi, 1974, p. 61), a context in which

> What a man *has* becomes as important as what he *is* and *does*. Wealth has thrown lineage into confusion… The democratic thrust turns against the traditional values of wealth and good family, and it redefines virtue in terms of man's actual behavior rather that his name and lineage. (p. 55)

Plutarch tells us that to counter the effect of the Periclean democracy, the old oligarchy carried out political actions, for the oligarchy "would not suffer those who were called the honest and good (persons of worth and distinction) to be scattered up and down and mix themselves and be lost among the populace, as formerly, diminishing and obscuring their superiority amongst the masses" (Plutarch, ca. 100 A.D.).

Plato grew up in the closed circle of his relatives Charmides and Critias, two members of the oligarchy who were both leaders of the antidemocratic government that the Spartans installed in Athens after the defeat of the Peloponnesian war. Because of his aristocratic linage, Plato was destined to become a member of Athens' ruling class. As he says in the *Seventh Letter*, in his youth he was certainly "full of enthusiasm for a political career" (Plato in Bluck, 1949, p. 154). Plato dreamt of correcting the deplorable political situation "for our city was no longer managed in accordance with the traditions and practices of former generations" (Plato in Bluck, 1949, p. 154). His philosophy was indeed a commitment towards the restoration of the old aristocratic values. It is within this cultural and political context that we can understand his attacks against the Sophists, who flourished in Periclean times and who were asserting the values of conventionalism through an epistemology and ethics inspired "by an almost Heracleitean sense of relativity and temporal flux" (Levi, 1974, p. 61) (For an account of Plato's fight against the sophists, see Catonné, 1998).

The Academy was a strategic pedagogical response. It was the place to educate people according to aristocratic values. "Young men of high birth who were preparing to take a leading part in the government of their native cities came to the Academy from all over Greece" (Bluck, 1949, p. 32). There, they did not learn rhetoric,–which was taught by the rival school of Isocrates–, for rhetoric "is a creator of persuasion" (Plato, *Gorgias,* 452e–453a, 1963, p 236), rhetoric "is a creator of a conviction that is persuasive but not instructive about right and wrong" (Plato, *Gorgias*, 455a–455a, 1963, p. 238). The young Greek aristocrats were instructed in dialectic, which "attempts through discourse of reason and apart from all perceptions of sense to find his way to the very essence of each thing" (Plato, *Republic*, Book 7, 532a–532b, 1963, p. 764).

Plato's general aim was based in a metaphysics based on "two principles which are aristocratic in the deepest and most essential sense: …. the *principle of hierarchy* and the *principle of permanence*" (Levi, 1974, p. 92). While the first one expresses the idea of rule conducted by one or a very few learned people (see e.g., *Republic* or *Laws*), the second one — which constitutes the center of gravity of Platonism — poses the basic problem of an aristocratic vision of the world in terms of the struggle against temporality and change. The theory of *Forms* is Plato's formidable weapon in this struggle to salvage the old aristocratic values. Indeed Plato's theory of *Forms*

> is the product of an act of pure supposition — that behind the phenomenal world which is temporal and in perpetual process lies a world of ideal forms, a system of necessary relations, patterns fixed in the nature of things which are eternal, not subject to the ravages of time; unchanging, ungenerated, and indestructible. (Levi, 1974, p. 94)

To sum up, the defeat of Athens in the Peloponnesian War was followed by an intensified questioning of Periclean populist sociopolitical principles. Plato's epistemology was one of the efforts to rescue the traditional aristocratic values. Politically, it was formulated as a kind rationality that opposes changes. Greek Mathematics was based on these aristocratic ideas and offered the paradigmatic example of the unchangeable, the permanent, and the eternal.

As I mentioned previously, the interest of this example for our inquiry into mathematical thinking from an anthropological viewpoint is that it offers us an instance of the intricate relationships between power and knowledge in culture. But there is more. Greek mathematics shaped the Western idea of mathematics and attitudes towards it. Even today, to a large extent, mathematics continues to be considered as dealing with matters that are beyond culture, geographic location and time. Mathematics is conceived of as something universal whose objects have always been there, waiting to be discovered.[10]

SYNTHESIS AND CONCLUDING REMARKS

The conception of mathematics elaborated by Western rationalist paradigms led to a conception of mathematical thinking as a cognitive activity based on the "innate rules of logic" (Leibniz, 1705/1949) — supposedly the rules capable of ensuring one the attainment of the universal truths of mathematics. Although the rationalist paradigm was challenged by 17th-century British empiricism, which contended that ideas were the result of impressions that external things imprint on us (Radford, 2004), it was only the advent of theories of evolution in the 19th century that made it possible to formulate the problem of thinking in new terms. However, the arrival of evolutionist theories only complicated the Western difficulty to understand other cultural traditions further. Indeed, within the evolutionist context, historical conceptual developments as well as different cultural conceptualizations were systematically seen through the lenses of the West, which did not hesitate in positing itself at the summit of the evolutionary line. To a very large extent, "civilization" was equated with technological progress. As we saw in the previous sections, a rhetoric anchored on the idea of "primitiveness" accounted for the variety of ways of thinking mathematically, as reported by missionaries, travelers, diplomats and so on. The least that can be said is that Western epistemologies have been terribly bad at recognizing that lines of conceptual development may be varied. Even Vygotsky and Luria, who were among the first to appreciate the role of culture in cognition, fell pray of a narrow view of culture and reduced it to its technological dimension.

Talking about Vygotsky's concept of culture, van der Veer (1996) says:

> It is quite clear that in dealing with the psychological significance of cultural objects he did not include the full range of cultural phenomena that was analysed by his contemporary ethnographers. Thurnwald, Durkheim and others investigated different systems of law, moral thinking, religion, art, kinship systems, etc. but Vygotsky chose to concentrate upon counting, writing and language (speech in his terms) at large […] Selecting these aspects of culture thus nicely fitted in with the dominant Soviet theme of social and cultural progress. (p. 256)

The path leading us to realize that there are various types of mathematics, irreducible to each other, has not been a short one. This situation can hardly be attributed to chance. One of the West's most confident products, one of its cornerstones and distinctive characteristics has been its mathematics. If it has been difficult to accept that there is a diversity of manners in which to live and to construct knowledge (Feyerabend, 1987), it has been even more difficult to accept that there is a diversity of types of mathematical knowledge and that they are *genuine* on their own.[11]

The rather long presentation of ethnographic data in the second section of this chapter — although irremediably selective and unrepresentative — gives at least a hint of the diversity of cultural forms of mathematical thinking. But in order to go further, the crucial problem of the relationship between knowledge, culture and thinking had to be addressed.[12]

Since the mentalist and individualist conception of thinking elaborated by classical cognitive science is too restrictive, I needed to formulate a broader concept of thinking tuned to a suitable concept of culture. I found Geertz's concept of culture particularly appealing. For him a culture is

> an historically transmitted pattern of meanings embodied in symbols, a system of inherited conceptions expressed in symbolic form by means of which men communicate, perpetuate, and develop their knowledge about and attitudes towards life. (Geertz, 1973, p. 89)

Before dwelling into the formulation of a non-mentalist and non-individualist concept of thinking, I thought it important to present an example of two cultures that developed two different forms of mathematical thinking (one numerically and one non-numerically oriented). At the same time, I stressed the role played by their corresponding cultural meanings and conceptions that Geertz was talking about and that, for me, as mathematics is concerned, relate (although not exclusively) to the ontological domain. Thus, I tried to show that both Loboda and Renaissance mathematical thinking were driven by two different ontologies (i.e., beliefs about the way the world is) and their corresponding historical-economical contexts. The temptation to see the Loboda's mathematical thinking as "a primitive" form of the other, led me to discuss the problem of the "primitive mind" as elaborated by evolutionary theories.

Drawing on anthropological and cultural schools of thought, I suggested that mathematical thinking is a form of reflective, mediated social praxis underpinned by the form of activities and modes of knowing as afforded by the historical-economical context of the culture in question and its semiotic system of cultural significations. Finally, in order to emphasize the heterogeneous social nature of thinking, I touched upon the question of power in culture. In addition to showing that mathematical thinking is much more than something merely "situated" or "conversational" — something almost epiphenomenal — the discussion of power in culture illustrates the fact that thinking is embedded in what Foucault called "regimes of truth" (Foucault, 1980), i.e., cultural mechanisms of production of facticities and claims about truths that are rendered unproblematic. I discussed the politics of Plato's *Forms* not only because of the urgent "need to anthropologize the West" if we want to understand our own practices, but also because it shows, in a clear way, how "claims to truth are linked to social practices and … become effective forces in the social world" (Rabinow, 1986, p. 241).

Although my interest in this chapter was not to deal with the challenges that multicultural societies are facing nowadays — challenges that arise in particular as a result of increasing migratory movements — the anthropology of mathematical thinking here presented may shed some light, I think, on the problem of cultural diversity in educational systems in general and in the classroom in particular.

First, semiotic systems of cultural significations relate, as we saw, to beliefs about knowledge itself and how mathematics fits in it. Commenting on paradigms of knowledge in Western and North American Aboriginal cultures, Gill (1999) says:

> these two ways of conceiving of knowledge differ from each other. The one seeks knowledge primarily as an end in itself, while the other pursues it as a means to the end of a meaningful and fruitful way of life. It would appear that these paradigms are largely incommensurable. (p. 425)

Second, semiotic systems of cultural significations relate to beliefs about what is relevant and how to deal with relevance. Thus, children who have grown up in cultures in which numbers are not considered relevant to the understanding of the world will encounter many difficulties in dealing with a type of mathematics that values reflections based on numeric and analytic relations. The same is true of geometry. As Harris (1991) notes,

> In Western European cultures, *shape* is considered to be both interesting and important… As a result, there is an abundance of terms for abstract geometric shapes in the English language, whereas Australian Aboriginal languages have relatively few terms for abstract shapes. By contrast, in Aboriginal cultures, *place* is very important, and Aboriginal people have many words which pinpoint particular places and many ways of being very specific in ordinary speech about the place where something happens. (p. 20)

Third, semiotic systems of cultural significations mediate the cultural kinds of relationships between subject and knowledge (e.g. attitudes towards mathematics). As Sfard and Prusak (2005, p. 1–41) note in their classroom study of new-comers to a culture, "the OldTimers and NewComers differed in a consistent manner both in the way they learned and in the results attained." The NewComers, immigrants from the former Soviet Union brought with them attitudes about homework, relevance, etc. that differed considerably with the Israeli OldTimers' attitudes.

Fourth, semiotic systems of cultural significations legitimate forms of knowledge representation that may vary from one culture to another. Because of the West's emphasis on the relevance of writing, mathematics is often reduced to the written dimension. Although incontestably important in Western cultures, orality, which is valued in other cultures, is often seen as a means (e.g., in classroom discourse) to reach the sphere of the written.

The written and oral traditions bring forward another related question — the problem of legitimate forms of knowledge communication. Again, Gill's reflections are of interest for us here:

> The primary means of communicating these traditions and teachings in the Native culture is by means of oral story-telling on the part of the elders and parents. Such stories are taken as fully authoritative sources, and thus as reliable bases of knowledge by the members of any given tribal community … the power and authority of traditional Native teachings as they are embodied in orally transmitted stories derives from their connection with the combined experience and wisdom of the tribal community in the past. (Gill, 1999, p. 427)[13]

It is beyond the scope of this chapter to propose solutions to the previous important problems. An anthropology of mathematical thinking may help us to realize, nonetheless, that the search for solutions should be framed by a sensitivity to other cultural traditions and new efforts to understand the *Other*. However, this sensitivity should neither be understood as a gesture of generosity nor as the result of our acknowledging the shortcomings of the "imperial eyes" (Pratt, 1992) and the limits of representation. It is rather a question of realizing that the understanding of the *Other* is at the same time the understanding of ourselves, for we can only construct ourselves through others.

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NOTES

1. This chapter is a result of a research program funded by The Social Sciences and Humanities Research Council of Canada (SSHRC/CRSH).
2. The variety of bases is amazing. To give but another example, the Kewa people of Papua New Guinea use a body-sign 68 base arithmetical system (Pumuge, 1975; Swetz, 1994, p. 52).
3. "The Ekoi people of Cameroon are among the few African peoples who actually do count on their toes" (Zaslavsky, 1973, p. 51).
4. In our own mathematical terms, we would say that they are number systems having a base of 2, 3, 4, or 5.
5. Since in the Oksapmin language there are no designated mathematical names for 6 and 8, the question in the version using concrete objects — more specifically, coins (1 coin = 1 shilling) — was asked as follows: "You have wrist shillings" (the informant gestured around the wrist and pointed at the first set of coins). "A friend gives you elbow shillings" (the informant gestured around the elbow and pointed at the second set of coins). "How many do you have altogether?" (after Saxe's, 1982a, p. 587, description).
6. As Feyerabend (1987) argues, like language or art, thinking is universal, but like language or art, it has many forms.
7. The translation comes from (Durkheim, 1965, pp. 620–621). I have slightly corrected it to conform to the original.
8. In (Radford, 2003) I called this superstructure *Semiotic Cultural Systems*. The expanded name that I am introducing here better reflects, I think, the idea behind it.
9. We know that, in his genetic epistemology, Piaget (1970) emphasized the action of the subject in the formation of the *schema* of a concept. What our discussion adds to this is that the schema also includes, in a decisive manner, the *cultural meaning* of the action as it is carried out in a specific sociocultural activity (Radford, 2005).
10. In 1935, Bernays remarked that "it is not an exaggeration to say that Platonism reigns today in mathematics" (Bernays, 1935, p. 56). In 2004, Patras said that "There is almost no professional mathematician who does not recognize himself a Platonist" (Patras, 2001, p. 35). In light of these and other old and recent remarks (see, e.g., Brown, 1999, p. 24), we should conclude that things have not changed very much.
11. And I am sure that disagreements on this issue are still far from being resolved (see, e.g,. Rowlands & Carson, 2002; Adam, Alangui, & Barton, 2003; Rowlands & Carson, 2004). Indeed, it is not infrequent that the mere idea of there being a plurality of genuine forms of mathematics gets Platonist-minded scholars irritated, mainly because the universality with which the rationalist tradition has endowed formal, academic mathematics seems suddenly to be put into question. Usually, the attempts at understanding other mathematics as mathematics in their own right are judged as being motivated by a love for the exotic, the rare and the aboriginal. Those critics thus fail to grasp the meaning of the anthropological enterprise: they reduce it to an anthropology of curiosities. Anthropological attempts at understanding other mathematics are then charged with "demagogy" and as being the products of "missionary zeal" and, more importantly, are seen as part of a plot against logical thinking. As one unhappy reviewer of this chapter suggested, papers like this convey "the common prejudice that exists against formalized, abstract academic mathematics." But this is to miss the entire point. My reviewer asks: "Could it not be said that logical thinking is perhaps one of the most remarkable products of human achievements?" Right after, my reviewer also asserts, "The author would obviously disagree". No, I would not. In fact, I do not. The problem is to understand the historical, political, economic, cultural, and social conditions that made such a form of thinking possible. And, to a modest extent, a contribution to the anthropological investigation of these conditions is what I was seeking to provide in the section about the politics of Plato's forms. But my reviewer finds this move unacceptable, for it leads me to posit Plato's conception of forms not as a certain and unquestionable fact but as a mere *basic cultural belief.*
12. Talking about scientific knowledge in general and its relationship to culture, Shapin says:

    The mere assertion that scientific knowledge 'has to do' with the social order or that it is 'not autonomous' is no longer interesting. We must now specify how, precisely, to treat scientific culture as social product. (Shapin, quoted in Woolgar, 1981, p. 366)

13. See also Dzobo, 1980; O. Kawagley, 1990; O. A. Kawagley, 2001.

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