LUIS RADFORD

THE ETHICS OF BEING AND KNOWING: TOWARDS A CULTURAL THEORY OF LEARNING[1]

This chapter sketches a theory of teaching and learning that takes its inspiration from some anthropological and historico-cultural schools of knowledge—the theory of knowledge objectification. Within this theory, the problem of learning is formulated in such a way that rationalist or individualists views of cognition and social interaction are avoided. The theory of knowledge objectification posits, indeed, the problem of learning as a social process through which students become progressively conversant with cultural forms of reflection. Arising in the course of sensuous mediated cultural praxes embedded in historically formed epistemes and ontologies, learning, it is argued, is not just about knowing something but also about becoming someone. The formulation of learning as a process where knowing and being are mutually constitutive leads to a non-utilitarian conception of the classroom: entrenched in unerasable ethical concerns, the classroom appears as a space for the growth of intersubjectivity and the nurturing of what is called here the *communal self*.

The chapter is divided into six sections. In the first section, I discuss some problematic assumptions often adopted by many contemporary theories of teaching and learning, in particular assumptions related to the learner, the content to be learned and process of learning. In Section 2, I introduce a non-mentalist, culturally embedded, concept of thinking that neither reduces the thinking subject to the mere product of discursive structures, nor posits it as a culturally-detached *res cogitans*. Section 3 is devoted to a discussion of the epistemological and ontological bases of the cultural theory here advocated. The concepts of learning and the mathematical classroom portrayed in this theory are presented in Sections 4 and 5, respectively. The main ideas of the previous sections are brought together in Section 6, where the educational questions surrounding the ethics of being and knowing are discussed.

1. THEORIES OF TEACHING AND LEARNING

Theories of teaching and learning differ from each other mainly in their conceptions about: (a) the content to be learned; (b) the learner; and (c) how learning actually occurs.

Concerning the third point, most contemporary theories have adopted the view according to which the student constructs his or her own knowledge (Lesh, Doerr, Carmona, & Hjalmarson, 2003). Although, in their account of learning, these theories do not necessarily exclude the role of the social, the social dimension of

L. Radford, G. Schubring, and F. Seeger (eds.), Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture, 215–234.

knowing is often reduced to a kind of external environment to which the cognitive activity of the student has to adapt. In these theories, assumed universal mechanisms of knowledge formation—e.g., the logical-mathematical structures of thinking in Piaget's genetic epistemology—account for the supposedly universal patterns of conceptual development. However, recent research in psychology, anthropology and other disciplines has pointed out the contextual nature of knowing and being (de Haan, 1999; Lave, 1988; Radford, 1997, 2003a, 2008a; Shweder and LeVine, 1984).[2] What this research makes clear is that cognition is much more complex than standard adaptive epistemologies intimate: cultural environments do indeed play a significant role in the ways we come to know and to be.

As to the second point, more often than not, theories that explain learning in adaptive terms share the same idea of the cognizing subject—an intrinsic rational auto-sustained individual maturing as it interprets and refines allegedly ethically neutral environmental feedback. This conceptualization of the cognizing subject leads to a narrow idea of the learner that Canadian psychologist Jack Martin describes in the following terms: a self-regulated individual whose "most vital resources are apparently available within its detached internality . . . a self that already knows its business, one that requires only a facilitative grooming to become more fully socialized and intellectually engaged" (Martin, 2004, p. 197). In short, what these theories convey is the problematic idea of a learner who "naturally" acts in a scientific, rational mindful manner.

Last but not least, adaptive explanations of learning mechanisms lead to important difficulties concerning the type of knowledge produced by adaptation. Indeed, the cognitive regulatory mechanisms of adaptation are usually conceived of in biological terms, with peripheral room for cultural considerations, leading to ahistorical and acultural accounts of knowledge. Often, biological premises are supplemented with a subjectivist interpretation of the individual's realm of experience. The result is a subjectivist interpretation of knowledge production. The best example is perhaps radical constructivism. In this theory, knowledge is merely made up of *personal* viable constructs. For many, however, this move is unconvincing: on the one hand, the idea of personal viability of knowledge leads to the unavoidable problem of solipsism (Lerman, 1996); on the other hand, radical constructivism gives up ontology (of any kind) and posits the *subjective* experiential realm as the limits of reason and knowledge.

At the educational level, radical constructivism has been criticized, among other things, for failing to account for the dissymmetric distribution of knowledge in the classroom. In a recent plenary lecture, Brousseau (2004) argued that "In didactics, radical constructivism is an absurdity." What Brousseau finds absurd in the radical constructivist position is not the claim that legitimate knowledge can only be the result of the individual's own achievement and deeds. What he finds erroneous is the idea that students' constructions necessarily lead to the institutional form of mathematical knowledge (*le savoir savant*). As Brousseau was able to observe over and over again in the classrooms of the Michelet School in Bordeaux, the students' subjective conceptual constructs require that an external perspective, among other things, *institutionalize* the knowledge arising from classroom mathematical activity.

The students cannot become aware of the cultural epistemic status of, say, a method arising as the result of their enquiring activity or, as Brousseau puts the matter, the students may not know that they know. The teacher hence has to encourage and highlight the kind of reasoning and the methods valued by the mathematicians' community.

These few comments provide an idea of some of the theoretical differences in current perspectives in mathematics education. Of course, the differences between theories are subtler than hinted at here. My interest is not to delve into these differences.[3] Rather my aim is to mention some focal points from where theoretical differences arise. In the rest of this paper, I present some elements of a theory of teaching and learning that takes its inspiration from some anthropological and historico-cultural schools of knowledge. This theory—the *theory of knowledge objectification*—relies on a non-rationalist epistemology and ontology, which gives rise, on the one hand, to an anthropological conception of thinking, and on the other, to an essentially social conception of learning.

2. A NON-MENTALIST CONCEPTION OF THINKING

2.1 Thinking as a Mediated Praxis Cogitans

Typically, thinking is understood as a kind of interior life, a series of mental processes on ideas carried out by the individual. This conception of thinking, as "mental activity" (de Vega, 1986, p. 439), comes from Augustine's interpretation of Greek philosophy at the end of the fourth century. For Augustine, ideas refer to something situated *inside of the individual*, contrary to the Greek tradition, where the term idea (*eidos*) referred to something external. Influenced by the Augustinian transformation of the Greek term, seventeenth-century rationalists such as Descartes and Leibniz believed that mathematics could be practiced even with one's eyes closed. As Leibniz put the matter, the principles that we need to understand objects or see their properties, the internal rules of reason, are "interior principles" that is, they are within our interior (Leibniz, 1704/1966, pp. 34–37). Anthropologists such as Clifford Geertz have demonstrated the limitations of the conceptualization of ideas as "things in the mind" or of thinking as an exclusively intracerebral process. Geertz (1973) claims that "The accepted view that mental functioning is essentially an intracerebral process, which can only be secondarily assisted or amplified by the various artificial devices which that process has enabled man to invent, appears to be quite wrong" (p. 76). He argues that "the human brain is thoroughly dependent upon cultural resources for its very operation; and those resources are, consequently, not adjuncts to, but constituents of, mental activity" (p. 76).

The conception of thinking as a kind of interior life has been very influential in the investigation of cognition in mathematics education. Written questionnaires, interviews, and drawing exercises have often been used to get a glimpse of what is going on *in* the head. To avoid the pitfalls of this mentalistic approach, some theories have simply discarded any psychological considerations. They simply avoid any talk about psychological constructs.

The theory of knowledge objectification adopts a non-mentalist position on thinking and intellectual activity. This theory suggests that thinking is a type of a social practice (Wartofsky, 1979), a *praxis cogitans*. To be more precise, thinking is considered to be *a mediated reflection in accordance with the form or mode of the activity of individuals*.

The Mediated Nature of Thinking The mediating nature of thinking refers to the role, in the Vygotskian sense, played by artifacts (objects, instruments, sign systems, etc.) in carrying out social practice (Bartolini Bussi & Mariotti, 2008). Artifacts are neither merely aids to thinking nor simple amplifiers, but rather constitutive and consubstantial parts of thinking. We think with and through cultural artifacts. The following example will help clarify this idea.

In a first-grade class in elementary school, the pupils had to solve a problem about a numeric sequence. The teacher introduced the problem through a story in which a squirrel, at the end of the summertime, brings two nuts to his new nest every day in preparation for the coming winter. In one part of the problem, the pupils had to determine how many nuts the squirrel had collected in his nest by the end of the tenth day, given the fact that there were already 8 nuts in the nest when the squirrel found it and that the squirrel never ate nuts from his winter provision. Christina, one of the pupils, began counting two by two: “ten, twelve, fourteen, sixteen.” When she noticed that she was not keeping track of the number of days that had passed, she started the count again. However, doing things simultaneously ended up being quite a difficult task. Addressing herself to Michael, her group mate, Christina said, “let’s do it together!” While the rest of the class continued working on the problem in small groups, Christina and Michael went to the blackboard and, using a large wooden ruler, Christina began counting two by two while Michael counted the days out loud.

In Figure 1 (left), when Michael says “nine,” Christina points with a wooden ruler to the number 26 on a number line placed above the blackboard, being the number of nuts the squirrel had collected by day 9. In Figure 1 (right), Michael, who continued counting the days, says “ten,” while Christina moves the ruler to the right and points to the number 28, finding the answer to the question in this way.

Figure 1. (Left picture) Michael says 9, and Christina points to number 26. (Right Picture) Michael says 10, and Christina points to number 28

The wooden ruler, the number line, the mathematical signs on the piece of paper that Michael holds up as he reads behind Christina, all are artifacts that *mediate* thinking. They are not merely aids: their mediating role is such that they *orient* and *materialize* thinking and, in so doing, become an *integral* part of it. Indeed, according to the theory advocated here, Christina and Michael's thinking is not something occurring merely in the students' mental plane. Thinking also occurs along the social plane, in a region that, paraphrasing Vološinov (1973), I want to call the *territory of artifactual thought*. It is within this territory that subjectivity and cultural objectivity mutually overlap and where the mind extends itself beyond the skin (Wertsch, 1991).

Thinking as Re-flection The reflexive nature of thinking means that the individual's thinking is neither the simple assimilation of an external reality (as the Empiricists and Behaviorists suggested) nor an *ex nihilo* construction (as certain constructivist schools claim). Thinking is a *re-flection*, that is, a dialectical movement between a historically and culturally constituted reality and an individual who refracts it (as well as modifies it) according to his/her own subjective interpretations, actions and feelings.

One of the roles of culture is to suggest ways of perceiving reality and its phenomena to students: literally, ways of intending (*manières de viser*), as Merleau-Ponty (1945) would say, or ways of intuiting, as Husserl (1931) might have it. In more general terms, the *re-flexivity* of thinking, from the phylogenetic point of view, consists in individuals giving rise to thinking and to the objects that thinking creates. However, at the same time, from the ontogenetic point of view, the thinking of individuals is, from the outset, subsumed by their cultural reality and by the historically formed concepts that they encounter in their environment. This is why we originate thinking, but at the same time become subsumed by it (Eagleton, 1997).

2.2 The Anthropological Dimension of Thinking

Thinking is not merely generated in the course of human activity. The *form* of the activity imprints its mark on thinking and in its product–i.e., knowledge. Now, the form that all activity takes depends on *symbolic superstructures*. These symbolic superstructures, which elsewhere I have called *Semiotic Systems of Cultural Signification* (Radford 2003a), include cultural conceptions surrounding mathematical objects (their nature, their way of existing, their relation to the concrete world, etc.) and social patterns of meaning production (see Figure 2). In their interaction with activities (their objects, actions, division of labour, etc.) and with the territory of artifactual thought, the *Semiotic Systems of Cultural Signification* give rise, on the one hand, to forms or *modes of activities*, and, on the other hand, to specific *modes of knowing* or *epistemes* (Foucault, 1966). While the first interaction gives rise to the particular ways in which activities are carried out at a certain historical moment, the second interaction gives rise to specific modes of knowing which allow for the identification of "interesting" situations or problems and the methods, reasoning, evidence, etc. that will be considered culturally valid.

Here is an example. The difference between the thinking of a Babylonian scribe and that of a Greek geometer cannot be reduced only to the kinds of problems with which they were respectively occupied or to the artifacts they used to think mathematically. The difference between their modes of thinking cannot be reduced to the fact that the Babylonian scribe was reflecting in a context tied to political and economic administration, whereas the Greek geometer was thinking within an aristocratic and philosophical context. The difference between the thinking of the Babylonian mathematician and that of the Greek one has to do with the fact that each one of these forms of thinking was underpinned by a particular *symbolic superstructure*. The thinking of the Babylonian scribe was framed by a realist pragmatism where mathematical objects such as "rectangle," "square," and so forth—objects which the Greek geometer of Euclid's time conceptualized in terms of Platonic forms or Aristotelian abstractions—acquired their meaning. The manner

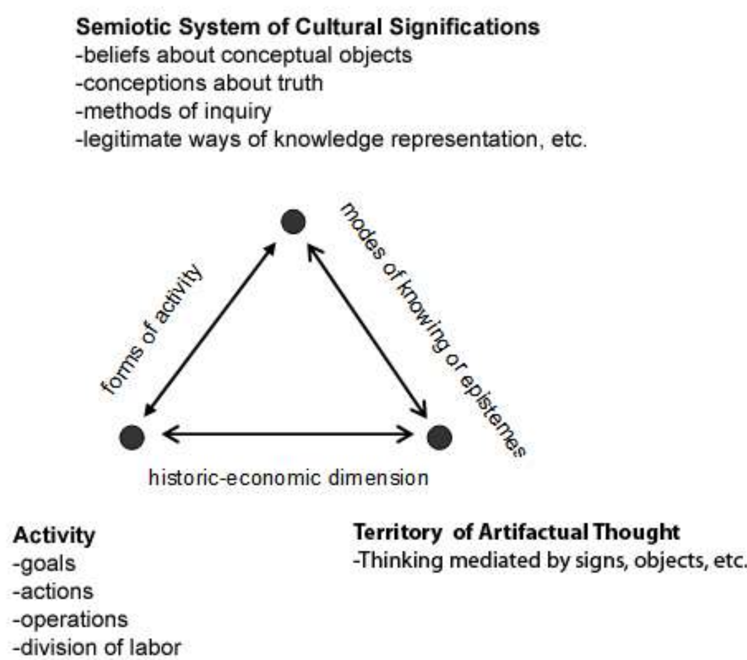

Figure 2. The arrows show the interaction between a Semiotic System of Cultural Significations, Activity and the Territory of the Artifactual Thought. The interaction generates the forms of activity and the modes of knowing on the base of the specific historic-economic dimension. In a dialectic process, forms of activity, modes of knowing, and the historic-economic dimension alter the triangle's vertices

in which the Babylonian scribe, the Greek geometer and the Renaissance abacist ended up thinking about and knowing objects of knowledge, the way in which they tackled their problems and considered them to be solved, all were framed by the very form of their activity and the corresponding cultural episteme (Radford, 1997, 2003a, 2003b).

Rather than seeing these (and other) historical and contemporary forms of mathematical thinking as "primitive" or "imperfect" versions of current mathematical thought (ethnocentrism), the anthropological dimension of the theory of knowledge objectification considers these forms as belonging to particular, genuine types of mathematics in their own right.

3. THE EPISTEMOLOGICAL AND ONTOLOGICAL BASES OF THE THEORY OF KNOWLEDGE OBJECTIFICATION

Any didactic theory, at one moment or another (unless it voluntarily wants to confine itself to a kind of naïve position), must clarify its ontological and epistemological position. The *ontological* position consists in specifying the sense in which the theory tackles the question of the nature of conceptual objects (in our case, the nature of mathematical objects, their forms of existence, etc.). The *epistemological* position consists in specifying the way in which, according to the theory, these objects can (or cannot) end up being known.

Often, contemporary didactic theories that start from an application of mathematics to the experiential world, adopt—even if it is not mentioned explicitly—a realist ontology and deal with the epistemological problem in terms of abstractions. Naturally, the situation is not that simple, as Kant himself recognized. As for Realism—which, in an important way, is the Platonist version of the instrumental rationalism (Weber, 1992) which emerged during the Renaissance—the existence of mathematical objects precedes and is independent from the activity of individuals. Like the Platonist, the Realist believes that mathematical objects exist independently of time and culture. The difference is that, whereas Platonic objects do not mix with the world of mortals, the conceptual objects of the Realist govern our world through "natural laws." According to realist ontology, this explains the miracle that is the applicability of mathematics to our phenomenal world (Colyvan, 2001). Naturally, in order to achieve this, Realism makes a leap of faith that consists in believing that the abstractive ascent from the concrete objects of sensuous experience to general, pre-existing objects is certainly possible. The faith that Plato placed in reasoned social discourse (*logos*) and which Descartes placed in cogitating with oneself are subjected to scientific experimentation by Realism.

The ontological and epistemological position of the theory of knowledge objectification moves away from Platonist and realist ontologies and their corresponding conception of mathematical objects as eternal objects preceding the activity of individuals. It also moves away from Rationalist ontologies and their conception of mathematical objects as products of a mind that works folded in onto itself working in accordance to the laws of logic. The theory of knowledge objectification suggests that mathematical objects are historically generated during the course of the mathematical activity of individuals. More precisely, mathematical objects *are fixed patterns of reflexive human activity incrusted in the ever-changing world of social practice mediated by artifacts*.

The conceptual object "circle," for example, is a fixed pattern of activity whose origins cannot be found in the intellectual contemplation of (more or less) round objects that the first individuals would have encountered in their surroundings. The conceptual object "circle" must rather be found in the *sensual* and *practical* activity that led individuals to notice the emergent object:

> People could see the sun as round only because they rounded clay with their hands. With their hands they shaped stone, sharpened its borders, gave it facets. (Mikhailov, 1980, p. 199)

This sensual experience of labour has remained fixed in language which encapsulates original meanings, so that

> the meaning of the words "border," "facet," "line" does not come from abstracting the general external features of things in the process of contemplation. (p. 199)

but rather comes from the activity of labour that has been taking place since the origins of humanity. Far from surrendering itself completely to our senses, our relationship with nature and the world is filtered through conceptual categories and cultural significations so that

> man could contemplate nature only through the prism of all the social work-skills that had been accumulated by his predecessors. (p. 199)

4. LEARNING

In the previous sections we have seen that, from a phylogenetic point of view, conceptual objects are generated in the course of human activity. From an ontogenetic point of view, the central problem is to explain how acquisition of the knowledge deposited in a culture can be achieved: this is a fundamental problem of mathematics education in particular and of learning in general.

As mentioned in the Introduction, classical theories of mathematical education posit the problem of learning in terms of a construction or re-construction of knowledge on the part of the student. I already mentioned some of the difficulties that arise from such a perspective at the ontological and epistemological levels (for a detailed discussion, see Radford, 2008d). Here, I just want to add that the idea that knowledge has to be "constructed" by each individual has its own history. Actually, such an idea only became thinkable after the Renaissance and did not receive a full and explicit articulation before the 18th century. It was in Kant's work that the idea was expressed with unprecedented detail. For Kant, the individual is not only an introspective thinker whose mental activity brings him mathematical truths, as upheld by the rationalists (Descartes, Leibniz, etc.); nor is he only a passive individual who receives sensory information in order to formulate ideas, as proposed by the Empiricists (Hume, Locke, etc.). For Kant, the individual is a craftsman of his/her own knowledge (Radford, 2006a). In formulating the relationship between subject and object in this way, Kant expressed, in a coherent and explicit manner, the epistemological change that had been gradually taking place since the appearance of manufacturing and the emergence of capitalism during the Renaissance. The reasons of this epistemological change can be summarized in the following way. The modern era is marked by a displacement in the conception of knowledge clearly manifested in a shift that went from a focus on "the what" (the object of knowledge) to "the how" (the process). Unlike the medieval individual, the modern individual can only know that which he/she himself/herself has made (Arendt, 1958). This idea arose out of the political and economic climate surrounding the Enlightenment, one of its chief characteristics being the opposition it presented to tradition and religion. From the 18^{th} century onward, knowledge was no longer conceived of as something to be received or passed on, but something made by the autonomous, rational, auto-sufficient, culturally-detached Enlightened self: within this individualist tradition, knowledge was hence considered as a purely personal "construction".

Although historically interesting, the idea that every piece of knowledge is necessarily a personal construction has been subjected to a series of critiques, in particular because of the tremendous subjectivism that it entails (Adorno, 2001). If we were really meant to construct everything we know, we would still be trying to light some fire in front of a dark cavern, as the French biologist and philosopher Henri Laborit (1985) once suggested. And, of course, the idea of knowledge as a personal construction is even more problematic in education (Lerman, 1996; Radford, 2008d). Constructing knowledge by oneself is certainly *one* form of knowing among others. But stating that this is the only possible one fails to capture the diversity of cognitive forms of learning, knowing and being that can be found in the mathematics classroom.

According to the theory of knowledge objectification, learning does not consist in constructing or reconstructing a piece of knowledge. It is a matter of actively and imaginatively endowing the conceptual objects that the student finds in his/her culture with meaning. It is what we will later call a process of objectification. For the moment, we need to discuss two important sources of meanings that underlie all forms of learning.

4.1 The Knowledge Deposited in Artifacts

One of the sources of learning results from our contact with the material world, the world of cultural artifacts which surrounds us (objects, instruments, etc.). The Grade 1 example discussed previously illustrated the artifact mediated nature of thinking and led us to argue that thinking occurs in a zone that was called the Territory of Artifactual Thought. But the role of artifacts is more than materializing thinking and making it thinking-with-and-through-artifacts. Artifacts, indeed, are bearers of historically deposited knowledge from the cognitive activity of previous generations. Although it is true that some animals are able to use artifacts (Boesch & Boesch, 1990; Torigoe, 1985), nevertheless, for animals, artifacts do not end up acquiring a durable meaning. The wooden stick that a chimpanzee uses to reach a piece of fruit looses its meaning after the action has been executed (Köhler, 1951). It is for this reason that animals do not preserve artifacts.[4] Furthermore—and this is a fundamental element of human cognition—unlike animals, the human being is profoundly altered by the artifact: by making contact with it, the human being restructures his/her movements (Baudrillard, 1968) and new motor and intellectual skills are formed such as anticipation, memory, and perception (Vygotsky & Luria, 1994).

The world of artifacts appears, then, to be an important source for the process of learning, but it is not the only one. Objects cannot make clear the historical intelligence that is imbedded in them. This requires that they be used in activities as well as in contact with other people who know how to "read" this intelligence and help us to acquire it. Symbolic-algebraic language would otherwise be reduced to a group of hieroglyphics. The intelligence that symbolic-algebraic language carries would not be noticed without the social activity that takes place in the school. It is this social dimension which constitutes, for the theory of knowledge objectification, the second essential source for learning.

4.2 Social Interaction

Even though the importance of the social dimension has been underlined by a great number of recent studies on classroom interaction, there are subtle differences with regards to its cognitive contribution (see, e.g., Kidron, Lenfant, Bikner-Ahsbahs, Artigue, and Dreyfus, 2008; Yackel and Cobb, 1996; Sierpinska, 1996; Steinbring, Bartolini Bussi and Sierpinska, 1998). Often, interaction is considered as a negotiation of meanings occurring in an environment that simply offers the stimuli of adaptation that are required for students' cognitive development. The problem is that the classroom is not merely a material space where the students negotiate and find an environment to adapt themselves; it is not only a matter of "external" conditions to which the subject must accommodate his/her activity. The crucial point is that the classroom is a *symbolic space*. It is a space that conveys scientific, aesthetic, ethical and other historically constituted cultural values impressed in "social languages" (Bakhtin, 1986) such as the scientific, the artistic, etc. that end up affecting individuals' actions and reflections. As was mentioned in the first part of this article, the actions that individuals carry out are submerged in cultural modes of activity. It is for this reason that the classroom cannot be viewed as an enclosed space, folded over against itself, where knowledge and rules of interaction are negotiated anew. In fact, knowledge and rules of social interaction have a whole cultural history behind them and therefore pre-exist the interaction that takes place in the classroom.

According to the sociocultural perspective advocated here, interaction plays a different role. Rather than performing a merely adaptive function—a catalyzing or facilitating one—interaction is consubstantial to learning. Therefore, we see that the material world and the social dimension play a basic role in learning. The allocation of meaning that rests on these dimensions has profound psychological importance inasmuch as it is both a progressive immersion into cultural forms of thinking as well as the process of development of the specific capacities of the individual—cognitive, ethical, subjective, etc. It is for this reason that learning is not merely appropriating something or assimilating something; rather, it is the very process by which our human capacities are formed (I shall come back to this point below).

4.3 Objectification and Subjectification

In the previous sections it was suggested that learning consists of endowing conceptual cultural objects with meaning. In fact, learning is much more than that. Learning rests on an attitude of open-mindedness: it is an opening movement towards others and the objects of culture. It is worth noticing that this is, in fact, the etymological sense of the term *acquisition*. Acquisition comes from the Latin *adquaerere,* which means *to seek*. In this context, to learn is not merely to acquire something in the corrupted sense of possessing it or mastering it, but to go to culture to find "something" in it. This is why the outcome of the act of learning is not the construction, re-construction, re-production, re-invention or mastering of concepts: its true outcome is to be found in the fact that, in this encounter with the other and cultural objects, the seeking individual *finds herself*. This creative process of finding or noticing something (a dynamic target) is what I have termed elsewhere a process of *objectification* (Radford, 2002).

As understood here, objectification thus is more than the connection of the two classical epistemological poles, subject and object: it is in fact a transformative and creative process between these two poles, where, in the course of learning, the subject objectifies cultural knowledge and, in so doing, finds itself objectified in a reflective move that can be termed *subjectification*. The making of the subject, the creation of a particular (and unique) subjectivity is thus a process of subjectification that is made possible by the activity in which objectification takes place, and by the *re-flective* nature of thinking and the possibilities that e.g. language and other cultural instruments of thought offer to distinguish between an "I" and its surroundings (I/non-I; I/you; I/it; we/them, the impersonal discourse of science, etc.). In the culturally mediated experience that a subject '*s*' makes of an object '*o,*', '*s*' comes to know '*o'* within the possibilities and constraints offered by the dynamic and ever-changing cultural-normative sphere of knowledge. In knowing '*o'*, '*s*' enters into a historically mediated relationship with, '*o'* and other subjects 's_i'. This historically mediated relationship not only makes the object '*o'* *noticeable* to '*s*' but also '*s*' to itself through the available forms of subjectivity and agency of the culture. This is why learning is both a process of knowing and a process of becoming.

5. THE MATHEMATICS CLASSROOM

5.1 Learning Activity

A central element of the concept of activity is its objective (Leont'ev, 1978). Even though the objective of classroom activity is clear for the teacher, this is not necessarily the case for the students. If the objective were to be clear to them, then there would be nothing left for them to learn. Within the didactic project in the class, the teacher suggests a series of mathematical tasks to the students so that a given objective can be achieved. Solving these problems becomes an end that directs the actions of the students. However, from the perspective of the theory of knowledge objectification, doing mathematics is more than doing tasks and solving problems. Without devaluing the role of problems in knowledge formation (see, for example, Bachelard, 1986), for us, problem solving is not the end but rather one of the means for achieving the type of *praxis cogitans* or cultural reflection that we call mathematical thinking. Behind the objective of the lesson, there lies a greater and more important objective—the generally held objective for the teaching and learning of mathematics—namely, the elaboration on the part of the student of a reflection defined as a *communal* and active relationship with his/her cultural-historical reality. Unfortunately, the learning of mathematics has often been reduced to merely obtaining a certain conceptual content. Knowledge has been reduced to a sort of *commodity*. This fetishist conception of learning operates a separation between knowing and being and ends up favouring an alienating form of being. The cultural theory of teaching-and-learning advocated here resists such a separation and argues for a reconnection between knowing and being. In other words, learning mathematics is not simply learning *to do* mathematics (problem solving), but rather it is learning *to be* in mathematics. This theoretical stance has important consequences, not only for the designing of activities, but also for the organization of the class itself and the roles that students and teachers play within it.

5.2 Layers of Generality

Teaching consists of generating and keeping in movement contextual activities which are heading toward inter-subjectively engaging the students with conceptual objects—fixed patterns of reflexive activity incrusted in the culture. This movement has three essential characteristics. First, the conceptual object is not a monolithic or homogenous object. It is an object made up of layers of generality. Second, from the epistemological point of view, these layers will be more or less general depending on the characteristics of the cultural meanings of the fixed pattern of activity in question (for example, the kinaesthetic movement that forms a circle; the symbolic formula that expresses it as a group of points at an equal distance from its centre, etc.). Third, from the cognitive point of view, the layers of generality are noticed in a progressive way by the student.

For the student, the learning process consists in becoming receptive to others and fluidly conversant with the various layers of generality of the object and their enabling forms of action—e.g. techniques and reflections on these techniques

(Bosch & Chevallard, 1999), modelling (Lesh et al., 2003), etc. Now, in order to get to know objects and products of cultural development, it is "necessary to carry out a determined activity around them, that is to say, a kind of activity that produces its essential characteristics, embodied, 'accumulated' in said objects" (Leontiev, 1968, p. 21). Thus, for the teacher, the teaching process consists in offering the students rich activities featuring, in a suitable manner, the encounter with other voices and the various layers of generality of the cultural object, making sure that this encounter is supported by the two sources of meaning discussed in Section 3—namely, meaning in artifacts, and meaning arising out of social interaction between students and between students and the teacher.

Because of the artifactual and embodied nature of thinking (Arzarello, Edwards, and Radford, 2008; Nemirovsky, 2003; Roth, 2001; Seitz, 2000), in the course of their objectification of knowledge, students and teachers use signs and artifacts of different sorts (mathematical symbols, graphs, words, gestures, calculators and so on). We call these artifacts and signs used to objectify knowledge *semiotic means of objectification* (Radford, 2003c). In previous works, we have discussed the prominent role of gestures and language in students' processes of knowledge objectification. We have provided evidence of the key role of deictic activity, both at the level of gestures, such as pointing, and at the level of language, such as when students use indexical terms such as *this* and *that* (Radford, 2002) and showed how, through various types of semiotic means of objectification, the students reach different layers of generality (Radford, 2000, 2003c, 2006b, 2008e; Radford, Bardini, & Sabena, 2007; Sabena, Radford, & Bardini, 2005). The investigation of the students' and teachers' interaction and use of semiotic means of objectification is indeed a methodological strategy to account for the processes of learning in the classroom. It provides a broad, but sufficiently specific, frame with which to track students' progressive acquisition of cultural forms of mathematical being and thinking.

6. THE ETHICS OF BEING AND KNOWING

The classroom is the symbolic space in which the student elaborates a communal and active relation with his/her historical-cultural reality. It is here that the aforementioned encounter between the subject and the object of knowledge occurs. The objectification that allows for this encounter is not an individual process but a social one. The sociability of the process, nevertheless, cannot be understood as a simple business "negotiation" during which each stakeholder invests some capital (e.g. some meaning) in the hopes of ending up with more of it. Here, sociability means the process of the formation of consciousness which Leont'ev (1978), paraphrasing Vygotsky, characterized as *co-sapientia*, that is to say, as knowing in common or knowing-with others.

Naturally, these ideas imply a re-conceptualization of the student and his/her role in the act of learning. Insofar as current theories in mathematics education draw on the concept of the individual as formulated by Kant and other Enlightenment philosophers, education justifies itself by guaranteeing the formation of an autonomous subject (understood in the sense of being able to do something for oneself without the help of others). Autonomy is, in effect, a central theme of modern education (Piaget, 1973) that has served as a basis for the theorizing of socio-constructivism (see, for example, Yackel and Cobb, 1996) and the theory of didactic situations (Brousseau, 1986; Brousseau and Gibel, 2005). The rationalism that weighs heavily on this concept of autonomy comes from its alliance with another key Kantian concept: that of freedom. There can be no autonomy without freedom and, for Kant, freedom means the convenient use of Reason according to its own universal principles (Kant, 1797/1974).

Since the Enlightenment did not put forward the possibility of there being a multiplicity of reasons, but rather postulated that western reason was The Reason, community coexistence implies respect for a duty which, in the end, is nothing but a manifestation of that alleged universal reason, whose epitome is mathematics. It was this supposed universality of reason that led Kant to fuse together the ethical, political and epistemological dimensions of life and to affirm that "to do something for the sake of duty means obeying reason" (Kant, 1803, p. 37).

For the theory of knowledge objectification, classroom functioning and the role of the teacher are not meant to promote the Enlightened individualistic idea of autonomy. The theory of knowledge objectification pleads for an idea of subjectivity and the self that goes beyond the ahistorical individualism inherited from the Enlightenment. It seeks rather to promote a concept of the autonomous person that is sensitive to the importance of history, the context and others, and where autonomy is both self-fulfilment and social commitment.

In her studies of Ancient Greek and Roman cultures, Arendt has shown that, in opposition to the modern idea of autonomy as something that comes from within—a personal and individual attribute coterminous with free will—autonomy for the Greek and Roman citizen had a social-civic connotation: it was related to action in the public sphere; it was a characteristic of human existence in the world. Unfortunately, Arendt comments, "Our philosophical tradition is almost unanimous in holding that freedom begins where men have left the realm of political life inhabited by the many, and that it is not experienced in association with others but in intercourse with one's self" (Arendt, 1993, p. 157).

Despite its legendary endurance in Western thought, non-individualistic conceptions of autonomy and freedom are frequently reported in contemporary anthropological research. Anthropologist Richard Shweder (1991) notes that

> not all cultures socialize autonomy or redundantly confirm the right of the individual to projects of personal expression, to a body, mind, and room's of one's own. . . . Linked to each other in an interdependent system, members . . . take an active interest in one another's affairs and feel at ease in regulating and being regulated. Indeed, others are the means to one's functioning, and vice versa. (p. 154)

It is indeed along the lines of a communal engagement, displayed in the public sphere, that autonomy might be better conceptualized, for, as the French philosopher Emmanuel Lévinas (1976) reminds us, "It is not through a relationship with the self, but through a relationship with another self, that man can become complete" (p. 31) and also, certainly, free and autonomous.

Instead of the idea of the self-regulated Enlightened individual common to many contemporary theories in education, the theory of knowledge objectification suggests the idea of a communitarian self, one busy with learning how to live in the community that is the classroom, learning how to interact with others, to opening oneself up to understanding other voices and other consciousnesses, in brief, *being-with-others*.[5]

The intrinsic social nature of knowledge and mathematical thinking has brought us then to conceiving of the classroom as an ethical and political space—the ethico-political space of the continuous renewing of being and knowing.[6] In our research with teachers, we encourage the students to work towards the creation of opportunities for the personal achievement of each individual, fostering respect, critique and mutual understanding (which includes understanding disagreements). We encourage them to show commitment to others and their community. Their community has to be flexible in its ideas and its forms of expression and be open to resistance and subversion in order to insure: modification, change and its transformation.

Being a member of the community, however, is not something that comes as a matter of course. Thus, in one of the classes we have worked with (a Grade 5 class), we encouraged the students to discuss mathematical problems in small groups (usually groups of 2 to 4 students). It was not unusual to invite the small groups to contrast their mathematical arguments in order to end up with more sophisticated ideas. During these inter-group encounters, the students were supposed to listen to the other groups' arguments, make sense of them and explain whether or not the argument could be improved. A student from one group whispered to a student from another group: "We'll let them fight for it!" It takes time (a long time indeed!) to make students aware of other non self-interested and alienating modes of classroom participation. In order to be a community member, students are encouraged to: share in the objectives of the community; involve themselves in the classroom activities; communicate with others (Radford and Demers, 2004). The abovementioned guidelines are not simply codes of conduct. On the contrary, they are indexes of forms of being in mathematics (and, as a consequence, of knowing mathematics) in the strictest sense of the term.

CONCLUDING REMARKS

The theory of knowledge objectification suggests a view of teaching and learning anchored in the idea that learning is a social activity (*praxis cogitans*) deeply rooted in historically constituted cultural forms of thinking and being. Its fundamental principles are articulated according to five interrelated concepts. The first of these is a concept of a psychological order: the concept of *thinking*, elaborated in non-mentalist terms. The second concept of the theory is of a socio-cultural order. This is the concept of learning. The third concept of the theory is of an epistemological nature and deals with those super-epistemic aspects that frame learning in the form of *semiotic systems of cultural signification*—cultural systems that "naturalize" the ways that one questions and investigates the world. The aforementioned concepts come to be completed by a fourth concept of an ontological nature—that of mathematical objects, which we have defined as *fixed patterns of reflexive activity incrusted in the ever-changing world of social practice mediated by artifacts*. To render the theory operational in its ontogentic aspect, it was necessary to introduce a fifth concept of a semiotic-cognitive nature—that of objectification, or a subjective awareness of the cultural object. In this context, and in light of the previous fundamental concepts, learning is defined as the social process of *objectification* of those external patterns of action fixed in the culture. Objectification entails another process, the process of *subjectification*—i.e., the becoming of the self. Subjectification has received very little attention in the literature on mathematics education (in addition to M. Fried's chapter in this volume, some works are: Brown, in press; Lerman, 1996; Popkewitz, 2004). However, its importance is easy to grasp as soon as we resist the temptation to reduce mathematics to its technical dimension and become aware that learning is much more than constructing logico-mathematical mental structures or picking up ready-made knowledge, that is to say, if and when we become aware of the fact that learning is about knowing *and* being. Objectification and subjectification should in fact be seen as two mutually constitutive processes leading to students' engagement with cultural forms of thinking and a sensibility to issues of interpersonal respect, plurality, inclusiveness and other main characteristics of the *communitarian self* (Radford, 2006a).

Such a view requires us to move away from traditional epistemologies where learning and knowing are the outcomes of a detached self, moved by its own business and interests—as, e.g., in radical constructivism, where the Self reduces the Other to a practical concern: one grounded in the individual subject's need for other people to corroborate its own constructions (see von Glasersfeld, 1995, p. 126). Moving away from traditional epistemologies means acknowledging that there is much more to others than the echoes they refract of our own cogitations. In order to teach and learn mathematics, we need to move beyond the standard noetic-noematic correlations between subject and object and to acknowledge that alterity, that is, the relationship to the Other, is not, as Levinas (1989) noted, of a conceptual order; for, the other cannot be dealt with through the same methods and forms of representation as conceptual objects. The relation to the other is one of solidarity, commitment and engagement, a relationship that challenges the reduction of the other to the same.

ACKNOWLEDGMENTS

A part of this paper was presented at the Working Group of Theories in Mathematics Education at the *Fifth Congress of the European Society for Research in Mathematics Education* (CERME 5), Cyprus, 2006. A partial version of it appeared in *Revista Latinoamericana de Matemática Educativa* (2006). I am grateful to the CERME Working Group participants and the reviewers for their insightful comments and critiques.

NOTES

[1] This article is a result of a research program funded by The Social Sciences and Humanities Research Council of Canada / Le Conseil de recherches en sciences humaines du Canada.

[2] See also Crombie (1995), Høyrup (2007) and Lizcano (1993).

[3] For some recent discussions on these differences see e.g. Bikner-Ahsbahs and Prediger (2006), Cobb (2007), Lerman (2006), Lester (2005), Niss (1999), Radford (2008b, 2008c), Silver and Herbst (2007), and Sriraman and English (2005).

[4] New Caledonian crows are an exception (see Bluff, Weir, Rutz, Wimpenny, & Kacelnik, 2007). I am indebted to Michael Roth for bringing this fact to my attention.

[5] The essence of Being, argues Jean-Luc Nancy, is *to-be-with*. “The one/the other is neither ‘by’, nor ‘for’, nor ‘in’, nor ‘despite’, but rather “with’.” (Nancy, 2000, p. 34).

[6] The moral and ethical dimensions of the classroom and everyday human praxis have been recently emphasized by Roth (2007a, 2007b). Valero and Zevenbergen (2004) have stressed the importance of taking into account the social and political dimensions in mathematics education.

REFERENCES/BIBLIOGRAPHY

Adorno, T. W. (2001). *Kant’s critique of pure reason*. Stanford, CA: Stanford University Press.

Arendt, H. (1958). *The human condition*. Chicago: The University of Chicago Press.

Arendt, H. (1993). *Between past and future*. (Reprint of the 1968 ed. published by The Viking Press). New York: Penguin Books.

Arzarello, F., Edwards, L., & Radford, L. (2008). Gesture and multimodality in the construction of mathematical meaning. Special Issue, *Educational Studies in Mathematics* (forthcoming).

Bachelard, G. (1986). *La formation de l’’esprit scientifique*. Paris: Vrin.

Bakhtin, M. (1986). *Speech genres and other late essays*. Austin, TX: University of Texas Press.

Bartolini, B.ussi M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective. In L. English (Ed.), *Handbook of international research in mathematics education* (2nd ed.ition). Mahwah, N.J.: Lawrence Erlbaum Associates. (in press).

Baudrillard, J. (1968). *Le système des objets [The system of objects]*. Gallimard, Paris: Gallimard.

Bikner-Ahsbahs, A., & Prediger, S. (2006). Diversity of theories in mathematics education - How can we deal with it? *ZDM - The International Journal on Mathematics Education*, *38*(1), 52–57.

Bluff, L. A., Weir, A. A. S., Rutz, C., Wimpenny, J. H., & Kacelnik, A. (2007). Tool-related cognition in new Caledonian crows. *Comparative Cognition & Behavior Reviews*, 2: 1–25.

Boesch, C., & Boesch, H. (1990). Tool use and tool making in wild chimpanzees. *Folia Primatologica*, *54*, 86–99.

Bosch, M., & Chevallard, Y. (1999). La sensibilité de l’’activité mathématique aux ostensifs. *Recherches en Didactique des Mathématiques, 19*(1), 77–124.

Brousseau, G. (2004). *Une modélisation de l’enseignement des mathématiques*. Plenary Lecture delivered at Convegno di didattica della matematica, September 24–25, 2004, Locarno, Switzerland.

Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques,. *Recherches en Didactique des Mathématiques*, *7*(2), 33–115.

Brousseau, G., and & Gibel, P. (2005). Didactical handling of students’ reasoning processes in problem solving situations. *Educational Studies in Mathematics*, *59*, 13–58.

Brown, T. (in press). Lacan, subjectivity, and the task of mathematics education research. *Educational Studies in Mathematics*.

Cobb, P. (2007). Putting philosophy to work. Coping with multiple theoretical perspectives. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 3–38). Greenwich, CT: Information Age Publishing.

Colyvan, M. (2001). The miracle of applied mathematics. *Synthese*, *127*, 265–277.
Crombie, A. C. (1995). Commitments and styles of European scientific thinking. *History of Sciences*, *33*, 225–238.
de Haan, M. (1999). *Learning as cultural practice*. Amsterdam: Thela/Thesis.
de Vega, M. (1986). *Introduccion a la psicologia cognitiva [Introduction to cognitive psychology]*. Mexico: Alianza.
Eagleton, T. (1997). *Marx*. London: Phoenix.
Foucault, M. (1966). *Les mots et les choses*. Paris: Éditions Gallimard.
Geertz, C. (1973). *The interpretation of cultures*. New York: Basic Books.
Høyrup, J. (2007). The roles of Mesopotamian bronze age mathematics tool for state formation and administration. *Educational Studies in Mathematics*, *66*(2), 257–271.
Husserl, E. (1931). *Ideas*. London: George Allen & Unwin.
Kant, I. (1797/1974). *Anthropology from a pragmatic point of view*: (Translated by Mary J. Gregor, Trans.). The Hague: Martinus Nijhoff.
Kant, E. (1803). *On Eeducation*. Boston: D. C. Heath. (Retrieved, on June 11, 2006, from: http://oll.libertyfund.org/EBooks/Kant_0235.pdf)
Kidron, I., Lenfant, A., Bikner-Ahsbahs, A., Artigue, M., & Dreyfus, T. (2008). Social interaction in learning processes as seen by means of three theoretical frameworks. *ZDM - The International Journal on Mathematics Education*. (in press).
Köhler, W. (1951). *The mentality of apes*. New York: Routledge and Kegan Paul.
Laborit, H. (1985). *Éloge de la fuite*. Paris: Gallimard.
Lave, J. (1988). *Cognition in practice*. Cambridge: Cambridge University Press.
Leibniz, G. W. (1704/1966). *Nouveaux essais sur l'entendement humain*. (published 1765). Paris: Flammarion.
Leont''ev, A. N. (1978). *Activity, consciousness, and personality*. New Jersey: Prentice-Hall.
Leontiev [or Leont''ev], A. N. (1968). El hombre y la cultura. In *El hombre y la cultura: pProblemas téoricos sobre educación* (pp. 9–48). México: Editorial Grijalbo.
Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education*, *27*(2), 133–150.
Lerman, S. (2006). Socio-cultural research in PME. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future*. Rotterdam: Sense Publishers.
Lesh, R., Doerr, H., Carmona, G., & Hjalmarson, M. (2003). Beyond constructivism. *Mathematical Thinking and Learning*, *5*(2–3), 211–233.
Lester, F. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *ZDM - The International Journal on Mathematics Education*, *37*(6), 457–-467.
Lévinas, E. (1976). *Noms Propres*. Paris: Fata Morgana.
Levinas, E. (1989). *The Levinas reader*. (Edited by (S. Hand, Ed.)). Oxford: Blackwell.
Lizcano, E. (1993). *Imaginario colectivo y creación matemática [Collective imaginary and mathematical creation]*. Barcelona: Editorial Gedisa.
Martin, J. (2004). The educational inadequacy of conceptions of self in educational psychology. *Interchange: A Quarterly Review of Education*, *35*, 185–-208.
Merleau-Ponty, M. (1945). *Phénomenologie de la perception*. Paris: Gallimard, Paris.
Mikhailov, F. (1980). *The riddle of the self*. Moscow: Progress Publishers.
Nancy, J.-L. (2000). *Being singular plural* (R. D. Richardson & A. E. O'Byrne, Trans.). Stanford, CAalifornia: Stanford University Press.
Nemirovsky, R. (2003). Perceptuo-motor activity and imagination in mathematics learning. In N. Pateman, B. Dougherty, & J. Zilliox (Eds.), *Proceedings of the 27th Cconference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 103–-135). University of Hawaii.

Niss, M. (1999). Aspects of the nature and state of research in mathematics education. *Educational Studies in Mathematics*, *40*, 1–-24.

Piaget, J. (1973). *To understand is to invent. The future of education*. New York: Grossman.

Popkewitz, T. (2004). School subjects, the politics of knowledge, and the projects of intellectuals in change. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education* (pp. 251–-267). Boston: Kluwer.

Radford, L. (1997). On psychology, historical epistemology and the teaching of mathematics. *for the learning of mathematics*, *17*(1), 26–-33.

Radford, L. (2000). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis. *Educational Studies in Mathematics*, *42*(3), 237–-268.

Radford, L. (2002). The seen, the spoken and the written. A semiotic approach to the problem of objectification of mathematical knowledge. *for the learning of mathematics*, *22*(2), 14–23.

Radford, L. (2003a). On culture and mind. A post-Vygotskian semiotic perspective, with an example from Greek mathematical thought. In M. Anderson, A. Sáenz-Ludlow, S. Zellweger, & V. Cifarelli (Eds.), *Educational perspectives on mathematics as semiosis: From thinking to interpreting to knowing* (pp. 49–-79). Ottawa: Legas Publishing.

Radford, L. (2003b). On the epistemological limits of language: Mathematical knowledge and social practice in the Renaissance. *Educational Studies in Mathematics*, *52*(2), 123–-150.

Radford, L. (2003c). Gestures, speech and the sprouting of signs. *Mathematical Thinking and Learning*, *5*(1), 37–-70.

Radford, L. (2006a). Communication, apprentissage et formation du *je communautaire*. In B. D'Amore & S. Sbaragli (eEds.), *20th Italian national conference Incontri con la Matematica*, November 3–-5, 2006. (Plenary Lecture available at: http://laurentian.ca/educ/lradford/).

Radford, L. (2006b). Algebraic thinking and the generalization of patterns: A semiotic perspective. In S. Alatorre et al. (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, North American Chapter* (Vol. 1, pp. 2–21),. Mérida, Vol. 1, pp. 2-21.

Radford, L. (2008a). Culture and cognition: Towards an anthropology of mathematical thinking. In L. English (Ed.), *Handbook of International Research in Mathematics Education (2nd Edition)* (pp. 439 - 464). New York: Routledge, Taylor and Francis.

Radford, L. (2008b). Connecting theories in mathematics education: Challenges and possibilities. *ZDM - The International Journal on Mathematics Education, 40*(2), 317-327.

Radford, L. (2008c). Theories in Mathematics Education: A Brief Inquiry into their Conceptual Differences. Working Paper. Prepared for the ICMI Survey Team 7. The notion and role of theory in mathematics education research. Retrieved from http://www.laurentian.ca/educ/lradford/.

Radford, L. (2008d). Di Sé e degli Altri: Riflessioni su un problema fondamentale dell'educazione [The self and the other: Reflections on a fundamental problem in education]. *La Matematica e la sua didattica, 22*(2), 185-205.

Radford, L. (2008e). Iconicity and contraction: A semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM - The International Journal on Mathematics Education*, *40*(1), 83–-96.

Radford, L., Bardini, C., & Sabena, C. (2007). Perceiving the general: The multisemiotic dimension of students' algebraic activity. *Journal for Research in Mathematics Education*, *38*, 507–-530.

Radford, L., & Demers, S. (2004). *Communication et apprentissage. Repères conceptuels et pratiques pour la salle de classe de mathématiques*. Ottawa: Centre franco-ontarien des ressources pédagogiques.

Roth, M.-W. (2001). Gestures: Their role in teaching and learning. *Review of Educational Research*, *71*(3), 365–-392.

Roth, W.-M. (2007a). Identity in scientific literacy: Emotional-volitional and ethico-moral dimensions. In W.-M. Roth & K. Tobin (Eds.), *Science, learning, and identity: Sociocultural and cultural historical perspectives* (pp. 153–-184).

Roth, W.-M. (2007b). The ethico-moral nature of identity: Prolegomena to the development of third-generation cultural-historical activity theory. *International Journal of Educational Research*, *46*, 83–-93.

Sabena, C., Radford, L., and & Bardini, C. (2005). Synchronizing gestures, words and actions in pattern generalizations,. *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 129–136),. Australia, Vol. 4, pp. 129–-136.

Seitz, J. A. (2000). The bodily basis of thought. *New Ideas in Psychology*, *18*, 23–-40.

Shweder, R. A. (1991). *Thinking through cultures*. Cambridge: Harvard University Press.

Shweder, R., & LeVine, R. (1984). *Culture theory. Essays on mind, self, and emotion*. Cambridge: Cambridge University Press.

Sierpinska, A. (1996). Interactionnisme et théorie des situations : Format d'interaction et contrat didactique. In D. Grenier (Ed.), *Didactique et technologies cognitives en mathématiques, Séminaires 1996* (pp. 5–-37). Grenoble: Université Joseph Fourier.

Silver, E., & Herbst, P. (2007). Theory in mathematics education scholarship. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 39–-67). Greenwich, CT.: Information Age Publishing.

Sriraman, B., & English, L. (2005). Theories of mathematics education: A global survey of theoretical frameworks/trends in mathematics education research. *ZDM - The International Journal on Mathematics Education*, *37*(6), 450–-456.

Steinbring, H., Bartolini Bussi, M., and & Sierpinska, A. (1998). *Language and communication in the mathematics classroom*. Reston, Virginia: NCTM.

Torigoe, T. (1985). Comparison of object manipulation among 74 species of non-human primates. *Primates*, *26*(2), 182–-194.

Valero, P., & Zevenbergen, R. (eEds.). (2004). *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology*. Boston: Kluwer.

Vološinov, V. N. (1973). *Marxism and the philosophy of language*. Harvard: Harvard University Press.

von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. London: The Falmer Press.

Vygotsky, L. S., and & A. Luria,A. (1994). Tool and symbol in child development. In R. van der Veer and & J. Valsiner (eEds.), *The Vygotsky Rreader* (pp. 99–-174). Oxford: Blackwell.

Wartofsky, M. (1979). *Models, representation and the scientific understanding*. Dordrecht: D. Reidel.

Weber, M. (1992). *The protestant ethic and the spirit of capitalism*. London: Routledge.

Wertsch, J. V. (1991). *Voices of the mind*. Cambridge, MAa.: Harvard University Press.

Yackel, E., and & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458–-477.

Luis Radford
École des sciences de l'éducation
Université Laurentienne
Sudbury, Ontario
Canada