

Sensuous Cognition

Luis Radford

Université Laurentienne, Canada

Abstract: In the first part of this chapter I sketch an approach where human cognition is conceptualized in non-dualistic, non-representational, and non-computational terms. The basic idea is that cognition is a feature of living material bodies characterized by a capacity for *responsive sensation*. The sketched approach, which I term *sensuous cognition*, refers precisely to this view where sensation is considered to be the substrate of mind, and all psychic activity (cognitive, affective, volitional, etc.). I argue that, as far as humans are concerned, responsive sensation evolves—both at the phylogenetic and ontogenetic levels—interwoven with the material culture in which individuals live and grow. As a result, human cognition can only be understood as a culturally and historically constituted sentient form of creatively responding, acting, feeling, transforming, and making sense of the world. In the second part of the chapter, I present classroom experimental data involving 7–8-year-old students dealing with pattern recognition. The classroom data allow me to illustrate the interplay of the various sensuous modalities in mathematical cognition. I end the chapter suggesting that a sensuous-based materialistic monistic view of cognition needs to attend not only to the plethora of sensorial modalities that teachers and students display while engaging in mathematical activities, but also to the manner in which sensorial modalities come to constitute more and more complex psychic *wholes* of sensorial and artifactual units.

Key Words: Cognition, sensation, plasticity, multimodality, material culture, gestures, semiotic node, pattern recognition.

1. Introduction

Mathematics has been enduringly renowned for its impressive use of symbols. Or at least this has been the case since the progressive invention of a succinct symbolism in the Renaissance by mathematicians such as Piero de la Francesca, Rafael Bombelli, and François Viète. Since then, mathematics has been conceived, implicitly or explicitly, as an essential activity mediated by written signs. However, mathematics can also be seen as an activity profoundly mediated by artifacts and signs other than those of the written register. This is certainly the case in classroom mathematics activity, where not only written signs, but also speech, gestures, body posture, kinesthetic actions, and artifacts mediate students' activities in a substantial manner. Figure 1 shows three Grade 11 students during a trigonometry lesson where they devised a formula to describe the position $P(x(t), y(t))$ of a train that moves at a constant speed along a circular route. The student to the left measures time with a chronometer; following the train, the student in the middle measures space with a pen; the student to the right coordinates the other two students' action and takes notes on a field sheet.



Figure 1. Grade 11 students investigating the equation of the point $P(x(t), y(t))$ of a train that moves around a circle at constant speed.

Now, what exact role do we ascribe to the artifacts to which student resort? What exact role do we ascribe to speech, body posture, gestures, and signs? Customarily, in traditional cognitive psychology, artifacts are considered convenient devices that reveal the functioning of the mind; they are hence thought to play a secondary role in cognition in general, and mathematical cognition in particular (see, e.g., Piaget's use of artifacts in his research). Body posture, gestures and other embodied signs have suffered the same fate. In some branches of traditional cognitive psychology, the mind is conceived of as a computational device; the research focus is on the linguistic formats through which information is transmitted and decoded (e.g., declarative vs. procedural sentences). In other trends, when attention has been paid to embodiment and the sensorial realm—like in Piaget's epistemology—the body appears to play a role in the early stages of intellectual development only (the Piagetian sensory-motor stage), apparently disappearing in more advanced stages.

A new research trend, however, offers a different approach to the understanding of human cognition. In this trend our tactile-kinesthetic bodily experience of the world and our interaction with artifacts and material culture are considered as much more than merely auxiliary or secondary elements in our cognitive endeavours. For instance, Sheets-Johnstone (2009) argues that, as a result of our biological makeup, we are naturally equipped with a range of archetypal corporeal-kinetic forms and relations that constitute the basis on which we make our ways into the world. Echoing an increasing number of neuroscientists and linguists, Seitz (2000) contends that the basis of thought is to be found *in* the body. Yet, it is clear that there is not just one way in which to theorize the cognitive role of the body. As a result, it is not surprising to find a variety of perspectives on what has come to be termed “embodied cognition.”¹ Thus, in her review of current perspectives, Wilson (2002, p. 626) highlights six claims to which, she argues, theorists of embodied cognition resort in their work:

- (1) cognition is situated;
- (2) cognition is time-pressured;

¹ For some embodied perspectives in mathematics education, see the special issue of *Educational Studies in Mathematics* edited by Edwards, Radford, and Arzarello (2009), and the special issue edited by Radford, Schubring, and Seeger (2011) in the same journal. See also Bautista and Roth (2011) and the seminal book of Lakoff and Núñez (2000).

- (3) we off-load cognitive work onto the environment;
- (4) the environment is part of the cognitive system;
- (5) cognition is for action;
- (6) off-line cognition is body-based.

The most problematic of those claims, Wilson finds, is the fourth. Indeed, the fourth claim requires a completely different point of departure about our ordinary dualistic conceptualizations of thinking. As long as we keep a *dualistic* approach to mind, Wilson's fourth claim remains problematic. This is why it is not enough to merely put the body and material culture back into thought. What we need is a different starting point where thinking and environment are not conceptualized as separate entities.

Dwelling upon Vygotsky's and Leont'ev's work and enactivism (Maturana and Varela, 1998), in this chapter I elaborate on what I have previously termed *sensuous cognition* (Radford, 2009). Sensuous cognition refers to a non-dualistic, non-representational, and non-computational view of the mind. Starting from the premise that cognition and environment are intertwined entities, the basic idea is that cognition is a feature of living material bodies characterized by a capacity for *responsive sensation*. In subsequent sections I argue that, as far as humans are concerned, responsive sensation evolves—both at the phylogenetic and ontogenetic levels—intertwined with the material culture in which individuals live and grow. As a result, cognition can only be understood as a culturally and historically constituted sentient form of creatively responding, acting, feeling, transforming, and making sense of the world.

However, I should hasten to make clear that my interest is not purely cognitive. As a mathematics educator I am deeply interested in exploring how a non-dualistic view of mind translates into teaching and learning contexts. The chapter is divided into two parts. The first part is of a theoretical nature. The goal is to present a cogent sensuous-based monistic view of cognition. To reach this goal, I need to present in some detail the concept of sensuous cognition and to discuss some theoretical constructs, such as sensation—its plasticity and multimodal nature—as well as the entanglement of sensuous cognition and material culture. In the second part of the chapter, I present classroom experimental data involving 7–8-year-old students dealing with pattern recognition. The classroom data allows me to illustrate the interplay of the various sensuous modalities in mathematical cognition. I close the chapter by suggesting that a sensuous-based monistic view of cognition needs to attend not only to the plethora of sensorial modalities that teachers and students display while engaging in mathematical activities, but also to the manner in which sensorial modalities come to constitute more and more complex psychic *wholes* of sensorial and artifactual units.

2. Sensuous cognition

The idea of sensuous cognition that I would like to advocate here rests on a non-dualistic view of the mind. In dualistic accounts, the mind is conceived of as operating through two distinctive planes, one internal and one external. The internal plane is usually considered to include consciousness, thought, ideas, intentions, etc., while the external plane refers to the material world—which includes concrete objects, our body, its movements, and so on. In opposition to this dualistic view, drawing on Vygotsky and

Leont'ev (1978, 2009), and Maturana and Varela (1998), I adopt a monistic position according to which mind is a property of matter. More specifically, mind is conceptualized as a feature of living material bodies characterized by a capacity for *responsive sensation*.

Sensation is a phylogenetically evolved feature of living organisms through which they *respond to, reflect* or *act* on their environment. Since the organism is itself a part of the material world, any reflection of reality is nothing other than a function of a material, corporeal organism (Leont'ev, 2009, p. 12) against a material milieu. As a result, reflection and action do not occur in two *separate* planes. They occur in the same plane—the plane of life.

Now, reflection, as understood here, cannot be considered a passive act of receiving sensorial impressions, as 17th and 18th century empiricists hold. As its etymology suggests, in reflection the organism “bends back” something—the “reality” or the “environment,” as the organism perceives or feels it. Reflection hence involves both (1) something that transcends the organism as such (something that, in order to differentiate it from the subject itself, we can call *objective*, namely *the object of reflection*), and (2) the *reflected object*, something that is *subjective* (in the sense that reflection depends on the specific organism reflecting the environment).²

It is worth noticing that, phylogenetically speaking, the relationship between the subjective world and objective reality is not absolute (Maturana and Varela, 1998); nor is it something that is given *a priori* (Leont'ev, 2009). As far as humans are concerned, this relationship is *dialectical*, where the objective world and its subjective reflection co-evolve. On the one hand, mind can only arise from the progressive complexity of processes of life; on the other hand, more complex conditions of life require organisms to have the capacity to reflect reality through more complex forms of sensation. This is why mind is not just something added to the organisms' vital functions: mind “arises in the course of [the organisms'] development and provides the basis for a qualitatively new, higher form of life—life linked with mind, with a capacity to reflect reality” (Leont'ev, 2009, p.18).

The historical origin of the printing press is a good example to illustrate these ideas. The apparition of the printing press can only be understood within the context of a complexification of previous forms of human labour, the unprecedented systematization and mechanization of actions in various spheres of life in the late Middle Ages, and the ensuing transformation of the human senses—e.g., mainly vision and tactility (McLuhan, 1962). Reciprocally, for such a complexification to occur, the capacity for psychic reflection of material reality was required. In short, the human mind, as a culturally and historically evolved form of sensation, and human consciousness—that is to say, the manner in which the individual's reality is *subjectively reflected*—can only be understood in light of the co-evolution of the nervous system, more evolved forms of sensation, and the concomitant complexity of social practices and material culture.

² My use of the adjective *objective* does not refer to claims about truth. It is rather a claim about something that is distinct from the organism, something that *objects* the organism; in other words, the term *objective* refers to something that we can term *Otherness*.

To sum up, instead of being purely “mental,” reflection and its products remain, one way or another, intertwined with the environment that is being reflected and with the organism’s capacities for sensation. Mind, in this context, is the ability of organisms to reflect, and act on, the reality around them. Thinking, memory, imagination and other cognitive functions are directly and indirectly related to a large range of sensorimotor functions expressed through the organism’s movement, tactility, sound reception and production, perception, etc. What I term *sensuous cognition* refers precisely to this view where sensation is considered to be the substrate of mind, and of all psychic activity (cognitive, affective, volitional, etc.). In the next section I dwell in more detail on this point.

3. The plasticity of human sensation

A sensuous approach to mind cannot avoid noticing the specific *plastic* nature of human sensation alluded to in the previous section, and without which the co-evolution of life, social practices, material culture, and sensation would be meaningless. Indeed, as the German social theorist Arnold Gehlen (1988) argues, animals are endowed from birth with specific instincts and highly developed sensorial systems that make them fit to survive in specific environments. The human senses, by contrast, are highly unspecialized. Thus, the hearing, smelling, and perceptual sensitive organs of the deer trigger an alert signal when presented with recognized clues in the environment, clues that would remain beyond the range of human attention. Similarly, the acute visual perception of the eagle and the thermal sense of some predators surpass humans’ sensorial acuity.

To cope with this *lack* of particular instinctual and environment-specific sensorial systems of animals, humans *develop* their highly unspecialized sensorial functions into complex forms that allow them to adapt to virtually any environment. Tactility, for instance, becomes a means to distinguish between temperature differences, soft and rough surfaces, and distances; in this way, tactility is transformed in “an intelligence in action” (Le Breton, 2007, p. 152). Through their hands humans grip things and explore and palpate the environment in movements that can become extremely specialized (Wilson, 1998). In short, the specific instincts and highly developed sensorial systems that we find in animals are compensated for in humans by the *plasticity* of their senses and the achievable levels of specialization that the senses can acquire.

4. Multimodal sensuous cognition

In the previous section we have mentioned that one chief characteristic of the human senses is constituted by their plasticity. Another central characteristic is the human senses’ *interrelated* development and functioning. What this means is that the various senses develop in *integrative* manners and come to collaborate in ways that are truly specific to humans. The result is that the human mind and cognition are not merely sensuous but also *multimodal*. This idea is certainly a cornerstone in the new approaches to the human mind (Gogtay et al., 2004; Lickliter & Bahrick, 2000; Lewkowicz & Lickliter, 1994).

Indeed, the senses *collaborate* among themselves, allowing us to come up with a complex perception of reality. Touch and sight, for instance, collaborate with each other. Through a tactile experience, I can feel the weight of an orange; through a perceptual one, I can have a sense of its relative chromatic characteristics. Later, I can feel its porous skin even if it is out of my actual tactile reach. Touch and sight collaborate at close distance in their experience of the world. Sight and language, by contrast, collaborate at a long distance. Knowing hence is ensured through a multi-modal sensorial experience of the world. The sensorial modalities are *integrated* into a complex of properties that bring together different sensuous modal experiences (e.g., shape from perception and rigidity from tactility). Referring to vision as a complex modal experience, Varela says:

vision is a patchwork of visual modalities, including at least form (shape, size, rigidity), surface properties (color, texture, specular reflectance, transparency), three-dimensional spatial relationships (relative positions, three dimensional orientation in space, distance) and three-dimension movement (trajectory, rotation). It has become evident that these different aspects of vision are emergent properties of concurrent subnetworks that make a visual percept coherent. (Varela, 1999, p. 48)

This multi-sensory characteristic of cognition is not specific to humans; it is shared by insects (Wessnitzer and Webb, 2006) and other primates as well. However, compared to the case of insects and other primates, human sensorial organs collaborate to a greater extent (Gómez 2004; Köhler, 1951), so that what we perceive or touch is endowed with a variety of sensuous coordinated characteristics. For instance, the human hand does not only feel the trace of the object. We can say that the hand also “perceives its colour, its volume, its weight” (Le Breton 2007, p. 151).

5. The cultural shaping of senses

A third chief characteristic of the human senses is the manner in which they co-evolve with culture. Indeed, our senses are not merely part of our biological apparatus. The raw range of orienting-adjusting biological reactions we are born with is transformed into complex, historically constituted forms of sensing. The cultural nature of this transformation can be illustrated through the example of a child who was found in the woods of Aveyron, between Montpellier and Toulouse in France, in 1800. The young child

who trotted and grunted like the beasts of the fields . . . was apparently incapable of attention or even of elementary perceptions . . . and spent his time apathetically rocking himself backwards and forwards like the animals at the zoo. (Humphrey in Itard, 1962, p. vi)

The so-called wild boy of Aveyron—or Victor as he was called later—was placed under the care of Dr. Jean-Marc-Gaspard Itard, who designed a series of exercises to teach Victor to speak and catch up with the development of his intellectual faculties. Thus, the boy started distinguishing incrementally between the sound of a bell and that of a drum, went on to discern among the tones of a wind instrument, and later distinguished between vowels. Itard comments:

It was not without difficulty and much delay that I succeeded at last in giving him a distinct idea of the vowels. The first that he distinguished clearly was O, next the vowel A. The other three presented greater difficulty and for a long time he confused them. (Itard, 1962, p. 58)

The training of the sense of hearing was followed by the training of the sense of sight and touch. Although from a developmental viewpoint Victor was not able to catch up completely, the example shows clearly that without life in society the raw biological ensemble of orienting-adjusting reactions with which we are born remains undeveloped. As we live in society, interact with others, and participate in more or less specialized forms of training, the biological orienting-adjusting reactions undergo cultural transformation and are converted into complex historically constituted forms of sensing, leading to specific features of human development and the concomitant forms of cultural reflection. This is why in the process of development the child not only matures, but is also equipped with sophisticated ways of seeing, touching, hearing, tasting, and so on.

The ontogenetic process of the cultural transformation of the senses has been investigated in great detail in the past few years. To mention but one example, Zaporozhets (2002) reports research with three- to five-year old preschoolers who were learning to discriminate between variants of two geometric figures: triangles and quadrilaterals. In the beginning, the preschoolers were making a substantial number of errors. Then, they were invited to trace systematically with a finger the outline of the figure, paying attention to directional changes of the motions at angles, and accompanying the tactile exploration with side counting (one, two three...). The investigator reports that at this stage perception was accomplished through the tactile experience, while the eye performed an auxiliary role. "Later," Zaporozhets says, "the eye developed the ability to solve these types of perceptual tasks independently, consecutively tracing the outline of a figure, as it was earlier done by a touching hand" (2002, p. 31). During this process, the eye undergoes a transformative change: "initially, the eye motions have an extremely extensive nature, consecutively tracing the entire outline of the perceived figure and simulating its specifics in all details" (p. 32). In a subsequent stage, the eye's motions "gradually begin to decrease and to focus on the individual, most informative attributes of the object" (p. 32).

6. The artifactual dimension of sensuous cognition

A closer look at the previous examples shows that the new cultural forms of sensation are deeply interrelated with the use of *artifacts*. Indeed, in the first example, Itard makes recourse to two artifacts —a drum and a bell—and the artificial sounds that they produce. Victor learns to distinguish between them. Aural discrimination is consequently *shaped* by the cultural sounds that the ear meets. Itard also uses the cultural distinction between vowels of the human artifact par excellence—speech. In the second example, preschool children develop a mathematical form of perception that allows them to distinguish between cultural categories of geometrical figures. In doing so, the children have recourse to the material objects whose contours they cover with a finger while using numbers to count aloud. What these examples show is that our individual senses evolve intertwined not only one with the other senses (which is the claim I made in Section 4), but also with the *materiality* of the objects in our surroundings. The materiality that shapes our senses is not, however, reduced to inert matter, but, as the examples show,

matter already endowed with meaning (e.g., ‘triangularity,’ ‘quadrilaterality,’ the bell’s sounds, etc.).

It is this key role of artifacts in the constitution and evolution of forms of sensing and reflecting that Luria and Vygotsky underlined in their work. The use of artifacts, they contended, constitutes the first phase in cultural development (Luria & Vygotsky, 1998; Vygotsky, & Luria, 1994). Such a phase marks the emergence of new forms of actions and reflection and the concomitant appearance of psychic functions. Vygotsky and Luria paid particular attention to the question of memory and argued that the construction of artificial signs, like a knot, transforms “natural” or eidetic memory (i.e., memory based on the recording of external impressions with great photographic precision). In some cultural formations knot-use appears as a material mechanism used to register events in information encoding systems, and gives rise to a new, cultural form of memory. Naturally, knots or writing are not the only historical artifacts at the base of the transformation of memory and other cognitive functions. Artifacts in general create *dispositions* through which to think, perceive, remember, etc. For instance, current electronic media and their forms of dynamic visualization are creating new *dispositions* through which to engage the world, much as pictorial representations and arithmetical calculations did in the Renaissance. In both cases, the senses are transformed to respond to new possibilities opened up by changes in material culture. For us mathematics educators, the challenge is to understand the sensuous possibilities of the new material culture so as to exploit it in design contexts as well as in teaching-learning cyber- and visually-based activities.

All in all, these examples amount to making a point about the embedded nature of artifacts in the evolution of our ways of sensing and reflecting. They stress the fundamentally cognitive role of artifacts and material culture in the ways we come to know. The claim that I am making, hence, goes beyond the conceptualization of artifacts as merely *mediators* of human thinking and experience, or as prostheses of the body. Artifacts do much more than mediate: they are a *constitutive part* of thinking and sensing. Behind this view lies, of course, the general concept of mind as a property of matter. This property expresses the enactive relationship between materiality and mind that inspired Vygotsky’s, Luria’s, and Leont’ev’s work and that Bateson (1973) illustrates so nicely in his example of the blind person’s stick. It is in this context that anthropologists Malafouris and Renfrew (2010) claim that we can speak of things as having a cognitive life. They say: “things have a cognitive life because minds have a material life” (p. 4).

Sensuous cognition is hence a perspective that highlights the role of sensation and materiality as the substrate of mind and of all psychic activity. But in contrast to other approaches where the focus remains on the individual’s body, sensuous cognition offers a perspective where sensation and its cultural transformation in sensing forms of action and reflection are understood to be interwoven with cultural artifacts and materiality at large. Sensuous cognition calls into question the usual divide between mind and matter and casts in new terms the classical boundaries of mind.

7. A classroom example

As mentioned in the introduction, my interest is not purely cognitive. I am first of all interested in exploring how the idea of sensuous cognition translates into teaching-

learning contexts. It is in this spirit that in this section I would like to discuss an example from a Grade 2 class (7–8-year-old students) involving the generalization of an elementary figural sequence. The example comes from the first of a series of lessons that were intended to introduce the students to a cultural-historical form of thinking that we recognize as algebraic. The first step is getting acquainted with what matters and what has to be attended to in the terms of a figural sequence. Figure 2 shows the terms of the sequence given to the students.

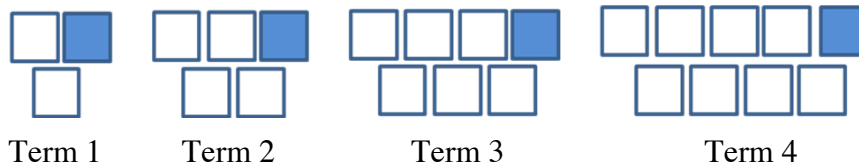


Fig. 2. The first terms of a sequence that 7–8-year-old students investigate in a Grade 2 class.

Mathematicians would attend without difficulty to those aspects of the terms that are relevant for the task at hand: they would, for instance, see the terms as divided into two rows and notice the immediate relationship between the number of the term and the number of squares in each one of the rows. The perception of those variational relationships usually moves so fast that mathematicians virtually do not even notice the complex work behind it. They would also extend without difficulty the noticed property of the rows to other terms that are not present in the perceptual field, like Term 100, and conclude that this term has $100+101$ squares, that is 2001 (see Figure 3). Or even better, that the number of squares in any term, say Term n , is $2n+1$.

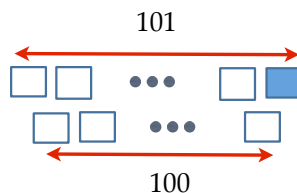


Figure 3. A frequently reported quick imagination of Term 100 by the trained eye.

Yet, the novice eye does not necessarily see the sequence in this way. Figure 4 shows an example of how some Grade 2 students extend the sequence beyond the four given terms shown in Figure 2.



Figure 4. Terms 5 and 6 as drawn by a Grade 2 student.

The student focuses on the *numerosity* of the squares, leaving in the background the *spatiality* of the terms (Radford, 2011a). We cannot say, I think, that the student's

answer shown in Figure 4 is wrong. The answer makes sense for the student, even if it is probably true that by focusing on the numerosity of the terms of the sequence, it might be difficult later on to end up with a general formula like $2n+1$. This is in fact what we have observed again and again in our research with older students (13–17-year-old students). In the latter case, the students tend to rely on trial-and-error methods that, as I have argued elsewhere, are not algebraic, but arithmetic in nature (Radford, 2008, 2010).

The issue is not that the students do not see the two rows of the terms. In Figure 5, we see a Grade 2 student pointing with his pen to the top row, then to the bottom row, after moving the pen across the top and bottom rows to properly distinguish between them. However, when the student draws Term 5, the *spatial* dimension of the terms is relegated to a second plane and does not play an organizing role in the drawing of the term. He draws a *heap* of rectangles. The issue is rather about not realizing yet that the spatiality of the terms provides us with clues that are interesting from an algebraic viewpoint.

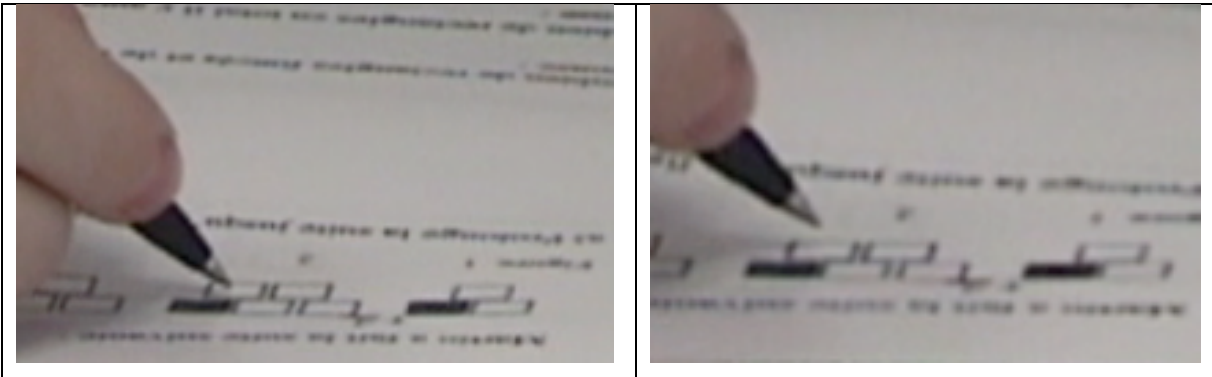


Figure 5. A student pointing to the top row (left) and to the bottom top (right) of Term 2

To become sensitive to the cultural-historical algebraic forms of perceiving terms in sequences like the one discussed here, students engage in processes that are far from mental. They engage with the task of exploring the sequence in a sensuous manner. I would like to illustrate this point by discussing the way in which the teacher and a group of students reflect on Term 8 of the sequence. As mentioned previously, the first question of the mathematical activity consisted in extending the terms of the sequence up to Term 6. Then, in questions 2 and 3, the students were asked to find out the number of squares in terms 12 and 25. In question 4, the students were given a term that looked like Term 8 of the sequence (see Figure 6). They were told that this term was drawn by Monique (an imaginary Grade 2 student) and encouraged to discuss in small groups to decide whether or not Monique's term was Term 8. The trained eye would not have difficulties in noticing the missing white square on the top row. The untrained eye, by contrast, may be satisfied with the apparent spatial resemblance of these terms with the other terms of the sequence and might consequently fail to note the missing square.

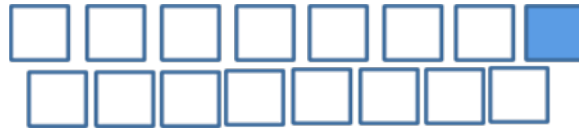


Figure 6. The students were requested to discuss whether Monique's term is Term 8 of the given sequence

Let me focus on the discussion that a group of students had with the teacher—a group formed by James, Sandra and Carla. When the teacher came to see their work the students had already worked together for about 32 minutes. They had finished drawing Terms 5 and 6, tried (unsuccessfully) to find the number of squares in Term 12 and 25, and answered the question about Term 8 (which they considered to be Term 8 of the sequence). The teacher engaged in collaborative actions to create the conditions of possibility for the students to perceive a general structure behind the sequence. She started by referring to the first four terms of the sequence that were drawn on the first page of the activity sheet (Monique's term, which the students examined previously, was drawn on the second page of the activity sheet and was hence not in the students' perceptual field in the first turns of the following episode):

1. Teacher: We will just look at the squares that are on the bottom (*while saying this, the teacher makes three consecutive sliding gestures, each one going from bottom row of Term 1 to bottom row of Term 4; Pics 1-2 in Fig. 7 show the beginning and end of the first sliding gesture*). Only the ones on the bottom. Not the ones that are on the top. In Term 1 (*she points with her two index fingers to the bottom row of Term 1; see Pic. 3*), how many [squares] are there?
2. Students: 1!
3. Teacher: (*Pointing with her two-finger indexical gesture to the bottom row of Term 2*) Term 2?
4. Students: 2! (*James points to the bottom row of Term 2; see Pic 4*).
5. Teacher: (*Pointing with her two-finger indexical gesture to the bottom row of Term 3*) Term 3?
6. Students: 3!
7. Teacher: (*Pointing with her two-finger indexical gesture to the bottom row of Term 4; see Pic 5*) Term 4?
8. Students: 4!
9. Teacher: (*Making a short pause and breaking the rhythmic count of the previous terms, as if starting a new theme in the counting process, she moves the hand far away from Term 4 and points with a two-finger indexical gesture to the place where one would hypothetically expect to find Term 8; see Pic 6*) How many squares would Term 8 have on the bottom?
10. Sandra: (*hesitantly, after a relatively long pause*) 4?

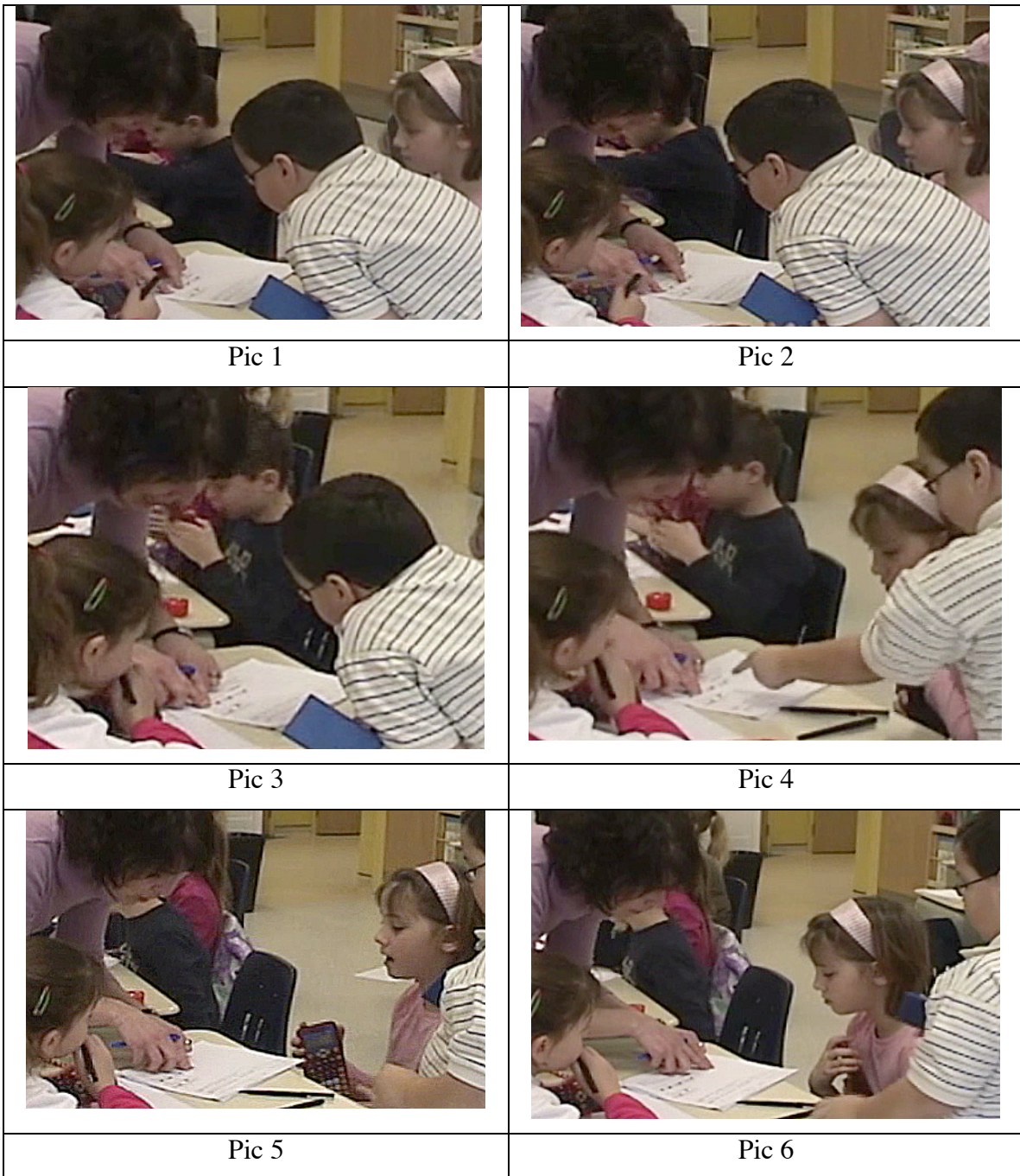


Figure 7. The teacher's and students' sensuous (perceptual, gestural, tactile, aural, vocal) engagement in the task.

In Line 1, the teacher makes three sliding gestures to emphasize the fact that they will count the bottom row of the four given terms. The gestural dimension of the teacher-students' joint activity is somehow similar to the material-tactile experience of the students who, in the aforementioned experiment reported by Zaporozhets (2002), follow the contour of shapes with their fingers. Here, the material-tactile dimension is carried out instead with gestures through which the teacher suggests a cultural form of perceiving the terms of the sequence—one in which the mathematical ideas of variable, and the relationship between them, are emphasized.

Now, the teacher does not gesture silently. Gestures are coordinated with utterances. This is why it might be more useful to consider the teacher's utterance as a *multimodal utterance*: that is to say as a bodily expression that resorts to various sensorial channels and different semiotic registers. In this case, the teacher coordinates eye, hand, and speech through a series of organized simultaneous actions that orient the students' perception and emergent understanding of the target mathematical ideas. In our previous work we have termed *semiotic node* this complex coordination of various sensorial and semiotic registers (Radford, 2009). The investigation of semiotic nodes in classroom activity, we have suggested (Radford, Demers, Guzmán, & Cerulli, 2003), is a crucial point in understanding the students' learning processes. The concept of semiotic node rests indeed on the idea that the understanding of multi-modal action does not consist in making an inventory of material signs and sensorial channels at work in a certain context. From a methodological viewpoint, the problem is to understand how the diverse sensorial channels and semiotic signs (linguistic, written symbols, diagrams, etc.) are *related*, *coordinated*, and *subsumed* into a new thinking or psychic *unity* (Radford, 2011a, 2012). Such a methodological problem makes sense only against the background of a conception of the mind that overcomes the dualistic view of internal-external processes. In our case, the methodological problem makes sense against the background of a concept of the human mind as *sensuous* through and through.

Methodologically speaking, we still need to account for the manner in which the new ideational-material psychic activity comes into being. And in order to do so, we need to pay attention to the manner in which individuals engage in the task, and they position themselves towards each other. In the classroom passage under discussion we note the teacher's posture and other means to which she resorts to engage the students in the task, not only asking explicit questions but also opening up a space for an *ethical engagement* to occur. That is, she becomes a *presence* and a *call* to which students are invited to respond (Radford & Roth, 2011). The students *respond* to the teacher by perceptually and aurally following her hands' movement and speech along the material terms, and answering her sequence of questions. We cannot fail to notice the tremendous role that *rhythm* comes to play here. Rhythm appears through different sensuous modalities: it appears in the aural modality through the flow of speech (in the regularly time-governed occurrence of the words "Term 1?," "Term 2?," "Term 3?," "Term 4?"); it appears in the kinesthetic modality of hands' movement through the regularly spatial-governed occurrence of the two-finger pointing gesture; it appears at a kind of supra-level, where the aural and kinesthetic modalities are coordinated so that they occur in a synchronized manner (Radford, Bardini, & Sabena, 2007). It is easy to imagine how disastrous a mismatching between these two forms of rhythm would be—e.g., producing the pointing

gestures faster or slower than the production of words. Indeed, it is rhythm that ensures an efficient link between the various sensorial modalities and material culture that paves the way for the students to become aware of the historical-cultural algebraic way of perceiving the given sequence. But, as mentioned previously, this link is not a mere connection between disparate and heterogenous sensuous-psychic elements. Rather, and this is the most important point, rhythm, psychologically speaking, is the token of the emergence of a new psychic unity: the unity of perception, gesture, symbol, and speech. Each one of these units has now been rearranged in a new plane of psychic activity where they do not operate in isolation, but along with the others as a *whole*.

From a semiotic viewpoint, let me note that rhythm is a sign, but of a very special sensuous sort. It does not point to an object as an indexical gesture or a linguistic term like 'this' does. That is, it does not have an existential relationship to its object. In other words, it is not an index. It is not a symbol either, in the Peircean sense of having an arbitrary relationship with its object (Peirce, 1931). Rhythm is an *icon* whose object is the embodied process that *incarnates* the target concept, in this case the relationship between mathematical variables. In our Grade 2 episode, rhythm appears as a complex icon embedded in various sensorial modalities (vocal, aural, kinesthetic, visual), wrapped in a composite supra coordination that emphasizes its object not by revealing it in an existential manner (as, for instance, when we point to a chair and say 'this chair') but by disclosing its *meaning*.

Yet, as Line 10 intimates, the passage from Term 4 to Term 8 was not successful. The objectification (that is, the becoming aware; see Radford, 2002, 2010b) of the algebraic manner in which sequences can be algebraically perceived has not yet occurred. The teacher hence decided to restart the process, with some important modifications, as we shall see.

As mentioned previously, Term 8 of the sequence was not materially drawn on the first page. Only the first four terms of the sequence were shown. In the previous excerpt, the teacher *pretends* that Term 8 is on the empty space of the sheet, somewhere to the right of Term 4. She points to the empty space, as she pointed to the other terms, to help the students imagine the term under consideration. During the second attempt, the teacher does not go from Term 4 to Term 8; this time she goes term after term until Term 8.

11. Teacher: We will do it again...
12. Teacher: (*Pointing to Term 1 with a two-finger indexical gesture*) Term 1, has how many?
13. Carla: (*Pointing with her pen to the bottom row*) 1, (*without talking to the teacher points to Term 2 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term 2*) 2, (*again without talking the teacher points to Term 3 with a two-finger indexical gesture; Carla points with her pen to the bottom row of Term 3*), 3, (*same as above*) 4, (*now moving to the hypothetical place of Term 5 would be expected to be and doing as above*) 5.
14. Teacher: Now it's Term 8! (*The teacher comes back to Term 1. She points again with a two-finger indexical gesture to the bottom row of Term 1*) Term 1, has how many [squares] on the bottom?
15. Students: 1.

16. Teacher: (*Pointing with a two-finger indexical gesture to the bottom row of Term 2*) Term 2?
17. Students: 2!
18. Teacher: (*Pointing with a two-finger indexical gesture to the bottom row of Term 3*) Term 3?
19. Students: 3!
20. Teacher: (*Pointing with a two-finger indexical gesture to the hypothetical place where bottom row of Term 4 would be*) Term 4?
21. Students: 4!
22. Teacher: (*Pointing as above*) Term 6?
23. Students: 6!
24. Teacher: (*Pointing as above*) Term 7?
25. Students: 7!
26. Teacher: (*Pointing as above*) Term 8?
27. Students: 8!
28. Sandra: There would be 8 on the bottom!

The teacher and the students counted together the squares on the bottom row of Monique's term and realized that the number was indeed 8. At this point the relationship between variables started becoming apparent for the students. The relationship started being objectified. The teacher then moved to a joint process of counting the squares on the top row:

29. Teacher: (*She turns the page and the students can see Monique's term*). Very, very good. Now, we will verify if Monique has the good amount [of squares] on top. We will just look at the top... (*like in the previous episode, she makes two sliding gestures, but this time pointing to the top row; see Figure 8, pic 1*). Term 1 has how many?
30. Students: 2!
31. Teacher: Term 2?
32. Students: 3! . . .
33. Teacher: Term 3?
34. Students: 4!
35. Teacher: Term 4?
36. Students: 5! (*see Pic 2*)
37. Teacher: Term 6?
38. James: 7
39. Teacher: (*Repeating*) 7 ... Bravo! Term 8, will have how many?
40. Students: 9!
41. Teacher: Ok. Oh! Excellent. Are there 9 [squares] here (*pointing to Monique's term*)?
42. Sandra: Yes, there are 9.
43. Teacher: We will count it together.
44. Students: (The teacher points orderly and rhythmically to the terms one after the other, while the Sandra says) 1, 2, 3, 4, 5, 6, 7, 8...! (*long pause following a general surprise. See Pics 3 and 4 in Figure 8*).

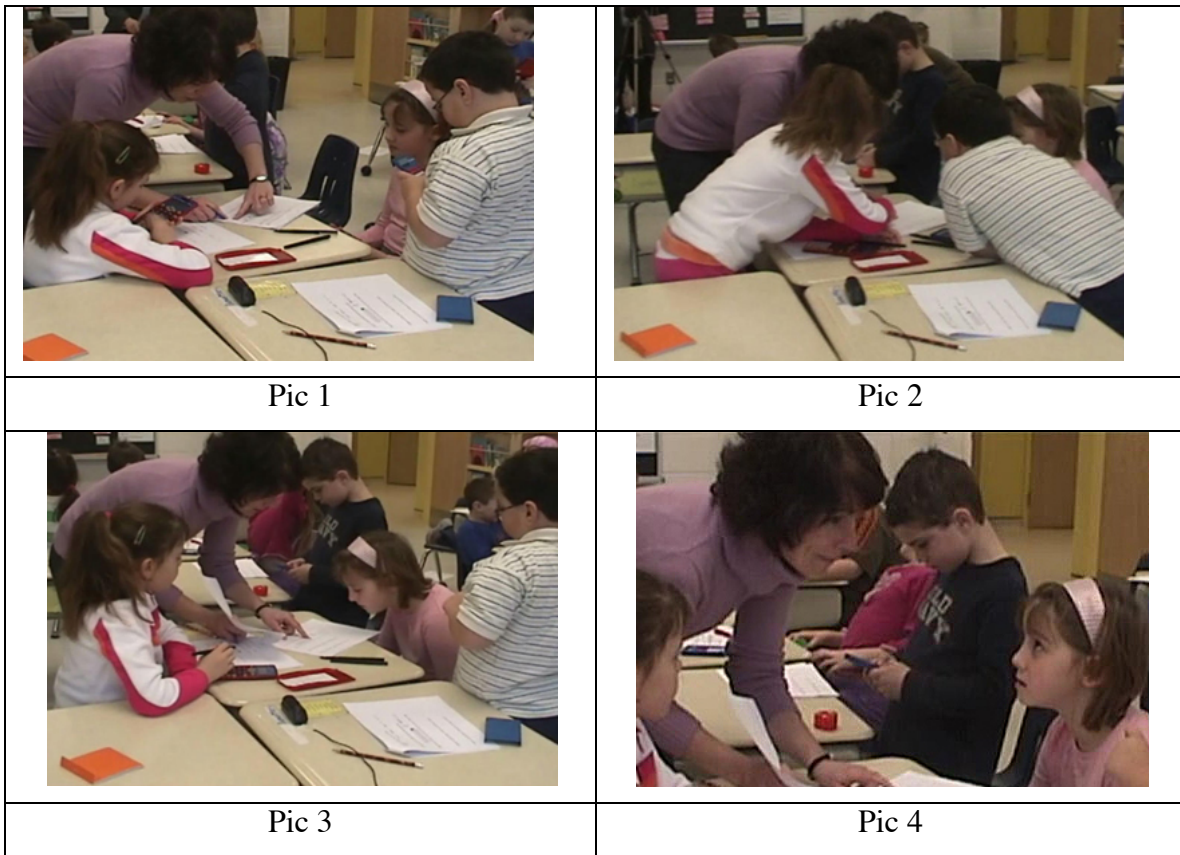


Figure 8. Using the four given terms of the sequence, in Pics 1 and 2, the teacher and the students count the squares on the top row of the visible Terms 1 to 4 and the imagined Terms 5 to 8. In Pics 3 and 4 the students count the squares on the top row of Monique's term.

The students were perplexed to see that contrary to what they believed, Monique's Term 8 did not fit into the sequence. Here the activity reached a tension. Pic 4 in Figure 8 shows Sandra's surprise. The students and the teacher remained silent for 2.5 seconds, that is to say, for a lapse of time that was 21 times longer than the average elapsed time between uttered words that preceded the moment of surprise (for details of this poetic moment in the teacher-students' objectification process, see Radford, 2010b).

Later on in the lesson the students were able to quickly answer questions about remote terms, such as term 12 and Term 25, which were not perceptually accessible. They refined the manner in which the terms of the sequence could be perceived. The number of squares on the bottom row was equated to the number of the term in the sequence, while the number of squares on the top row was equated to the number of the term plus one (identified as the dark square in the corner; see Radford, 2011b). Here is an excerpt from the dialogue of Sandra's group as they discuss without the teacher:

45. Sandra: (*Referring to Term 12*) 12 plus 12, plus 1.
46. Carla: (*Using a calculator*) 12 plus 12 ... plus 1 equal to ...

47. James: (*Interrupting*) 25.

48. Sandra: Yeah!

49. Carla: (*looking at the calculator*) 25!

At this point, the target cultural knowledge has been objectified and a new ideational-material psychic unity has been forged. The students no longer need to see the terms of the sequence or to touch them with their hands. What could only be made apparent through an intense interplay between material culture and the various sensorial modalities is now contracted, subsumed and reorganized in a new complex psychic unity where no reference is made to top or bottom rows (see lines 45-49).

8. Implications for Teaching and Learning in a Digital Era

As mentioned previously, the concept of sensuous cognition elaborated in this chapter is based on the idea that human sensation is consubstantial with material culture. This consubstantiality means, in particular, that our senses and the way we come to think about our world are shaped, oriented, and transformed by the ubiquitous presence of material culture in the activities we engage in. Our digital era is certainly provoking a transformation of the senses and our ways of thinking that we still need to better understand (Gee, 2003; Trend, 2010). Audiovisual forms of knowledge mediation are likely to lead to the creation of new dispositions or sensibilities in the ways we imagine, recall, reflect, visualize, and generalize. To give but one example, in her book *The skin of the film*, Canadian critic Laura Marks, shows that instead of being a flat screen, the skin of the film works rather as a “membrane that brings its audience into contact with the material forms of memory” (2000, p. 243) expressed through cultural forms of touching, smelling, and caressing, for example. The digital era and new material culture in general operates under new organizations of cultural sensoria with an increasing and definite “abstraction and symbolization of all sense modalities” (p. 244). Visual images and media are challenging the longstanding role that the written word has had for centuries in the Western world (see, e.g., Stephens, 1998). In a chapter entitled “The disappearing world,” Trend summarizes the ongoing transformation as follows:

The dawn of the digit era [in the 1990s] marked the biggest change in our relationship to language since the invention of the printing press, the rise of personal computing, networked communication, and other technologies at the turn of the millennium coincided with the death of Enlightenment thinking that separated the word from the image. . . [Written] Language is being taken over by images as experience itself becomes increasingly visual. (Trend, 2010, p. 31)

The implications for the teaching and learning of mathematics are evident. Hegedus and Moreno-Armella (2011), from the Kaput Research Center, are exploring the potentials of haptic devices, which work as semiotic mediators much as a dragged mouse does in dynamic geometry environments to mediate visual information. The haptic devices allow young students to sense mathematical structures, both physically and visually. Yet, we need to understand the reorganization of the senses and the creation of new sensibilities and forms of mathematical thinking that emerge from the contact and the engagement with digital environments and cyberlearning. Dynamic geometry, haptic devices and similar cultural artifacts provide the students with forms of exploration that are incommensurate with those offered by paper-and-pencil environments. The question

concerns more than economy of time. These technological devices offer room for the creation of an experimental space that might require the appearance of new sensibilities and new embodied ways of thinking—dynamic new literate ways of scrutinizing, enquiring, looking into, and thinking about, mathematics, mathematical objects and their relationships.

9. Synthesis and Concluding Remarks

In the first part of this chapter, drawing on Vygotsky and his cultural-historical psychological school (Vygotsky, 1987-1999; Luria, 1984; Luria and Vygotsky, 1998; Leont'ev, 1978, 2009), as well as on the work of Maturana and Varela (1998), I sketched a theoretical approach to cognition that highlights the role of sensation as the substrate of mind and of all psychic activity. The role of sensation in our cognitive endeavours is not, however, something new. It has been a recurrent theme in philosophical inquiries since the pre-Socratics. Since Plato and in fact since the Eleatic thinkers sensation was nevertheless understood in negative terms—as something that hinders the road to knowledge (see e.g., Radford, Edwards, & Arzarello, 2009). This is the sense with which rationalist epistemologies of the 17th and 18th centuries up to the present have endowed sensation. Thus, to give but one example, for Kant, sensations such as colour, sound, heat, and smell, “are connected with the appearance only as effects accidentally added by the particular constitution of the sense organs” (Kant, 1787, p. 72), and as such they are unable to yield true knowledge. They do not constitute an objective determination of the object, as they pertain to the subjective dimension of the sensing subject. As a result, sensations are not, according to Kant, necessary conditions of the object's appearance and consequently are not a constitutive part of the process of knowing (for a detailed discussion, see Radford 2011c). For some contemporary rationalists, sensation does play a cognitive and epistemic role. Yet, our sensing organs are considered as having little (if any) relation with culture and history. Their only history is the one of biology and natural development. Piaget's genetic epistemology is not, of course, the only example. By contrast, within the theoretical approach here sketched (which, I suggested, might be best captured by the term *sensuous cognition*) sensation is not merely part of our bodily and biological constitution. Sensation is rather conceived of as a culturally transformed sensing form of action and reflection interwoven with cultural artifacts (language, signs, diagrams, etc.) and material culture more generally. Sensuous cognition calls into question the usual divide between mind and matter and offers a perspective through which to cast the role of the body and artifacts in knowing processes.

In the second part of the article, I presented a short example that, I hope, gives an idea of the manner in which sensuous cognition may help us understand teaching-learning activity. Sensuous cognition, I argued, does not amount to claiming that our various senses come into play in classroom interaction. This is no more than a banal statement. The real question, I argued, is about understanding how, through classroom activity, our forms of sensing and reflecting are culturally transformed. The example discussed in the previous section intimates that as knowledge objectification proceeds, new ideational-material psychic unities emerge out of previous psychic formations. Within this context, the concept of semiotic node is a practical construct that may be useful in investigating this important aspect of teaching-learning and conceptual development. Thus, in the Grade 2 example we noticed how a complex psychic unity was

revealed by a semiotic node constituted by the teacher's and the students' intercorporeal and material activity. This complex psychic unity gave rise to a new psychic formation where students were able to quickly tackle questions about remote terms (like Term 25). The complex and dynamic unity of perception, language, gestures, rhythm, diagrams revealed by the semiotic node yielded place to a new, more compact unity where language and cultural artifacts predominated (see lines 45-49). The issue, however, is not that knowledge has become disembodied. Behind language and cultural artifacts, there still resonates the complex bodily, material, and semiotic activity of the previous sensuous actions and forms of sensing. They have been *contracted* (Radford, 2010a) and *reorganized*, and will re-emerge if difficult questions arise (see Radford, 2011b). Previous psychic formations do not disappear. They are subsumed in the new ones and are reactivated if necessary, although not in an intact form: the eye, for instance, cannot regain the primary purity and naivety through which it saw the world before. The same could be argued of hearing and touching. Human sensuous cognition is not an additive formation. Its structure is rather made up of new formations subsuming the previous ones by relations of a dialectical nature. As French philosopher Jean Hyppolite (1961) notes in his essay on Hegel's system, a word, in its truly dialectical nature, signifies what is not there by signifying what is there, and signifies what is there by signifying what is not there.

Acknowledgements

This chapter is a result of a research program funded by the Social Sciences and Humanities Research Council of Canada (SSHRC/CRSH). A previous version was presented at the ICME-12 Topic Study Group 22 (*Learning and cognition in mathematics*, co-chaired by Hsin-Mei Huang and Gaye William), South Korea, July 2012.

References

- Bautista, A., & Roth, W. -M. (2011). Conceptualizing sound as a form of incarnate mathematical consciousness. *Educational Studies in Mathematics*, 1-19.
- Bateson, G. (1973). *Steps to an ecology of mind*. Frogmore: Paladin.
- Edwards, L., Radford, L., & Arzarello, F. (Eds.) (2009). Gestures and multimodality in the teaching and learning of mathematics. Special issue of *Educational Studies in Mathematics*, 70(2), 91-215.
- Gee, J. P. (2003). *What video games have to teach us about learning and literacy*. New York: Palgrave Macmillan.
- Gehlen, A. (1988). *Man. His nature and place in the world*. New York: Columbia University Press.
- Gogtay, N., Giedd, J., Lusk, L., Hayashi, K., Greenstein, D., Vaituzis, A. et al. (2004). Dynamic mapping of human cortical development during childhood through early adulthood. *Proceedings of the National Academy of Sciences of the United States of America*, 101(21), 8174-8179.
- Gómez, J. C. (2004). *Apes, monkeys, children, and the growth of mind*. Cambridge: Harvard University Press.
- Hegedus, S., & Moreno-Armella, L. (2011). The emergence of mathematical structures. *Educational Studies in Mathematics*, 77(2-3), 369-388.
- Humphrey, G. (1962). Introduction. In J.-M.-G. Itard, *The wild boy of Aveyron* (v-xxiv). New

- York: Meredith Publishing Company.
- Hyppolite, J. (1961). *Logique et existence [logic and existence]*. Paris: Presses Universitaires de France.
- Itard, J.-M-G. (1962). *The wild boy of Aveyron*. New York: Meredith Publishing Company.
- Kant, I. (1787). *Critique of pure reason*. New York: St. Marin's Press. Second Printing, 1965.
- Köhler, W. (1951). *The mentality of apes*. New York, London: The Humanities Press; Routledge & Kegan Paul.
- Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from*. New York: Basic Books.
- Le Breton, D. (2009). *El sabor del mundo. Una antropología de los sentidos [The taste of the world. An anthropology of the senses]*. Buenos Aires: Ediciones Nueva Visión.
- Leont'ev, A. N. (1978). *Activity, consciousness, and personality*. Englewood Cliffs, NJ: Prentice-Hall.
- Leontyev [or Leont'ev], A. N. (2009). *Activity and consciousness*. Pacifica, CA: MIA. Retrieved August 29, 2009, from <http://www.marxists.org/archive/leontev/works/activity-consciousness.pdf>.
- Lewkowicz, D., & Lickliter, R. (Eds.) (1994). *The development of intersensory perception*. Hillsdale, N.J.: Routledge.
- Lickliter, R., & Bahrick, L. E. (2000). The development of infant intersensory perception: Advantages of a comparative convergent-operations approach. *Psychological Bulletin*, 126(2), 260-280.
- Luria, A. R. (1984). *Sensación y percepción [Sensation and perception]*. Barcelona: Ediciones Martínez Roca.
- Luria, A. R., & Vygotski, L. S. (1998). *Ape primitive man and child. Essays in the history of behavior*. Boca Raton, Fl.: CRC Press LLC.
- Malafouris, L., & Renfrew, C. (2010). The cognitive life of things: Archaeology, material engagement and the extended mind. *The Cognitive Life of Things: Recasting the Boundaries of the Mind* (pp. 1-12). Cambridge: Mcdonald Institute.
- Marks, L. U. (2000). *The skin of the film: Intercultural cinema, embodiment, and the senses*. Duke University Press Durham and London.
- Maturana, H., & Varela, F. (1998). *The tree of knowledge*. Boston: Shambhala. (Original work published 1987).
- McLuhan, M. (1962). *The Gutenberg galaxy: The making of typographic man*. Toronto: University of Toronto Press.
- Peirce, C. S. (1931). *Collected papers, vol. I-VIII*. Cambridge, MA: Harvard University Press.
- Radford, L. (2002). The seen, the spoken and the written. A semiotic approach to the problem of objectification of mathematical knowledge. *For the Learning of Mathematics*, 22(2), 14-23.
- Radford, L. (2008). Iconicity and contraction: A semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM - the International Journal on Mathematics Education*, 40(1), 83-96.
- Radford, L. (2009). "No! He starts walking backwards!": Interpreting motion graphs and the question of space, place and distance. *ZDM - the International Journal on Mathematics Education*, 41, 467-480.
- Radford, L. (2010a). Algebraic thinking from a cultural semiotic perspective. *Research in Mathematics Education*, 12(1), 1-19.
- Radford, L. (2010b). The eye as a theoretician: Seeing structures in generalizing activities. *For*

- the Learning of Mathematics*, 30(2), 2-7.
- Radford, L. (2011a). Embodiment, perception and symbols in the development of early algebraic thinking. In *Proceedings of the 35th conference of the international group for the psychology of mathematics education* (pp. 17-24). Ankara: PME.
- Radford, L. (2011b). Grade 2 students' non-symbolic algebraic thinking. In J. Cai & E. Knuth (Eds.), *Early algebraization* (pp. 303-22). Berlin: Springer-Verlag.
- Radford, L. (2011c). Phenomenological mediations: Changing the subject (and the object!). Working Paper presented at the Phenomenology and Mathematics Education ϕ -ME First Meeting, Turin, November 2-3, 2011.
- Radford, L. (2012). On the development of early algebraic thinking. *PNA*, 6(4), 117-133.
- Radford, L., Bardini, C., & Sabena, C. (2007). Perceiving the general: The multisemiotic dimension of students' algebraic activity. *Journal for Research in Mathematics Education*, 38, 507-530.
- Radford, L., Demers, S., Guzmán, J., & Cerulli, M. (2003). Calculators, graphs, gestures, and the production meaning. In P. Pateman, B. Dougherty, & J. Zilliox (Eds.), *Proceedings of the 27 conference of the international group for the psychology of mathematics education* (Vol. 4, pp. 55-62). University of Hawaii: PME.
- Radford, L., Edwards, L., & Arzarello, F. (2009). Beyond words. *Educational Studies in Mathematics*, 70(2), 91-95.
- Radford, L., & Roth, W.-M. (2011). Intercorporeality and ethical commitment: An activity perspective on classroom interaction. *Educational Studies in Mathematics*, 77(2-3), 227-245.
- Radford, L., Schubring, G. & Seeger, F. (Eds.) (2011). Signifying and meaning-making in mathematics thinking, teaching and learning: Semiotic perspectives. Special issue of *Educational Studies in Mathematics*, 77(2-3), 149-397.
- Sheets-Johnstone, M. (2009). *The corporeal turn*. Exeter: imprint-academic.com.
- Seitz, J. A. (2000). The bodily basis of thought. *New Ideas in Psychology*, 18, 23-40.
- Trend, D. (2010). *The end of reading. From Gutenberg to grand theft auto*. New York: Peter Lang.
- Varela, F. (1999). *Ethical know-how*. Stanford: Stanford University Press. (Original work published 1992).
- Vygotsky, L. S. (1987-1999). *Collected works*. New York: Plenum Press. New York: Plenum.
- Vygotsky, L. S., & Luria, A. (1994). Tool and symbol in child development. In R. V. D. Veer & J. Valsiner (Eds.), *The vygotsky reader* (pp. 99-174). Oxford: Blackwell Publishers.
- Wilson, F. (1998). *The hand. How its use shapes the brain, language, and human culture*. New York: Pantheon Books.
- Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, 9(4), 625-36.
- Wessnitzer, J., & Webb, B. (2006). Multimodal sensory integration in insects—towards insect brain control architectures. *Bioinspiration & Biomimetics*, 1(3), 63-75.
- Zaporozhets, A. V. (2002). The development of sensations and perceptions in early and preschool childhood. *Journal of Russian and East European Psychology*, 40(2), 22-34.