# THE EYE AS A THEORETICIAN: SEEING STRUCTURES IN GENERALIZING ACTIVITIES

LUIS RADFORD [1]

Many years ago, in a landmark article, Lettvin, Maturana, McCulloch, and Pitts (1959) identified some image operations in the frog's visual system. Endowed with these basic operations (which include curvature and motion) the frog can, within certain limits, make perceptual distinctions. Thus, while it seems unable to see motionless prey and hence starves in a cage filled with dead insects (Roth, 1986), the frog is able to distinguish moving prey of different size (Anderson, 1993) and colour (Hatle & Salazar, 2001). With a considerably more complex visual system humans can discern a greater and more extraordinary panoply of differences and similarities. Were we humans left without this discerning capability, concept formation would simply be impossible. The world in front of us would be reduced to myriads of single and incommensurable facts: everything would be different from everything else and resemblances between things would be impossible to imagine. We would not be able to generalize, for as Kant (1800/1974) contended, generalization rests on synthesizing *resemblances* between *different* things and also *differences* between *resembling* things. But how do we learn to make distinctions? How do we learn to tell the similar from the different?

The basic operations of the frog's visual system are genetically built in and constitute what Lettvin *et al.* call, using a Kantian term, the *physiological synthetic a priori* – *i.e.,* the physiological substratum that allows the frog to see the world the way it does. What about humans?

Research dealing with perception suggests that babies start noticing resemblances and differences in the early postnatal years. Although at birth the pupils are not yet fully dilated leading to a limited capacity for object fixation and discrimination, visual sensitivity develops gradually over the course of the first years. By 5 to 6 months, infants become visually aware of their environment and eye and hand movements become coordinated. By 5 to 7 years, the basic functions associated with the child's cortical sensory areas have completed their development. The child's basic sensory processing abilities match the adult's (Atkinson, 2000; Farroni & Menon, 2008). At this age, we can think, in principle the child can see the world as the adult does.

In fact, this is not exactly the case. What could be called the human's *physiological synthetic a priori* turns out to be highly sensitive to the social context and the development of other sensorial organs that come to affect the way we see the world. The development of vision includes indeed several cortical and subcortical areas. As a result, like all sensory experience, visual experience "can influence the way the brain wires itself up after birth" (Farroni & Menon, 2008, p. 5). In the end, what we see is not the result of direct inputs but of stimuli already filtered by meanings and information about objects and events in the world – meanings conveyed by language and other cultural semiotic systems. Thus, in contrast to the frog's perception, rather than being a purely biological act, human perception is a social process through and through. It is, as Wartofsky put it, "a cultural artifact shaped by our own historically changing practices" (1984, p. 865).

The understanding of the social manners in which we come to perceive concrete things and generalize them in synthesizing experiences is, of course, a major theme of educational research. That there are many such manners, some definitely incompatible with others, is largely attested to by results from research conducted in different fields such as history, sociology, anthropology (Dzobo, 1980; Kawagley, 1990) and mathematics (*e.g.,* Bowers & Lepi, 1975; Crump, 1990; Harris, 1991). The issue is that despite what Piagetian and other rational epistemologies and their associated theories of human development have claimed, there are uncountable manners of abstracting and generalizing the always individual and contingent facts intuitively given to us by the senses and filtered by culture. In the course of our ontogenetic development, the senses and our understandings *become* shaped in certain historically formed ways as we engage in sociocultural practices.

In this article, I focus on a topic that has gained more and more interest in the past few years, namely generalization of, and pattern search in, elementary sequences (Mason, 1996; Moss & Beatty, 2006; Rossi Becker & Rivera, 2006). I shall focus specifically on the way in which teachers create the possibility for students to perceive things in certain ways and encounter a cultural mode of generalizing. This new way of perceiving sequences in certain efficient cultural ways entails a transformation of the eye into a sophisticated theoretician organ. This article is about just such a transformation. Drawing on the work of Vygotsky (1987) and Husserl (1931, 1970), in the first part of the article I present some theoretical ideas. In the second part I discuss two episodes from a Grade 2 class.

## Objectification

At birth, we all arrive in a world that is already replete with concrete and conceptual objects. The world in front of us is not the Adamic world of untouched nature but a historical world which, through objects and practices, conveys meanings and forms of reasoning – aesthetic, ethical, mathematical, scientific, and so on. Now, precisely because cultural forms of reasoning have been forged and refined through centuries of cognitive activity, they are far from trivial for the students. Learning, I submitted elsewhere (Radford, 2008a), can be theorized as those processes through which students gradually become acquainted with historically constituted cultural meanings and forms of reasoning and action. Those processes are termed processes of *objectification* (Radford, 2002; Radford, Miranda, & Guzmán, 2008). They entail a moment of *poēsis*: a moment of 'bringing-forth' something to the realm of attention and understanding. *Poēsis* is a creative moment of disclosure – the event of the thing in consciousness.

## The social dimension of objectification

Generally speaking, in school settings, the event of the thing in consciousness, or the encounter and disclosure of new objects of knowledge, occurs through specific classroom activities. Through these activities, knowledge is correlated with its objects. Indeed, objects of knowledge do not exist as Kantian "things-in-themselves": *they exist only in the form of activity.* It is here that the pedagogical design of activities – which includes forms of social interaction, and collaboration, artifact use, choice of problems, their sequence, *etc.* – acquires great importance. Through all its components, the classroom mathematical activity defines the extension and borders of a communal *zone of proximal development* that houses the students' processes of objectification. Although the zone of proximal development (ZPD) is perhaps Vygotsky's most frequently used concept in sociocultural educational research, it is unfortunately the least understood of all Vygotskian ideas. Usually it is quoted as the "discrepancy between a child's actual mental age and the level he reaches in solving problems with assistance" (Vygotsky, 1986, p. 187). And often, it is understood as a simple space of knowledge transmission: the space where the teacher dispatches knowledge to the student. [2] In other no less unfortunate interpretations, the ZPD appears as something *intrinsic* to the student. Indeed, the concept of ZPD is often presented as if the student *has* his or her own ZPD, regardless of the sociocultural context within which he/she develops. This simplification of Vygotsky's original idea overlooks the fact that the ZPD was Vygotsky's construct to account for the problem of the relationship between instruction and development. It overlooks the fundamental insight that distinguishes Vygotsky's approach from others, namely that instruction leads the course of development and that such a course depends on the kind of relationship that is created between the student and her context. This is why, rather than an absolute concept, the ZPD is a *relational* one (see also Schneuwly, 2008). In particular, it is forged out of the interaction between students, and between the students and their teacher. The ZPD is not a kind of well-delimited and rigid region that belongs to one particular student but a social, complex system in motion with evolving tensions, a point to which I shall come back later. The classroom example that follows focuses on the ontogenetic transformation of the students' perception. It explores such a transformation as a result of participating in something that seems to be missing in frogs, namely sign and artifact-mediated multilayered dynamic zones of proximal development.

## A classroom example

The following example comes from an ongoing longitudinal classroom study involving a Grade 2 class (7- to 8-year-old students). I shall focus on two passages from a generalizing activity. What I want to discuss is the manner in which the students' perception is transformed into a theoretical cultural form of perception required to tackle generalizing questions.

### Intention and perception

At the beginning of a five-day activity, the students and the teacher explored some sequences together. Figure 1 shows one of the sequences used in the classroom mathematical activity.

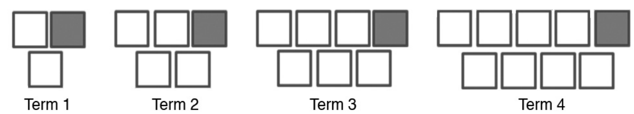


*Figure 1. The first four terms of a sequence investigated in a Grade 2 class*

In the first problem, the students were required to continue the sequence up to Term 6. In the next problem they had to discuss whether or not the drawing of Term 8 by Monique – an imaginary Grade 2 student – was exact (see fig. 2 for the statement of the problem and the accompanying figure).

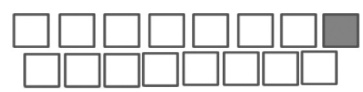

*Figure 2. Monique draws this figure and argues that this is Term 8 of the sequence. Do you agree?*

In dealing with problems 1 and 2, some students focused on the numerical relationship between consecutive terms, noticing that there were two more squares between one term and the next. They did not take advantage of the spatial clues suggested by the arrangements of the squares in each figure. Figure 3 shows two examples.

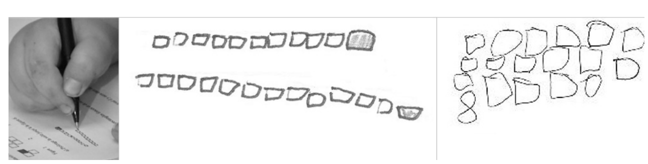

*Figure 3. (a) the moment in which a student (James) is drawing Term 6. (b) James' Terms 5 (top) and 6 (bottom). (c) Term 8 according to another student (Sandra)*

As shown on the middle part of Figure 3, Terms 5 and 6 of the sequence were drawn as having a *single* row each. In other cases, the figures were drawn as having *more than* two rows (see fig. 3, right). Why? Contrary to what empiricist psychology has claimed, the image of an object in consciousness is not the simple mapping of the object. What is grasped of an object in a perceiving act is not the object in its totality. This is true even of "simple" objects, for they present to us many attributes (color, shape, weight, odor, and so on). As Levinas (1989) remarked, the essential character of perception is to be *inadequate*. This is why it is not enough for the students to have the figures before their eyes. Instead of being complete, human perception is selective or, as Husserl said, *intentional*. In perception, Husserl (1931) argued,

> I am turned towards the object, to the paper, for instance…. Around and about the paper lie books, pencils, ink-well, and so forth … but whilst I was turned towards the paper there was no turning in their direction, nor any apprehending of them, not even in a secondary sense. (p. 117)

To apprehend the books, an intentional act (a form of *intuiting* it) has to come to the fore. In the same way, to apprehend the terms as divided into two rows, a specific intentional act has to lead perception, for "'intuiting' [an object] already includes the state of being turned towards" it (Husserl, 1931, p. 117). The problem for the students, then, is to attend to the figures in a certain intentional way. They have to go beyond the intentional stance focused on numerosity, which makes the figures appear in a certain way in consciousness, to a different one, based on rows.

It seems that here we enter a vicious circle. To perceive a feature $O'$ of an object $O$, a certain intentional act $I'$ is required – an intentional act that will make available the intentional object under the form of its feature $O'$. For, "[t]he intentional object … first becomes an apprehended object through a distinctively *'objectifying' turn of thought.*" (Husserl, 1931, p. 122, emphasis in original). But the point is, and Husserl (1970) repeats it over and over, the intentional act or manner of intuiting the object and the perceived object thus objectified do not constitute two distinct aspects of perception: they are together the very basic unit of perception. To put it differently, intention and object co-emerge in the objectifying perceptual process.

The question, however, remains: how do the students move from a 'mundane' or every-day phenomenological apprehension of the figures to a more sophisticated theoretical one? The question can be answered in two different ways. Since it is not just a matter of looking harder to the figures for the theoretical structure to become disclosed, one can argue that the mathematical problem or situation, if well designed, should by itself promote in the student the kind of scientific intention required. In other words, the intention and the mathematical structure should derive from the student's engagement with the mathematical problem. Adopting a strong rationalist epistemology, this way of reasoning supposes that the student's mind is somehow equipped with the necessary scientific intuitions to build intentions in a scientific way. The second answer rests on the idea of development: the students are not mature enough yet to perceive the scientific structure.

From the sociocultural theoretical perspective that I am advocating here none of these answers is satisfactory. The rationalist stance, upon which the first answer rests, makes it unconvincing. I mentioned in the introduction that recent research in anthropology and history casts serious doubts on the idea of a single "natural" line of conceptual development and questions the idea that human reason is moved by the logic of the Enlightened scientific-based Reason portrayed by Kant and the 17th-century rationalists that preceded him (Radford, 2008b). The second answer subjugates instruction to development. As mentioned previously, Vygotsky's idea of zone of proximal development shows that the relationship in fact goes the other way around.

### The domestication of the eye

Yet, to the experts' eyes, perceiving the figures as divided into two rows may seem a trivial endeavour. And surely it is. But it is so only to the extent that the mathematicians' eyes have been culturally educated to organize the perception of things in particular *rational* ways. The mathematicians' eyes have undergone a lengthy process of domestication. That such a process is not "natural" is proven not only by results from cross-cultural psychology (Geurts, 2002; Segall, Campbell, & Herskovits, 1966) but also by our young students' responses. The domestication of the eye is a lengthy process in the course of which we come to see and recognize things according to "efficient" cultural means. It is the process that converts the eye (and other human senses) into a sophisticated intellectual organ – a "theoretician" as Marx put it (Marx, 1998). Of course, I am not saying that the students did not see two rows. They surely did. But they did not deem it important to recognize the figures as being divided into two rows. Geometric clues were relegated to the background of attention to yield space to numerical matters. The capacity to perceive certain things in certain ways, the capacity to intuit and attend to them in certain manners rather than others, belongs to those *sensibilities* that students develop as they engage in processes of objectification.

Let me now get back to the students. As usual, the students worked in small groups of 2 or 3. When the teacher came to see the work of James, Sandra and Carla, the students had worked for about 31:50 minutes together. They had finished drawing Terms 5 and 6, answered the question about Monique's Term 8 (which they considered to be Term 8 of the sequence) and tried (unsuccessfully) to find the number of squares in Term 12. Noticing that the students were dealing with the sequences by adding two rectangles each time, the teacher engaged in collaborative actions to create the conditions of possibility for the students to perceive a general structure behind the sequence:

Teacher: Okay. … We are going to look at the squares at the bottom … just the squares at the bottom … [*emphasizing the word bottom and slowly moving her finger three times horizontally from Term 1 to Term 4, the teacher points to the bottom rows of the terms; see fig. 4a*], not those that are on the top [*pointing to the bottom row of Term 1*]. In Term 1, how many …?

Students: 1!

Teacher: [*pointing to the bottom row of Term 2; see fig. 4b*] Term 2?

Students: 2!

*Figure 4.* *(a) The teacher points slowly to the bottom rows of the terms. (b) The teacher and James point together to the bottom row of Term 2, while Carla (left) and Sandra (right) look attentively. (c) The teacher helps the student imagine Term 8 as being present in the sequence and points to its imaginary bottom row.*

Teacher: [*continuing to point and speak in a rhythmic manner, as she will do in the next interventions, she points to the bottom row of Term 3*] Term 3?

Students: 3!

Teacher: [*pointing to the bottom row of Term 4*] Term 4?

Students: 4!

Teacher: [*moving her hand to an empty space after Term 4, the space where Term 5 would be expected to be, she points to the imagined bottom row of Term 5*] Term 5?

Students: 5!

Teacher: [*moving her hand again to another space, she points to the imagined bottom row of Term 6*] Term 6?

Students: 6!

Teacher: [*similarly, pointing to the imagined bottom row of Term 7*] Term 7?

Students: 7!

Teacher: [*similarly, pointing to the imagined bottom row of Term 8; see Fig. 4c*] Term 8?

Students: 8!

Sandra: There should be 8 on the bottom!

Teacher: Excellent! Let's see if she [Monique] has 8 [squares] on the bottom [of her figure].

Sandra: [*counting the squares on Monique's figure*] 1, 2, 3, 4, 5, 6, 7, 8! Yes, she has 8!

Teacher: Very well. Now we are going to check the top [*twice makes a slow gesture to indicate the top rows of the figures*]. We'll look at the top.

The teacher repeated the same set of rhythmic pointing gestures as the students answered each of her questions. When they reached the top of Term 8 and figured out that there were 9 squares, she invited the students to verify Monique's drawing. The teacher pointed one after the other to the squares in the top row of Monique's figure while Sandra counted in a rhythmic way: "1, 2, 3, 4, 5, 6, 7, 8 …!?".

The students were perplexed to see that, contrary to what they believed, Monique's Term 8 did not fit into the sequence. Here activity reached a tension. Figure 5 shows Sandra's surprise. Sandra and the teacher remained silent for 2.5 seconds, that is to say, for a lapse of time that was 21 times longer than the average elapsed time between uttered words that proceeded the moment of surprise. [3]

### The poēsis of objectification

The previous interaction occurred in a zone of proximal development. This zone consisted of the various perspectives brought forward by the students and the teacher and was shaped by the manner in which the participants progressively switched, revised, and refined their theoretical stances towards the mathematical problem at hand. Zones of proximal development are indeed both relational and dynamic. They shift as the focus of attention shifts.

Now, while teachers cannot inject into the students' consciousness the object of knowledge, what they can do is to create the conditions of possibility for the students to transform the object of knowledge into an object of consciousness (Radford, 2006). In the classroom episode that I am discussing here, the teacher created the conditions of possibility for the students to perceive a general structure behind the sequence. And, as the example shows, the teacher did so by mobilizing key semiotic resources. The shift of attention from strict numerical matters to a more encompassing view of the sequence based on geometric-spatial clues was made possible through an intense recourse to pointing gestures, words, and rhythm. While gestures allowed the teacher to point to the bottom row of the figures and words to qualify the number of the term, rhythm

*Figure 5.* *(a) The teacher points to the last square in the top row of Monique's figure, while Sandra says "8". (b) Astonished by the result, Sandra stays still and silently looks at the teacher for 2.5 seconds.*

served the crucial cognitive function of creating a spatial-temporal order and expectation (You, 1994) that proved important in the synthesis on which generalization rested. The disclosing of the mathematical structure and the concomitant poetic moment of objectification resulted from the complex link of those *semiotic means of objectification* (gestures, words, rhythm) that accompanied and oriented the students' perceptual, aural, linguistic, and imaginative activity.

There is, I want to argue, an aesthetic experience in the poetic moment of the encounter with the emerging mathematical structure. The aesthetic experience and the poetic moment have unfortunately been overlooked by rationalist approaches that reduce them to a mere "cognitive conflict." There is much more to thinking than the cogitative ruminations of the mind. Sandra was deeply surprised to see that what had previously seemed so obvious (*i.e.*, that Monique's figure was *certainly* Term 8 of the sequence) turned out not to be so. This aesthetic experience is the opening towards a new form of seeing. The teacher's sustained look into Sandra's eyes in the long silent pause that followed the students' discovery is a crucial form of communication in which two consciousnesses meet in front of the cultural mathematical meaning. Becoming aware of these poetic moments may empower us teachers and mathematics educators to offer the students room to enjoy these precious instants.

### Counting the unseen

The previous episode took place towards the end of the math class. The next day, the Grade 2 teacher started the math lesson with a general discussion. She drew the figures on the blackboard and discussed with the class a counting method similar to the one used in Sandra's group at the end of the previous day. One month earlier, during the design of the activities with the teacher, we decided that it would be important to encourage several ways of perceiving the mathematical structure behind the sequence. With this idea in mind, the teacher appealed to a method that was devised by another group of students. The method consisted of conceiving of the figures as being divided into two rows, and counting separately the dark square. As one of the students put it in dealing with Figure 6, "We go 6 + 6 equal 12, plus one."

As on the previous day, the teacher illustrated the method through a complex use of gestures, words, and rhythm:

Teacher: [*pointing to the number of the figure*] Term 1 [*pointing to the bottom line*], one on the bottom [*pointing to the top*], one on top [*pointing to the dark square*], plus one.

Joined by the students, she counted in the same rhythmic way the other figures up to Term 5 (see fig. 6).

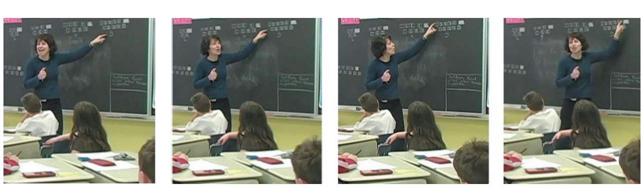

*Figure 6. The teacher and the students counting rhythmically say (a) "Term 5", (b) "5 on the bottom", (c) "5 on top", (d) "plus 1."*

The students were now able to tackle the activity's subsequent questions. Among these questions the students had to find out the number of squares in figures that were not perceptually accessible, such as Term 12 and Term 25. Here is an excerpt from the dialogue of Sandra's group as they discuss without the teacher:

Sandra: [*referring to Term 12*] 12 plus 12, plus 1.

Carla: [*using a calculator*] 12 plus 12 … plus 1 equal to …

James: [*interrupting*] 25.

Sandra: Yeah!

Carla [*looking at the calculator*] 25!

Then, a few minutes later, dealing with Term 25, Carla quickly says: "25 + 25 + 1 equals 51."

## Concluding remarks

In this article, I discussed the manner in which the students' perception was transformed into a higher and more sophisticated form of perception as required in the generalization of numerical-geometric sequences.

In the first part, I suggested that learning can be conceptualized in terms of processes of objectification, that is, those processes in the course of which, through action and reflection, the students come to notice and acquire fluency with certain cultural forms of mathematical reasoning. In the second part, I presented an example of becoming acquainted with a form of mathematical thinking required in generalization tasks. The generalization of numerical-geometric sequences like the one discussed in this paper required the students to perceive the terms of the sequence in a certain way. To attend something in a certain way, and to "intuit" it in a specific form, to use Husserl's concepts, requires a specific intentional act. The object as it appears in consciousness and the intention to perceive it are coterminous. The problem was then to account for that which would allow the students to move from an initial perception of the given terms of the sequence into a more sophisticated one. We saw that such a transformation was crucial to ensuring the students' increased fluency in a complex form of mathematical reasoning vital to their road towards algebra. This transformation, I suggested, is part of the *domestication of the eye*, or – if one so prefers – in educating the eye, that is to say, in converting it into a cultural-theoretical organ of perception.

Gaining fluency in complex mathematical forms of reasoning does not, of course, consist of a mere transmission of knowledge. It would be a mistake to see in the previous passages the teacher merely "transmitting" a generalizing method. The sophisticated form of generalization reached by the students during the activity required the emergence of certain *sensibilities*, such as perceiving the figures in certain ways that facilitate the calculations required to answer questions about Terms 12, 25, 100, or other figures beyond the perceptual sensorial realm. The *poetic moment* of disclosure of the general structure behind the sequence discussed in this paper was the result of a joint student-teacher interaction. This moment – the event of the thing in consciousness – was much more than a negotiation of meanings and an exchange of points of view. It was rather a Bakhtinian heteroglossic merging of voices, pointing gestures, perceptions, and perspectives. The poetic moment at the heart of the objectification process was made possible by the teacher's and the students' evolving shifts of attention in a zone of proximal development mediated by a complex configuration of semiotic resources – in particular, gestures, language, and rhythm.

Becoming aware both of how such resources can be mobilized and of the complexities behind zones of proximal development can be an important element in our understanding of the phenomena accompanying the learning and teaching of mathematics. This awareness might lead in particular to revisiting the teacher-student relationship formed in the act of knowing (Ligozat & Schubauer-Leoni, 2009). The students' accomplished generalization may not be as sophisticated as a symbolic generalization such as $2x + 1$, which could be expected from older students. Yet, although our young students' generalization is not expressed through alphanumeric signs, it is, I want to argue, algebraic in its own way. The "formula" through which such a generalization is expressed is not made up of letters and other signs (*e.g.,* "+"). It is made up of *actions*. It is better to see it as an *embodied formula* that, instead of being expressed through letters, is expressed through actions unfolding in space and time. The evolution of the students' embodied formula into more sophisticated, purely alphanumeric formulas requires, we can conjecture, a supplementary refinement of the eye and is, of course, a matter for further investigation.

## Notes

[1] This article is a result of a research program funded by The Social Sciences and Humanities Research Council of Canada / Le Conseil de recherches en sciences humaines du Canada (SSHRC/CRSH).

[2] This interpretation is embedded in a general and equally erring idea according to which teaching, in Vygotskian sociocultural approaches, amounts to knowledge transmission. Better than anyone else, constructivists have been instrumental in promoting such misunderstood interpretations (see, *e.g.,* Cobb & Yackel, 1996).

[3] These results come from an analysis that we conducted using the voice analysis dedicated software *Praat* v. 5.1.04, developed by P. Boersma and D. Weenink. See www.prat.org.

## References

Anderson, C. W. (1993) 'The modulation of feeding behavior in response to prey type in the frog *rana pipiens*', *Journal of Experimental Biology* **17**, 1–12.

Atkinson, J. (2000) *The developing visual brain*, New York, NY, Oxford University Press.

Bowers, N. and Lepi, P. (1975) 'Kaugel valley systems of reckoning', *Journal of the Polynesian Society* **84**(3), 309–324.

Cobb, P. and Yackel, E. (1996) 'Constructivist, emergent, and sociocultural perspectives in the context of developmental research', *Educational Psychologist* **31**(3/4), 175–190.

Crump, T. (1990) *The anthropology of numbers*, Cambridge, UK, Cambridge University Press.

Dzobo, N. K. (1980) 'The indigenous African theory of knowledge and truth', *The Conch* **7**(1–2), 85–102.

Farroni, T. and Menon, E. (2008) 'Visual perception and early brain development', in Tremblay, R. E., Barr, R. G., Peters, R. and Boivin, M. (eds.), *Encyclopedia on early childhood development,* Montréal, QC, Centre of excellence for early childhood development, retrieved May 5, 2009 from http://www.child-encyclopedia.com/documents/Farroni-MenonANGxp.pdf, pp. 1–6.

Geurts, K. L. (2002) *Culture and the senses*, Berkeley, CA, University of California Press.

Harris, P. (1991) *Mathematics in a cultural context: Aboriginal perspectives on space, time and money*, Geelong, AU, Deakin University.

Hatle, J. D. and Salazar, B. A. (2001) 'Aposematic coloration of gregarious insects can delay predation by ambush predator', *Environmental Entomology* **30**(1), 51–54.

Husserl, E. (1931) *Ideas. General introduction to pure phenomenology* (W. R. Boyce Gibson, trans., 3rd [1958] edn.), London, UK, George Allen and Unwin.

Husserl, E. (1970) *Logical investigations*, London, UK, Routledge and Kegan Paul.

Kant, I. (1800/1974) *Logic*, Indianapolis, IN, Bobbs-Merrill.

Kawagley, O. (1990) 'Yup'ik ways of knowing', *Canadian Journal of Native Education*, **17**, 5–17.

Lettvin, J. Y., Maturana, H. R., McCulloch, W. S., and Pitts, W. H. (1959) 'What the frog's eye tells the frog's brain', *Proceedings of the Institute of Radio Engineers* **47**(11), 1940–1951.

Levinas, E. (1989) *The Levinas reader* (Seán Hand, ed.), Oxford, UK, Blackwell.

Ligozat, F. and Schubauer-Leoni, M. L (2009) 'The joint action theory in didactics: why do we need it in the case of teaching and learning mathematics?', paper presented at the *Sixth Conference of European Research in Mathematics Education* (CERME 6), Université Claude Bernard, Lyon, FR, January 28–February 1, 2009.

Marx, K. (1998) *The German ideology,* New York, NY, Prometheus Books.

Mason, J. (1996) 'Expressing generality and roots of algebra', in Bednarz, N., Kieran, C. and Lee, L. (eds.), *Approaches to algebra*, Dordrecht, NL, Kluwer, pp. 65–86.

Moss, J., and Beatty, R. (2006) 'Knowledge building and knowledge forum: Grade 4 students collaborate to solve linear generalizing problems', in Novotná, J., Moraová, H., Krátká, M. and Stehlíková, N. (eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, Prague, CZ, **4**, pp. 193–199.

Radford, L. (2002) 'The seen, the spoken and the written: a semiotic approach to the problem of objectification of mathematical knowledge', *For the Learning of Mathematics* **22**(2), 14–23.

Radford, L. (2006) 'The anthropology of meaning', *Educational Studies in Mathematics* **61**, 39–65.

Radford, L. (2008a) 'The ethics of being and knowing: towards a cultural theory of learning', in Radford, L., Schubring, G. and Seeger, F. (eds.), *Semiotics in mathematics education: epistemology, history, classroom, and culture*, Rotterdam, NL, Sense Publishers, pp. 215–234.

Radford, L. (2008b) 'Culture and cognition: towards an anthropology of mathematical thinking', in English, L. (ed.), *Handbook of international research in mathematics education (2nd edn.)*, New York, NY, Routledge, Taylor and Francis, pp. 439–464.

Radford, L., Miranda, I. and Guzmán, J. (2008) 'Relative motion, graphs and the heteroglossic transformation of meanings: a semiotic analysis', in Figueras, O., Cortina, J. L., Alatorre, S., Rojano, T. and Sepúlveda, A. (eds.), *Proceedings of the Joint 32nd Conference of the International Group for the Psychology of Mathematics Education and the 30th North American Chapter,* Morelia, MX, Cinvestav-UMSNH, 4, pp. 161–168.

Rossi Becker, J. and Rivera, F. (2006) 'Establishing and justifying algebraic generalization at the sixth grade level', in: Novotná, J., Moraová, H., Krátká, M. and Stehlíková, N. (eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, Prague, CZ, **4**, pp. 465–472.

Roth, G. (1986) 'Neural mechanisms of prey recognition: an example in amphibians', in Feder, M. E. and Lauder, G. V. (eds.), *Predator-prey relationships: perspectives and approaches from the study of lower vertebrates*, Chicago, IL, The University of Chicago Press, pp. 42–68.

Schneuwly, B. (2008) *Vygotski, l'école et l'écriture*, Genève, FR, Université de Genève, Cahiers de la section des sciences de l'éducation, vol. 118.

Segall, M. H., Campbell, D. T. and Herskovits, M. J. (1966) *The influence of culture on visual perception*, Indianapolis, IL, Bobbs-Merrill.

Vygotsky, L. S. (1986) *Thought and language* (A. Kozulin, ed.), Cambridge, MA, MIT Press.

Vygotsky, L. S. (1987) *Collected works*, New York, NY, Plenum Press.

Wartofsky, M. (1984) 'The paradox of painting: pictorial representation and the dimensionality of visual space', *Social Research* **5**(4), 863–883.

You, H. (1994) 'Defining rhythm: aspects of an anthropology of rhythm', *Culture, Medicine and Psychiatry* **18**, 361–384.