

# ON THE INFLUENCE OF CONSTRUCTIVISM AND THE THEORY OF DIDACTICAL SITUATIONS ON THE THEORY OF OBJECTIFICATION

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*This article deals with the influence of North American constructivism and the French theory of didactical situations on the theory of objectification. Understanding "influence" in a dialectical Hegelian sense, the purpose of the article is to show how the concepts of knowledge and the concepts of the subject (i.e., the individual) of constructivism and the theory of didactical situations shaped the corresponding concepts in the theory of objectification. The shaping of these concepts is described through a process of sublation informed by a theoretical archaeological expedition of concepts in the early modern period.*

## INTRODUCTION

At the end of the 19<sup>th</sup> century, French philosopher André Lalande contended that "toute l'histoire de la philosophie montre qu'une théorie ne dépend pas moins de celles qu'elle réfute que de celles qu'elle continue" (1899, p. 1). I would argue that this claim is also true of theories in mathematics education. However, I would also argue that things are more complex. A theory B does not need to refute or continue another theory A to be profoundly affected by it. Theory A can be a great source of inspiration in the constitution of the key concepts of B even if it does not refute or prolong A. In other words, the way theory B relates to A and/or other previous theories seems to me to be much more complex than the lines of refutation/continuation. There are important influences that escape this schema.

As Hegel argued in his *Shorter Logic*,

This is the true meaning of a much misunderstood phenomenon in the history of philosophy—the refutation of one system by another, of an earlier by a later. Most commonly the refutation is taken in a purely negative sense to mean that the system refuted has ceased to count for anything, has been set aside and done for. Were it so, the history of philosophy would be, of all studies, most saddening, displaying, as it does, the refutation of every system which time has brought forth. Now although it may be admitted that every philosophy has been refuted, it must be in an equal degree maintained that no philosophy has been refuted. (Hegel, 2009, pp. 223-224)

In this paper I would like to discuss the influence that North American constructivism (as articulated for Cobb and Yackel (1998) and von Glasersfeld (1995) among others) and the theory of didactical situations (Brousseau, 2005) have had on the theory of objectification. In particular, I would like to focus on the concept of knowledge and the concept of the subject (le sujet, l'individu).

## INFLUENCES

“In the passage of different into different, the different does not vanish: the different terms remain in their relation.” (Hegel, 2009, p. 260)

In a naïve sense, everything is influenced by something else. The influence I am talking about here is more specific; it has a dialectical materialist sense. Let me try to explain it.

From a dialectic materialist perspective, when an entity B emerges, it emerges from something else, say A. In the emergence of B, links or relations between A and B are created. Think now of B as a theory emerging from a collection of theories. It means that between theory B and a collection of theories  $A_1, A_2, \dots$  that B considers while building its theoretical apparatus definite links between B and the  $A_i$  are created.

Through those links, theory B enters into a dialectical relation with theories  $A_1, A_2, \dots$ . These links allow B to see how a certain theoretical construct C (e.g., “knowledge,” “subject”) is framed, theorized, and thematized in  $A_1, A_2, \dots$ . Although B does not need to build the construct C (we’ll call it  $C_B$ ), as an expansion or generalization of the corresponding constructs as found in  $A_1, A_2, \dots$ , the dialectical link is such that the form and content of  $C_B$  is profoundly affected by them. Hegel provides us with a term and an idea to better express my argument. Hegel talks about sublation, the sublation (or preservation) of something into something else, which, by being something else appears as an “opposite” entity to that which is sublated. Let’s see these ideas in some detail.

In *Science of Logic*, Hegel says: “Nothing is immediate” (1969, p. 107; emphasis in the original). What Hegel means is that any entity is always derived from something else, it comes from something. “What is sublated . . . is the result of mediation” (p. 107; emphasis in the original). Then, he explains that “what is sublated . . . is not thereby reduced to nothing” (p. 107). The derived entity hence keeps in itself the “the determinateness from which it originates . . . Thus what is sublated is at the same time preserved; it has only lost its immediacy but is not on that account annihilated” (p. 107).

Along with the preservation of the source entity in the new entity, Hegel also recognizes a difference. The concept of difference is relational: A is different from B. And by being different, B does not only distinguish itself from A, but negates A—it negates A as something B is not, as fractions negate whole numbers. Hegel’s concept of negation, which attempts to go beyond the concept of negation of classical logic (a concept that does not tolerate that something can be and not be at the same time), sheds a new light on the idea of sublation. Sublation is the result of a mediation that not only emphasizes the preservation of the source entity in the new entity but also exposes the latter as an opposition or negation of the former; that is, as something the first is not. This is why “Something is sublated only in so far as it has entered into unity with its opposite” (Hegel, 1969, p. 107).

In the preface of the *Phenomenology of the Spirit*, the idea is presented through a biological metaphor: the bud, the blossom, and the fruit. First the bud disappears in the bursting-forth of the blossom, and one might say that the former is refuted by the latter; similarly, when the fruit appears, the blossom is shown up in its turn as a false manifestation of the plant, and the fruit now emerges as the truth of it instead. These forms are not just distinguished from one another, they also supplant one another as mutually incompatible. Yet at the same time their fluid nature makes them moments of an organic unity in which they not only do not conflict, but in which each is as necessary as the other; and this mutual necessity alone constitutes the life of the whole. (Hegel, 1977, p. 2).

In the language of the *Science of Logic*, the bud is sublated in the blossom as the blossom is sublated in the fruit. Not only the fruit comes from the bud and the blossom, but the fruit, by being different, negates them. This negation is not a simple opposition. It is an opposition from where the entities acquire new meanings. Indeed, the three entities create a new unity—a dialectical unity of units, each unit bestowing meanings on the others, meanings that they did not have before.

Michael Inwood uses the following example to explain these ideas:

Before the emergence of Protestantism, Catholicism is just (Western) Christianity as such. It then generates Protestantism, which negates it. Protestantism is not just non-Catholicism, but actively differentiates itself from it and bears the marks of the Catholicism that it negates. Catholicism in turn negates Protestantism, thereby ceasing to be simply Christianity as such and bearing the marks of its active self-differentiation from Protestantism. (Inwood, 1992, p. 201)

We could also use the example of intuitionism in mathematics (Brouwer, 1952). Intuitionism emerges from classical Aristotelian mathematics as something different; it negates its source. But by coming into life, intuitionism sheds light on classical mathematics, and vice versa. Žižek has famously used this Hegelian idea to understand how the present and the past continuously rewrite each other (see, e.g., Žižek, 2010).

Hegel's philosophy rests on an organic view of life that, contrary to other philosophical systems, conceives of living organisms, not only biological organisms but also ideas, and systems of ideas (theories, we may say), as mutually interrelated. Their interrelations come out of mediations that account for their historical transformations allowing us to see living organisms as "moments" (historical moments) of a whole in motion.

One of the limits of Hegel's biological metaphor is that, in matters of human phenomena, sublation—that is, the transformation of something into something else that keeps ineluctably in itself the traces of the sublated form—does not

follow a natural course. The sublation of concepts of the subject or concepts of knowledge cannot be approached as natural phenomena.

We must bear in mind that sublation is at the heart of the “dialectical moment,” the moment of movement and transformation. The dialectical moment, as Marx (2007) and Ilyenkov (1977) argued later, can only be understood in the comprehensive context of history and culture, for it is there that the mediations find their force.

To recap, the sublated form is a central part of the derived form that sublates it. One and the other appear as different, as one negating the other, but in the dialectical movement of this negation, they are both enriched by acquiring meanings they did not have before. This is the idea that I want to explore in discussing how the concepts of knowledge and subject (the teacher, the student) in the theory of objectification have been deeply shaped by those of constructivism and the theory of didactical situations. Although different, the theory of objectification’s concepts of knowledge and subjectivity could hardly have appeared in the way they did without a continuous reference to the concepts of constructivism and the theory of didactical situations (TDS).

## **THE SUBJECT IN CONSTRUCTIVISM AND THE TDS**

One of the chief characteristics of the learner in constructivism and the TDS is the learner’s autonomy. Autonomy is modernity’s answer to the question of the relation between the individual and the social. The relation that lies at the heart of the answer is one that, for the first time in Western history, gives precedence to the individual over the social. In premodern times, the individual was certainly a physical entity, but it was rather conceived as an integral part of a larger unit (Marcus & Fischer, 1999). Indeed, medieval political treatises portray the societies of the time as a sort of body arranged by a natural transcendental order; in this body the individuals found their own positions and roles to play (Adams, 2009). What was it then that made the individuals come to see their relationship with the social in a different way, to see themselves as autonomous beings?

The concept of autonomy grew up out of the progressive collapse of feudal structures and the rise of artisanal capitalism in the late Middle Ages. Indeed, the emergence of market towns and the bourgeois world led to the transition from what the German sociologist Norbert Elias (1991) describes as “external consciousness” to “self-consciousness,” the former referring to doing things as one is told, the latter to doing things as oneself decides to do. Elias says,

One can see more clearly in retrospect how closely this new form of self-consciousness was linked to the growing commercialization and the formation of states, to the rise of rich court and urban classes and, not least, to the noticeably increasing power of human beings over non-human natural events. (Elias, 1991, pp. 97–98)

Historically speaking, autonomy is the name of the societal movement of emancipation of the early modern individual from the medieval order. It consisted

in conceiving of oneself as the origin of authentic action and thought (Taylor, 2003). This concept of autonomy accompanied indefatigably the unfolding of modernity, finding a special place in Piaget's work. In his reflection on the future of education, we find him asserting that "The goal of intellectual education is not to know how to repeat or retain ready-made truths (a truth that is parroted is only a half-truth). It is in learning to master the truth by oneself" (Piaget, 1973, p. 106; my emphasis).

Von Glasersfeld's (1995) book, *Radical Constructivism*, draws heavily on Piaget's work. In this book von Glasersfeld purposely draws on the aforementioned conception of the autonomous individual. It does not then come as a surprise that, in this seminal book, to the eyes of the individual, the social world appears in a way that would have scandalized the ancient Greeks, namely as a kind of space comprised of independent autonomous monads, a space where these monads jump in and out at their convenience, like in a market. Naturally, this modern social world is made up of relations. As Jean-Jacques Rousseau (2012) noticed, these relations are of a very specific nature: they are contractual. Thus, individuals relate to each other, they collaborate among themselves; but they do it from their individual perspective and interests, exactly in the manner Piaget (1967) does when he tries to understand interaction in his *Études sociologiques*. This is why, as von Glasersfeld reminds us, autonomy has to do with "the need [of the individual] to manage with what is available" (1995, p. 147). Von Glasersfeld finds in this principle of autonomy the principle that "governs the construction of human knowledge, and therefore lies at the root of all epistemology" (p. 147). It is against the background of the autonomous subject that we need to understand the main theoretical claim of constructivism, namely that "knowledge is actively built up by the cognizing subject" (p. 51).

What happened when these ideas were transposed to education? For one thing, constructivists have emphasized the idea that it is the student and only the student who constructs her own knowledge; the teacher cannot just pass on knowledge to the student. The theoretical assumptions about the individual have given rise to a pedagogy where the teacher is pushed to the margins of the student's processes of learning. The teacher is there to help but she must be very cautious not to trespass into the student's autonomous space of action. If she trespasses on the thin frontier separating herself from the student, then she may find herself imposing her own view on the student, thereby ruining the autonomous learning project, the only possible one in the constructivist account (for details, see Radford, 2014).

Because constructivism and the TDS (Brousseau, 2005) have drawn on Piaget's epistemological principles, it is not surprising to see similarities in terms of the way they conceptualize the student. In both approaches autonomy is part of the theoretical backbone to witness the concepts of a-didactical situations, the Topaze Effect, devolution, and the TDS's conception of learning as something that the

student has to accomplish independently of the teacher. However, there are differences, some of which come from a different understanding of knowledge.

### **KNOWLEDGE IN CONSTRUCTIVISM AND THE TDS**

The constructivist emphasis on “the need [of the individual] to manage with what is available” (von Glasersfeld, 1995 p. 147) leads to a subjective radical concept of knowledge. Knowledge is what the individual does or produces in order to cope with the situations in which she finds herself immersed. All knowledge is simply “viable” (von Glasersfeld, 1995). Although the TDS adopts a Piagetian adaptive view of knowledge, it does not see this adaptation in the same way. The TDS adds that knowledge results from the optimal solution to a certain situation or problem. Of course, what is optimal is relative. Something can be optimal in one culture but not in others. The optimal in the TDS must be understood in terms of mathematically optimal; that is to say, in the sense of the optimal according to what came to be, historically speaking, standard Western mathematics.

We are ready to come back to Hegel’s ideas to better understand the question of the influence that theories have on each other.

### **THE PROCESSES OF MEDIATION**

One way to understand a theory is perhaps to start by considering what questions the theory tries to address. The research questions provide us with a sense of what the theory wants to do. Next, we can move on to how the theory tries to answer its questions. Finally, we want to understand the theoretical principles of the theory, from where the theory intends to start to reach its ends. Although useful to understand what a theory is, this strategy does not help much in understanding the genesis of a theory. So, the what-how-where strategy needs to be supplemented with something else. It is at this point where the processes of mediation are important.

As I mentioned before, in a process of mediation an entity (e.g., a concept) is sublated; that is, transformed into something else. Let us look first at the concept of knowledge in the TO.

#### **The role of the sublated forms**

Because the intention of the theory of objectification was to provide a cultural-historical account of learning, the concepts of knowledge conveyed by constructivism and the TDS did not seem to fit the theoretical project. In the case of constructivism, the subjectivist and individualist concept of knowledge was at odds with the sought-after concept. In constructivism, knowledge is a psychological entity. We needed something of a different order. In the case of the TDS, knowledge is not seen as a psychological entity. But because of its ubiquitous commitment to standard Western mathematics, it did not seem to have

the encompassing cultural generality we wanted to cover in the TO.<sup>7</sup> The encompassing cultural generality requires us to bear in mind that, like truth, knowledge is always embedded in cultural, symbolic, political, and economic structures of authority and assent (Fernandez-Armesto, 1998, p. 67; see also Shapin, 1994; Detienne, 1996). To know something is to enter into a rhetorical structure that Foucault called the society's regime of truth:

Each society has its regime of truth, its 'general politics' of truth: that is, the types of discourse which it accepts and makes function as true; the mechanisms and instances which enable one to distinguish true and false statements, the means by which each is sanctioned; the techniques and procedures accorded value in the acquisition of truth; the status of those who are charged with saying what counts as true. (Foucault, 1980, p. 131)

The mediation I am describing (although I am not attempting to offer a precise historical reconstruction of it) already puts into evidence how the sought-after concept of knowledge in the TO is influenced by the concepts of knowledge in constructivism and the TDS. The TO's concept of knowledge does not arise "immediately," to use Hegel's expression; it arises in a process in which it is shaped by those concepts. In this process of mediation, there is a dialogue that helps the nascent entity recognize itself by acquiring bit by bit determinations it did not have before.

The sublated forms contribute in different ways. They are permanent reminders of important aspects of a concept of knowledge. One reminds us of the subjective presence and role of the individual; the other reminds us of the cultural logic that organizes knowledge.

The nascent entity could be seen as a synthesis of the sublated forms, at least in the sense that these forms are constitutive parts of it. But a synthetic view may put us at risk of forgetting the dialectical movement that remains between these forms and the result of the process of mediation. Indeed, things are more complex. A synthetic view fails to capture the fact that new relations are crafted between the three entities—knowledge in constructivism, in the TDS, and the TO. In other words, the process of mediation is not merely a process of emergence of something from something else but also a process through which relations are created. We need to better understand this process. We need to consider the constant presence of the general project of the theory, which acts as a remote lighthouse.

### **The general project**

If there were no general project, all theories informed by constructivism and the TDS would end up at the same point. Obviously, this is not the case. The process

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<sup>7</sup> During a coffee break after a presentation I made at a conference organized by Bruno D'Amore in Locarno in 2004 (see Radford, 2005), in front of a table full of cookies and refreshments, Guy Brousseau told me that he liked the presentation but did not see much of "his" mathematics in it.

of mediation is animated and vitalized by its own telos. Its starting point is the concepts at hand from which the TO's concept of knowledge tries to differentiate itself. Now to move, to be put into motion, the process of mediation requires a reference point (un repère) so that it can move in a certain direction. However, rather than being something precise, definite and definitive, and worked out in all its details, the repère of the general project is refined as the process of mediation unfolds. To continue with our metaphor, the lighthouse moves. It is never still. In actual fact, it is produced in the very movement of the process of mediation—in this movement that is informed by the cultural-historical setting and the educational project it tries to respond to. It is also crucially informed by the confrontation of the theory with its effective reality; that is, with the educational settings (the school, academia, etc.).

One of the earliest explicit formulations of the TO's concept of knowledge appeared in 2004 through the idea of mathematical objects “seen as fixed patterns of activity embedded in the always changing realm of reflective, semiotic- and artefact-mediated social practice” (Radford, 2004, p. 4). This formulation includes the constructivist emphasis on the individual but articulates it in a way that the individual appears sublated in an activity that is more than merely her; it is an activity with others (Radford, 2022). Mathematical knowledge would be not the activity itself but something more general: a pattern of activity.

This formulation also includes the cultural-historical dimension of knowledge in the emphasis that this activity is situated in social practices of a different nature (not only the ones of standard Western mathematics) and the claim that these activities (and hence knowledge) is always changing. No wonder that, when presented at the CERME 7 group of theories in mathematics education in Poland in 2011, this formulation ended up with mixed results. Constructivist-oriented scholars did not recognize their concept of knowledge in it. The process of mediation is such that the sublated form is not easy to recognize. It is far from evident to see that “what is sublated is . . . preserved; it has only lost its immediacy but is not on that account annihilated” (Hegel, 1969, p. 107) as mentioned above. From the 2004 formulation to the present one, the process of mediation continued, highlighting and/or making explicit new features; for example, that knowledge is a cultural-historical entity, not a psychological one. Knowledge is a general entity in the Hegelian sense; it is a dialectical system of ways of thinking, action, and reflection constituted historically and culturally out of material, embodied, and sensible collective labour (Radford 2021).

The articulation of the question of the subject (the individual) in the TO followed a similar process of mediation. As discussed before, in constructivism and the TDS, one of the main features of the student is considered to be her autonomy, understood as an emancipating relation vis-à-vis authority—the authority of the teacher. From a Vygotskian perspective, the question of the student is put in different terms, as clearly illustrated in Vygotsky's (1987) concept of the zone of



proximal development where the teacher and the student work together to get things done. The challenge here was to come up with a new form of the subject where autonomy is not the leading principle. We had to completely rethink the leading type of relation (social relation) that would replace autonomy. Of course, this point led us to an altogether different conception of learning (Radford, 2018).

The departure point of the mediating process was the fundamental participation of the student in learning. But the process of mediation had to move in a direction that would show that this student finds herself in front of systems of mathematical thinking that were already there, in her culture, before her arrival in the world. Ontogenetically speaking, the object precedes the subject. We had hence to move from a conception of the student as one who creates or recreates knowledge to a conception in which the student encounters knowledge. One of the challenges was that the encounter with knowledge should not be seen as a mere receptive act, as in acquiring knowledge, which, as we were often told, seems to entail a passive subject. An invited seminar at the University of Ottawa around 2007 is a case in point. Objectification as a creative, active, sensible, and material encounter with cultural knowledge was our response. Learning, we argued at the Ottawa seminar, consists of actively and creatively noticing and making sense of cultural-historical knowledge (Radford, 2021, p. 81).

The sublated forms can be clearly discerned in the TO's emphasis on the active learner. However, after a lengthy process of mediation, this encounter has been theorized as one that involves more than merely recognizing cultural knowledge and its cultural logic; the encounter is also a site of transformation of teachers and students. The result is that in the TO, learning is not only about knowing but also about becoming. Learning moves around Knowledge and Being.

Out of the processes of mediation, the aforementioned concepts of the theories considered here enter into new relations, which give them meanings these concepts did not have before. We are, I believe, in a situation like the one mentioned by Inwood in his example of Catholicism and Protestantism. Each theory does not look the same; they form a new unity from where they see things under a new light, even if only to be in a position to better say what they are not. But this negation—this Hegelian negation—is part and parcel of what they are.

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