



Text

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On the epistemology of the Theory of Objectification

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In this paper I discuss some concepts that are part of the epistemology dimension of the Theory of Objectification—a Vygotskian theory of teaching and learning whose philosophical background comes from dialectical materialism. The epistemological concepts are presented in the first part of the paper. In the second part, they are illustrated through a classroom example.

Keywords: Knowledge, knowing, processes of objectification, learning, concept.

Introduction

This article seeks to contribute to the research field of theoretical perspectives in mathematics education. Its goal is to offer a short account of the manner in which a central, yet controversial, concept in mathematics education—the concept of learning—is conceived of in the Theory of Objectification (TO). To do so, this article describes the epistemological dimension of the TO as a dynamic whole system driven by dialectical relationships between knowledge, knowing, and learning. Drawing on a previous paper (Radford, 2013), I revisit a classroom example to illustrate the ideas presented in the article and to add a new epistemological element—the concept of *concept*. The example also provides insights about the interplay between the theoretical principles of a theory and its methodology and shows how the TO addresses a specific sensitivity of mathematical epistemology: one in which mathematical knowledge is considered as a cultural-historical entity in motion, an entity that is incessantly renewed and expanded and continuously brought to life by sensuous, practical, and material human activity.

Knowledge

In the Theory of Objectification, knowledge is understood as a *general entity* that, ontologically speaking, is already in the culture when we are born. Knowledge includes historical archetypes and culturally constituted processes of thinking, reflection, and action. Let us imagine a rural community that, in the course of time, has produced ways of thinking, reflecting, and doing things—for example, how to sow the earth, how to think about space, quantity, time, etc. These ways of thinking, reflecting upon, and doing things are general archetypes that constitute the knowledge of the culture. Let us now imagine a baby born at this moment in that culture. For this baby, those ways of thinking about the world, space, quantity, time, etc., appear as *possibilities*—possibilities of action and reflection. Another culture (e.g., a culture based on capitalist ways of commercial production in a contemporary European or North American country) will offer individuals born at this moment other possibilities; that is to say, other knowledge. These possibilities are *potential* actions/reflections, or *capacities* to do something. It is in this sense that knowledge can be considered as *potentiality*.

According to Aristotle, potentiality (δύναμις) is synonymous with power or disposition. Living beings and mechanisms have potentiality. A musical instrument, for example, has the ability to produce sounds. A fish has the ability to move in the water. *Actuality* (ἐνέργεια, *energía*), by

contrast, is the concrete happening of that which, before being put in motion, before being actualized, was potentiality. Potentiality is something undefined, without form, like a sound *before it is produced* or like the capacity of the fish *before it travels* in the water; something purely potential that, through *movement*, becomes materialized or actualized (*act*-ualized: transformed by an *act*-ion). However, the potentiality that living beings and mechanisms enjoy can be natural or acquired. The fish is biologically equipped to move in the water. Other potentialities or capacities are acquired, as Aristotle indicated (1998) in *Metaphysics* (1048a). This is the case of knowledge. Knowledge is this: *generative capacity, potentiality*. Algebraic knowledge, for example, is potentiality embedded in the culture: capacities that are offered to individuals in order to think, reflect, pose, and solve problems in a *specific way*.

Knowledge as the generative capacity of action and thinking changes from one culture to another and from one historical period to another. It would be a mistake, though, to think that the idea of knowledge I am outlining here stands in a Platonic line. The fact that when each one of us was born and was confronted with a series of scientific, ethical, aesthetic, legal and other forms of knowledge/thinking already established historically and culturally, does not mean that those forms of knowledge are Platonic forms, universal and timeless, independent of human labour. On the contrary, knowledge in each culture is produced by concrete people through their own labour—through their own actions, their own reflections, their joys, their suffering, and their hopes. To be more precise, I suggest that *knowledge is a system of embodied, sensible and material processes of action and reflection, constituted historically and culturally*. The adjectives *embodied, sensible, and material* mentioned in the previous definition signify that the processes of action and thinking are not mental cogitations occurring inside the head, but actions of real individuals who work and live in a social and cultural world. These actions are carried out through the body, the human senses, and through the use of physical objects and cultural artefacts.

Knowing

Knowledge is *related* to each one of its concrete instances or actualizations, but is, at the same time, *different* from each one of them. In its materialization or actualization, each of these concrete actualizations keeps in a sublated manner the generality of the ideal form that engenders it, but it *does not* coincide with the ideal form. In the TO, the actualization of knowledge has a specific name: *knowing*. Knowing is the concrete conceptual content through which knowledge is embodied and materialized or actualized. Although knowledge and knowing belong to two different ontological spheres—the former is *general*, the latter *singular*—they are interrelated in a dialectical manner and are part of a *dynamic whole system*. Knowing as the *actualization* of knowledge evokes indeed this *temporal* dimension of a whole in continuous movement. And what produces the movement is *activity*: knowledge and knowing are related through activity. Indeed, knowing can appear only through activity. This activity actualizes knowledge, brings it to life—like the activity of playing a violin brings musical notes to life, or the classroom activity of solving an algebraic equation brings algebraic knowledge to life. We can now state in a more precise way the relationship between knowledge and knowing: knowing is a sensible developed form of knowledge—much like the bud's example that Hegel offers in his *Phenomenology of the Spirit*: the blossom originates from the bud; it is the materialization or actualization of the bud, yet it does not

coincide with the bud. The blossom is a sensible developed form of the bud: although different, “their fluid nature makes [the bud and the blossom] moments of an organic unity . . . in which each is as necessary as the other; and this mutual necessity alone constitutes the life of the whole” (Hegel, 1977 p. 2). The *dialectical moment* is precisely the moment in which one becomes the other, the moment in which algebraic knowledge as *general* becomes transformed into something sensible, *singular*—that is, an object of consciousness. “It is of the highest importance,” Hegel notes, “to interpret the dialectical [moment] properly, and to [re]cognise it. It is in general the principle of all motion, of all life” (Hegel, 1991, p. 128). Up to here I have dwelled on concepts that have to do with aspects of the general theoretical stance of the TO. Now we enter into the educational-epistemological realm and deal with the concept of learning.

Learning

In student-centred pedagogies the student is considered to *construct* his/her *own* knowledge. No one can construct it for him/her. To *construct* a concept is equated to *learning* such a concept. In this conception, knowledge (K) appears as an *extension* of the subject (S). Since knowledge is not something different from the subject, but the subject’s *own* construction; in other words, since there is an identity between the thinking self and the products of its cogitations (Ilyenkov, 1977), this conception can be summarized through the equation: $S = K$. The intention behind the TO is to move beyond this individualistic stance.

To theorize learning, sociocultural theories have resorted to a series of concepts, such as enculturation (mainly formulated in anthropological research) and internalization (borrowed from Vygotsky’s work). I have argued elsewhere (Radford, 2018) that both concepts are insufficient to come up with an operational definition of learning from an educational perspective. To put it in a nutshell, the concept of enculturation adopts as its explanatory principle the idea of social practice, but leaves it uncritically analyzed. In enculturation approaches a social practice often amounts to what people *do*. Furthermore, in enculturation approaches the agentic dimension of individuals remains usually at the periphery. In Rogoff’s (1990) account, the individuals are certainly considered as active participants. But learning is conceptualized as *apprenticeship*; that is, something occurring through “the guidance and challenge of other people” (Rogoff, 1990, p. 19). In the end, learning is a process whose goal is to *adapt* oneself to existing social practices. Education is reduced to reproduction. There is little room to investigate education as transformation of people and the world. Likewise, there is little room to investigate the individuals as agentic entities, such as the manners in which the individuals come to position themselves and be positioned in social practices. There is little room to investigate the tensions that arise from the normative dimension of cultures (what Bakhtin, 1981, called a centripetal force) and the agentic movements of the individuals (the centrifugal force in Bakhtin’s terminology). A similar critique may hold for Vygotsky’s concept of internalization; that is, the “*transition of a [psychological] function from outside inward*” (Vygotsky, 1998, p. 170; emphasis in the original). It might be worth noticing that the content of Vygotsky’s concept of internalization (Вращивание – vraschivanie) is not learning, but the higher psychological functions (such as memory and perception). The problem that internalization seeks to explain is not how the child learns but how the higher psychological

functions arise from social relations, and how these functions evolve. How, then, is learning theorized in the TO? In the rest of this article I sketch the answer to this question.

Processes of objectification

As suggested earlier, in the TO, knowledge is considered as a culturally and historically constituted system of thinking and action. When each one of us was born, these systems (always in motion, always changing) were already there, existing in our culture in the form of knowing how to plant corn seeds, knowing how to calculate mortgages, etc. In other words, at birth, to each one of us, knowledge appeared as a cultural-historical generative, *latent capacity*. Our *encounter* with culturally and historically constituted systems of thought (e.g., mathematical, scientific, aesthetic, legal, etc.) is what in the TO is called *objectification*.

To understand the meaning of this encounter, let us bear in mind that the noun “objectification” tries to convey the idea that, before our encounter with knowledge, knowledge presents to us as something *different* from us: something that in its *alterity*, its own presence *objects* us; that is, resists or opposes us. The equation is: $S \neq K$. Before our encounter with knowledge, knowledge is the sign of a *difference*. *Object-ification* is the attempt to erase that difference. But because knowledge is an ideal (general) form always changing (constantly being recreated, refined, and expanded), the difference that the encounter tries to erase is not something that can happen totally. There is always a residue, a surplus that remains beyond our always local, situated, and concrete encounters with knowledge. As a result, objectification is always partial, a Sisyphean attempt at embracing knowledge—at becoming conscious or aware of it. “Object” in objectification does not refer to the verb “to objectify,” but to the verb “to object” (as when something, a desk, a chair, *objects* us). This is why, in the TO, in providing accounts of learning, instead of saying that students objectified knowledge, we talk about students engaged in *processes of objectification*. More precisely, *processes of objectification are those social, collective processes of becoming progressively conscious of a culturally and historically constituted system of thought and action*—a system that we gradually and partially notice and at the same time endow with meaning. Processes of objectification are those processes of attempting to notice something culturally significant, something that is revealed to the consciousness not passively but by means of the corporeal, sensible, affective, emotional, artefactual, semiotic, and creative activity of the individuals. In this context, learning is defined as the outcome of processes of objectification. And since systems of thought (mathematical, etc.) are always revealed partially, these processes are always endless—and hence, so is learning.

Processes of subjectification

Learning includes emotions and affect, not as merely concomitant phenomena of learning, but as *constitutive* parts of it. The educational implication is that instead of being a purely mental endeavour, learning mathematics involves emotions and affect in manners that touch and shape us profoundly. This is why classrooms do not produce knowledge only; they produce *subjectivities* (i.e., unique human beings) as well. In the TO, the investigation of the production of subjectivities in the classroom is carried out through the construct of *processes of subjectification*: the processes where, co-producing themselves against the backdrop of culture and history, teachers *and* students

come into presence. To come into presence refers to the idea of the student as someone who, through classroom activity, comes to *occupy a space* in the social world and to *be a perspective* in it. To come into presence is a *dialectical* movement between culture and the individual. The dialectical nature of this movement brings us to conceive of the individuals as entities in flux—entities who are continuously co-producing themselves and find in their culture the raw material of their own existence. Both the individual and culture are coterminous entities in perpetual change, one continuously becoming the other and the other the one. In this dialectical movement, students as well as teachers are considered as subjectivities in the making, openness towards the world. Teachers and students are conceptualized as unfinished and continuously evolving projects of life, in search of themselves, engaged together in the same endeavour where they suffer, struggle, and find enjoyment and fulfillment together.

Joint labour

In the TO, what makes learning possible is human *activity*. Processes of objectification and subjectification are embedded in activity. Now, the activity where learning occurs can be alienating. This is what happens in the classroom activity of both the traditional teaching and its pedagogy of knowledge transmission and the constructivist student-centred pedagogy of knowledge construction. In the first case, the students do not have room to *express* themselves. As a result, the activity *alienates* them from their own product—the knowledge that was produced in the classroom. In the second case, the student is involved in doing things and expresses herself. However, that expression remains confined to the subjective sphere of the self. Since knowledge is understood as that which is produced by the action of the student, the student is not in conversation with the world. There is a mere monological conversation of the subject with the subject itself. The student is alienated from the historical-cultural world and is confined to live in a “taken-as-shared” universe. The TO resorts to a different, non-alienating concept of learning activity. First, the teacher does not appear as a possessor of knowledge who is delivering or transmitting knowledge to the students or as someone scaffolding strategies to the students. Nor do the students appear as passive subjects receiving knowledge or as the authors of their own knowledge. Second, teaching and learning are not considered as two separate activities, one carried out by the teacher (the teacher’s activity) and the other carried out by the student (the student’s activity). In the TO, teaching and learning are conceptualized as a single and *same* activity: the same teachers-and-students’ activity. This concept of activity does not reduce activity to a series of actions that individuals perform, perhaps in coordination with each other, in the attainment of their respective goals. This line of thinking reduces activity to a functional and *technical* conception. In the TO, following Marx and Leont’ev, activity is a *form of life*, a kind of *energy* formed by the individuals in their pursuit of something common, an energy that is sensible and sensual, material and ideational, discursive and gestural. To avoid confusion with other meanings, in the Theory of Objectification, activity in the latter sense is termed *joint labour*. Joint labour is the chief ontological category of the TO and its unit of analysis. Sensuous, material joint labour is considered the ultimate field of aesthetic experience, subjectivity, and cognition. It asserts the fundamental ontological and epistemological role of matter, body, movement, action, rhythm, passion, and sensation in what it is to be human.

An example

I would like to refer to an example that comes from a Grade 4 class (9-10-year-old students) where the students were dealing with a sequence generalization problem based on the following story: “For his birthday, Marc receives a piggy bank with one dollar. He saves two dollars each week. At the end of the first week he has three dollars, at the end of the second week he has five dollars, and so on.” The teacher provided the students with bingo chips of two colours (blue and red) and numbered plastic goblets intended to represent Week 1, Week 2, etc., and invited the students to work in small groups to model the saving process until Week 5. Then, drawing on the model, the teacher invited the students to find the amount of money saved at the end of Weeks 10, 15, and 25. After some discussion, the students came up with an arithmetic strategy, the “doubling strategy”: they found the number of bingo chips in Week 5, doubled this amount and removed one bingo chip. The teacher came to see the students’ work and engaged in the conversation:

- 1 Teacher: (*Trying to make noticeable to the students the co-variational structure*) What do you remark about Week 5 (*She shows the goblet corresponding to Week 5*) and (*Pointing to the red bingo chips*) the number of bingo chips? (*Making the same actions*) The fourth week and the number of bingo chips?
- 2 Albert: (*Hesitantly and at the same time interested*) It’s always twice . . .
- 3 Teacher: (*Repeating*) It’s always twice.

This teaching-learning activity was the first one of a sequence of activities dealing with algebra. Algebraic knowledge already exists in the students’ culture. It is part of the school curriculum. However, until that morning, for the students, algebraic knowledge existed as a pure generative capacity of actions and thinking. Learning requires making algebraic knowledge something noticeable, an object of consciousness. The classroom activity was organized by the teacher so that, working collaboratively with the students, algebraic knowledge could be materialized or instantiated and so progressively it could manifest itself through one of its developed forms—i.e., as knowing. The three lines of the excerpt above show this progressive transformation of knowledge into knowing. Indeed, the mathematical variables started being noticed. They started becoming objects of consciousness. However, their co-variational algebraic nature remained unnoticed. Joint labour reaches here a tension that derives from the contradictory ways in which the terms of the sequence have been so far perceived (an arithmetical one, based on doubling, and an algebraic one, based on a co-variational approach to the problem). This contradiction is not a flaw of a didactical design: it is the very motor that keeps the activity unfolding. To encounter algebraic thinking as featured in the teacher’s didactical project, the teacher and the students have to keep working together to try to make the algebraic approach *appear* in the classroom and become an object of the students’ consciousness. Its appearance is a bit like the appearance of Beethoven’s 7th symphony: for it to become an object of consciousness it has to aurally appear through the activity of the orchestra. Since mathematics is simultaneously visual, tactile, aural, material, artefactual, gestural, and kinesthetic, it can only come into life here through the sensuous and artefactual joint labour of the teachers and the students.

After some discussion and failed attempts at making noticeable the algebraic structure of the bingo chips’ visual arrangement (see Figure 1, Pic 1), the teacher came back to an analysis of Week 5:

- 4 Teacher: (*Taking with her hand again the goblet of Week 5; see Pic 1*) What did you do here?
- 5 Albert: (*Takes a long breath while the teacher holds the goblet of Week 5; see Pic 2*) OK.
- 6 Teacher: (*Still holding the goblet, speaks softly*) 5 . . .
- 7 Albert: (*In sync with the teacher's gesture that points next to the red chips; see Pic 3*) Times 2 . . .
- 8 Krysta: (*Who has followed the discussion*) Times 2 equal . . .
- 9 Teacher: (*Pointing at the blue bingo chip; see Pic 4*) Plus 1.
- 10 Albert: (*Almost at the same time*) Plus 1.
- 11 Teacher: (*Now pointing to an empty space where Week 10 would be*) 10?
- 12 Albert: (*The teacher points silently at the place where the red bingo chips should be; see Pic 5*) Times 2.
- 13 Krysta: (*At the same time*) Times 2.
- 14 Teacher: (*Points silently at the place where the blue bingo chip should be; see Pic 6*)
- 15 Krysta: Plus 1.
- 16 Albert: (*Looking at the teacher*) Minus 1?, times 2, minus 1?, plus 1?

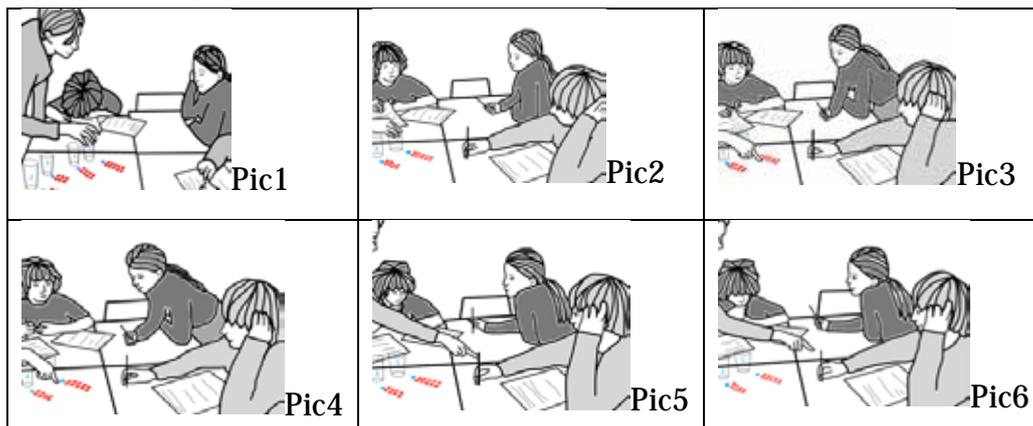


Figure 1: Joint labour and the appearance of an algebraic approach to solve the problem

The example illustrates three things. First, what joint labour *is*: a spatial-temporal *dynamic system* that is created by the students and the teacher. It is made up of the *energy* that the teacher and the students spend in trying to solve the problem and whose fabric includes language, gestures, perception, body position, and artefacts. It is a fluid carrier of half-expressed and half-understood intentions and motives. Second, the example shows some of the intricacies of the transformation of knowledge into knowing. Knowing is this specific manner in which algebraic knowledge is being *instantiated* from a particular problem and through joint labour (distinguishing the variables and their relational-functional nature, expressing it through language, gestures, etc.). Third, the example let us have a glimpse of how learning occurs. Indeed, the example shows how, wrapped in this energy of which joint labour is made up, the teacher moves her hand silently to indicate with an indexical gesture the imaginary position of the blue chip (see Pic 6). She is very tense, as the outcome of the joint labour is still uncertain. She awaits Albert's answer with a tension that is reflected in her body and language intonation. Her question is an *invitation* to Albert to come to position himself in a mathematical practice. The question is *already* a positioning. But still Albert

has to accept. He could have just given up. But he does not. Albert, who is also very tense, accepts the teacher's question/invitation and says "Minus 1? Times 2 Minus 1? More 1?" The answer attests to the fact that Albert is positioning himself and being positioned in a social practice where things are thought of in a certain manner. But the answer attests also to the fact that the co-variational algebraic manner by which to see the variables is progressively becoming intelligible to Albert's consciousness. More generally, in the joint labour we see the unfolding of a social process that is at the same time a process of subjectification and objectification. Albert is living an encounter with key aspects of algebraic knowledge. The difference between S and K in the equation $S \neq K$ is being reduced. Since knowledge is a *general*, the difference will never vanish. Even though, there is still room for Albert to perceive better the nuances of the algebraic variables and how they relate to each other. It did not take long. During the general discussion, which started right after the end of the previous excerpt, the teacher invited Albert to explain how to find out the number of bingo chips in Week 4. He said: "4 times 2 . . . plus 1, 4 times 2 plus 1 equals . . . 9."

Concept

As knowledge is put into motion through joint labour, it becomes materialized into something sensible; that is, knowing. In the course of its materialization, knowledge is *refracted* in the students' consciousness. This refraction is always different: it changes from student to student. A concept is precisely the subjective refraction of knowledge in consciousness through the mediation of knowing. A concept enables us to *do* things and to *think* about them in certain ways. While knowledge and knowing are historical-cultural entities, a concept is of a subjective order: the subjective and partial version of cultural knowledge. In the example discussed above, algebraic knowledge is refracted in Albert's consciousness—as the *concept* of algebraic generalization of (linear) sequences. Although still fragile, this concept, originated in activity, becomes an "organ" of Albert's body. It is something that allows and empowers him to perceive sequences and to think of them in a new way. No less important, the concept connects Albert with culture and history and transforms him at the same time into a cultural-historical subject.

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