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PLAY AND THE PRODUCTION OF SUBJECTIVITIES IN PRESCHOOL

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In this article, I explore the question of the production of subjectivities in preschool. The question is based on a well-known sociocultural principle according to which individuals are affected by their cultural-historical context. Following a Vygotskian idea, I claim, however, that this affection is not to be understood in a causal sense, but in a reflexive one. Hence, what this claim means is that while individuals are living agentic entities in a continuous process of transformation, the scope and parameters of their agentic dimension can only be understood against the backdrop of culture and history. It is in this sense that I investigate the manners in which preschool children produce themselves and, at the same time, are produced by their cultural setting. I draw on data involving preschool children playing a mathematical game. I focus in particular on the constitutive role (1) of rules in the making of the subjects, (2) the mathematical content, and (3) the teacher. The last part of the chapter is an attempt at showing that the question of the production of subjectivities is immersed in ethical issues that mathematics education can no longer avoid taking into account.

INTRODUCTION

Play has always been a popular topic in early childhood education. And, one way or another, it has been associated with the more general question of children's development. Indeed, despite the impressive variety of conceptions of play (see, e.g., Elkonin, 2005), play has usually been considered either as a *source* of development or as a *window* through which one can grasp the current state of the child's development.

In the latter view, play appears as a kind of methodological tool. This is the case of Piaget's conception of play. In observing children play, the children's understanding of rules can be made apparent. Reasoning along this line, Piaget (1948) suggested a series of successive stages which children undergo in play: children travel from a motor or individual understanding of rules where the driven force is the child's desires, to an egocentric stage where although playing together each child plays "on his own" (p. 16), to incipient and, later on, developed stages characterized by social forms of collaboration.

In the former view, by contrast, play appears as something that can potentially *influence* the child's development. For instance, Smirnova and Gudareva argue that "Play is of special importance for the formation of the child's motivational sphere and voluntariness" (2017, p. 252).

This chapter is about children playing mathematical games in a preschool setting. However, it goes in a different direction. Indeed, in this chapter, I am not interested in exploring how play allows children to develop mathematical ideas (the *play-as-a-source* view mentioned above that confines play to a mere facilitator of knowledge construction and intellectual growth). Nor am I interested in what we can learn about development in observing children play (the *play-as a-window* view that confines development to a natural unfolding process). I am interested in something different: I seek

to understand how, through mathematical ideas and play, children and their teachers co-produce themselves and, at the same time, are produced by their cultural-historical context.

To pose the problem of teachers and students as entities that co-produce themselves and, at the same time, are produced culturally and historically, is to adopt a theoretical position about humans that is at odds with the classical view articulated during the Enlightenment and that has come down to us through the work of Rousseau, Pestalozzi, Piaget, and the mathematics education movement of the 20th century epitomized in constructivism. In the enlightened tradition—that is, the European intellectual movement of the late 17th and 18th centuries that broke with tradition and emphasized individualism and reason (Horkheimer & Adorno, 2002)—the individual is portrayed as a constructor of ideas and the origin of her feelings, meaning, and intentionality. Kant, perhaps the most enlightened philosopher of the Enlightenment, illustrates better than anyone else this idea of the individual: the Kantian individual is a subject of reason, the crafter of her/his own destiny, the origin and source of meaning and knowledge. The result is a self-sufficient and substantialist conception of the individual: the *self-made* subject. In this context, the child appears as a *give entity*; that is, someone who, in order to develop her own intellectual capacities, simply needs a stimulating social environment (Martin, 2004).

In this chapter, I take a different route: I draw on a dialectical materialist philosophy and its conception of the human. Instead of being the origin of knowledge, feelings, meaning, and intentionality, the individual is conceptualized as an entity in flux, in perpetual becoming—an entity who, through practical activity (like play) is continuously inscribing herself in the social world and, in doing so, she is continuously produced and co-producing herself within the limits and possibilities of her culture. In the first part of the chapter I consider some theoretical ideas—such as subjectivity, subjectification, being, and becoming. These ideas frame the dialectical understanding of the child and her production in play offered here. In the second part, I discuss some video data that come from my current research in preschool settings. The last part of the chapter is an attempt at showing that the question of the production of individuals is immersed in ethical issues that mathematics education can no longer avoid taking into account.

THE PRODUCTION OF INDIVIDUALS IN AND THROUGH PLAY

At first sight, exploring the production of individuals in and through play may seem an esoteric endeavor. Why, indeed, could such a problem be interesting from the point of view of mathematics education? Two of the major theories in our field—constructivism and the theory of didactic situations (see, e.g., Radford, 2018a)—charted a research agenda for themselves and the theories that followed where our problem at hand hardly finds a niche. While constructivism is oriented towards the investigation of the child's "construction of increasingly powerful conceptual structures and the development of intellectual autonomy" (Cobb, 1988, p. 100), the theory of didactic situations is oriented towards the creation of the didactical conditions that are conducive to the diffusion of mathematical knowledge (Brousseau, 1997). As we can see, the problem of the individual is left unproblematized in both theories.

I draw here on the theory of objectification (TO)—a Vygotskian theory of teaching and learning (Radford, 2008, 2018b)—that inscribes itself in a different educational project: it posits the goal of mathematics education as a political, societal, historical, and cultural endeavor aimed at the dialectical

creation of reflexive and ethical individuals who critically position themselves in historically and culturally constituted mathematical practices, and who ponder new possibilities of action and thinking.

As a result, in the TO, the focus is not on the mathematical content alone; the focus is not only on *knowing* (the dimension of knowledge) but also on *becoming* (the dimension of the subject or the individual). As a result, a cogent understanding and explanation of how learning happens should include accounts of how students come to know (knowing) and to be (becoming). Therefore, instead of being something esoteric, the problem of the production of individuals in and through play (or other educational settings) appears as something of great importance.

To avoid misunderstandings, I hasten to say that I do not see the production of individuals as the deterministic result of social forces shaping an inert *tabula rasa* subject. However, I do not see the production of individuals as the mere auto-production of the self either. What I have in mind is a production of individuals whose most distinctive feature is to be *dialectical*: individuals are projects of life in the making; they produce reality as much as reality produces them.

To look at children and teachers in this dialectical manner is to depart from the view of the world as "some eternal and objective network of causal factors converging on [the individuals] to shape an unresisting, passive blob to their external pregiven [cognitive] structures" (Wartofsky, 1983, p. 188). To look at children and teachers in a dialectical manner is also to depart from the view that conceives of individuals in general and children in particular as "self-contained homunculus, radiating outward in development from some fixed configuration of traits, dispositions, or preformed potencies" (Wartofsky, 1983, p. 188). That is, a view where children and teachers appear as the origin of their own experience and the product of their own life. Unfortunately, we tend to believe that the experiences through which we allegedly auto-craft ourselves are something direct. We tend to forget that the way we experience ourselves and come to constitute ourselves as subjects is mediated by culture and history. As Michel Foucault notes

> The experience we make of ourselves seems to us to be the most immediate and the most original; but it has in fact its historically formed patterns and practices. And what we believe to see so clearly in us and with such transparency is given to us in fact through deciphering techniques painstakingly constructed throughout history. (Foucault, 2017, pp. 29-30)

I want to contend that it is only through a genuine dialectical understanding of individuals and their social, cultural, and historical contexts that we can unravel what Stetsenko and Ho (2015) call "one of the most complex paradoxes of human existence" (p. 224).

> This paradox is about being one among many, that is, about being a unique individual in an essentially communal world shared with others. The paradox involved is that human beings are singular and unique individuals, yet they are also profoundly relational and deeply social, sharing with other people no less than the existential grounding of life in all of its expressions and forms. (Stetsenko & Ho, 2015, p. 224)

It is against the background of this most complex paradox of human existence that, in this chapter, I want to continue exploring a line of inquiry that I outlined in previous papers (Radford, 2014, 2018c), where the central idea is that all educational settings—play included—ubiquitously produce not only knowledge but individuals too. Since we are entering almost uncharted territory, I need to introduce some theoretical constructs. I need to delve into more detail on the question of the individual and the role cultures play in the process of knowing and becoming. To do so, I need to start from the

beginning. I need to start with a brief discussion of a symbolic structure that, in each culture, defines the space of agentic maneuvering of the individuals and provides them with a definite sense of personhood.

SEMIOTIC SYSTEMS OF CULTURAL SIGNIFICATION

The starting point of the theoretical position that I want to explore here is that human subjectivity is entangled with its social, cultural, and historical contexts. Cultures, indeed, provide their individuals with the raw material of what they are. For instance, the very fabric of human subjectivity in ancient Mesopotamia was intertwined with the individuals' participation and their positioning in social and cultural activities such as agriculture, animal husbandry, or participation in religious events or military campaigns. These social and cultural activities out of which a sense of self emerged, were, in turn, shaped by the political, religious, and economic structures that provide the individuals with meaning to their life. It is in this context that individuals in ancient Mesopotamia learned to live and die (see, e.g., Crawford, 1991; Kramer, 1963; Reade, 1991). And so do we, in our own cultural-historical context. And because these contexts are different, we find ourselves confronted by a different range of possibilities concerning rights and obligations from those encountered by the Mesopotamians, the ancient Greeks and Chinese, etc. We find ourselves in front of a world with different political, economic, and legal apparatuses and, as a result, with a different space of *agentic maneuvering*. The scope of the space of agentic maneuvering is both facilitated and constrained by a *symbolic superstructure*. This symbolic superstructure encapsulates the distinctive features of a culture—e.g., its thematization of meaning production, the relationship between mind and reality, and the understanding of reality itself.

Symbolic superstructures have always puzzled philosophers, sociologists, and anthropologists. For example, adopting a Kantian position, Ernst Cassirer speaks of *symbolic forms*. Symbolic forms operate ubiquitously. They structure experience. For Cassirer (1955), language is the symbolic form *par excellence*: it is through language that, according to Cassirer, all forms of thought find meaning and expression. Abandoning the Kantian perspective, Hegel proposes a more dynamic vision in which the mind is considered as advancing historically (Hegel, 2001). Writing from a sociological perspective, Castoriadis (1987) speaks of the collective creation of symbolic webs that provide the individuals with the means to overcome the real and imagine new things. From the social, historical and cultural educational perspective in which the theory of objectification is inscribed, the question of the symbolic superstructure is articulated around the material production of life in all its spheres, and particularly around the production of knowledge, mainly around dominant forms of knowledge production and their political-economic character. In the theory of objectification, the symbolic superstructure is termed *Semiotic Systems of Cultural Signification* (SSCS). They are dynamic systems that originate in the practical and sensuous activity of the individuals. They comprise ideas about:

(a) the nature of the world (e.g., the nature of mathematical objects and their way of existing),

(b) truth (e.g., how truth is and can be established), and

(c) the nature of the individuals.

SSCS are full of tensions, as are the activities from where they emanate. They have a (implicit, explicit, or both) normative function and necessarily convey political and ethical views; for example,

how we show ourselves to others, how we are expected to behave socially and to be recognized by others.

To understand the operativity of SSCS and how the individuals' deeds are embedded in a web of historical, political, legal, and economic relations that circumscribe the concept of self, let me mention an example from premodern times. The example comes from a county court in medieval England, where a prestigious blacksmith individual, Richard Bourdeaux, was offended publicly by a lower-status butcher, William Webbe (for details, see Shaw, 2005). This act, which was socially sensed as a disruption of rules governing the honour ethic and hierarchically structured social order that defined premodern life, went to court. The insult was seen as an offense against God and the hierarchical status of the town. The sentence included a repentance about social behavior and a monetary penalty. While the repentance was issued as a means to protect and validate the structural relationships between the social categories of people involved, the monetary penalty was a way to repair the offense to society in the form of a charitable donation to help with the restoration of a church. An excerpt from the court record reads as follows:

> For this reason, the said William begged (*supplicavit*) the said Richard, out of respect for God and for charity's sake, in view of the entire meeting, that he earnestly hoped he would pardon him his abusive language (*maledictum*) and the slander (*verba de dicto Ricardo malelocuit*) he had spoken. Then the said Richard, at the request of the master and burgesses, remitted and relaxed to the said William all the said fine and evil deed (*malefactum*) on condition that he never in the future publicly or openly say or proclaim defamatory words about Richard, such as he previously spoke so violently and harmfully, on threat of 40s. sterling to be paid to the current or future Master within two weeks of the relapse. And the said money should be applied to the restoration of St. Cuthbert's Church. (Shaw, 2005, p. 127)

Commenting on this medieval example, Diehl and McFarland note that "being successful in disputes over honor was predicated, at least partly, on the ability of disputants to justify their [social] position by appealing to cultural beliefs about what persons like them (and their opponents) should or should not do" (Diehl & McFarland, 2010, p. 1735). Those cultural beliefs about persons and what they can do can only make sense through the effects of *Semiotic Systems of Cultural Signification*. They operate through a complex web of political, legal, and economic relations and come to shape the concept of self, offering a spectrum of socially recognized positions and providing them with an agentic space for human action. The agentic space for human action is organized through a conception of the nature of the individuals that demarcates, in particular, the ethics of situated actions. Such agentic space is enforced through a legal system that vigilantly seeks to keep society and its individuals in a certain harmony.

BEING, BECOMING, AND SUBJECTIVITY

Now, the relationship between the cultural "raw material" conveyed by the SSCS and the concrete individuals should not be seen in a causal or mechanical sense. On the one hand, as humans, we are unavoidably affected by our cultural-historical concrete context. This is part of our ontological makeup. It is part of what it means to be human. This is the point that the 17th-century philosopher Baruch Spinoza (1989) made in his *Ethics*—a book that had a tremendous influence on Vygotsky and Marx (Fischbach, 2014). However, individuals are not simply affected. They are affected in a *reflexive* manner. What reflexivity means here is that, in addition to being affected by their cultural-historical concrete context, individuals *react agentically* to such context. Vygotsky used to say that

what distinguishes us most from other species is not intelligence, but free will (del Rio & Alvarez, 1995; see also Tappan, 1998).

Thus, while what emerges from the effects of affection—i.e., the subject—bears the imprint of its culture, it always emerges as something different—different to others and to itself: the resulting subject is an "I" whose formula is "I ≠ I."

This formula captures the conception of the individual as, on the one hand, a dynamic concrete living agentic entity always in flux, in transformation, and, on the other hand, an entity whose agentic dimension can only be understood against the backdrop of culture and history. To refer to the individual in the aforementioned sense, I shall use the term *subjectivity*. To specify its sense a bit further, I need to turn to two related terms first: *being* and *becoming*.

Being, as I understand it here, is a *generative capacity* constituted of *cultural conceptions* of living in the world: ways of conceiving of oneself and of being conceived; ways of positioning oneself and of being positioned. In the previous example from medieval England, *being* includes those ways in which blacksmiths, butchers, cathedral builders, priests, soldiers, etc. conceived of themselves and were conceived by others. Those ways of conceiving of oneself and of being conceived by others are continuously *materialized* in the deeds and activities of the individuals. What materializes, however, does not coincide with the capacity that engenders it, for this capacity is a cultural, general, latent capacity. *Being* can only show itself through its materializations in the concrete world, where it can be recognized as what it is. *Being* a butcher, for example, is materialized in the *deeds* of William Webbe, as *being* a blacksmith is materialized in the *deeds* of Richard Bourdeaux. The always unfolding materialization or instantiation of *being* is related to *being*, but it does not coincide with it. William Webbe's deeds do not coincide with "butcher" (in the same way as the idea of a triangle does not coincide with any of its materializations). The materialization of *being* has a technical name: its name is *becoming*.

Now we can come back to the concept of subjectivity. A *subjectivity* is a unique sentient cultural concrete subject (William Webbe, Richard Bourdeaux, or a student or a teacher in our case) whose specificity results from the fact that it is continuously *reflectively* affected by *being* through its concrete materializations—an entity always in a process of be-com-ing: an unfinished and unending project of life. Moreover, because it is constantly reflectively affected by *being*, a subjectivity is an entity that "is inseparable from the space of moral issues [of its culture and] from how one ought to be" (Taylor, 1989, p. 112). To be a subjectivity is "being able to find one's standpoint in this space, being able to occupy, to *be* a perspective in it" (p.112).

Empirically speaking, subjectivities are investigated through what I have termed in previous papers as *processes of subjectification* (Radford, 2012, 2018b). That is, the activity-bound processes where, co-producing themselves against the backdrop of culture and history, teachers and students (and individuals in general) *come into presence.*

In the next part of the chapter, I seek to understand how, through play, children and their teacher co-produce themselves and, at the same time, are produced by their cultural-historical context. I draw on video data that come from my current research in preschool settings.

PLAYING A MATHEMATICAL GAME

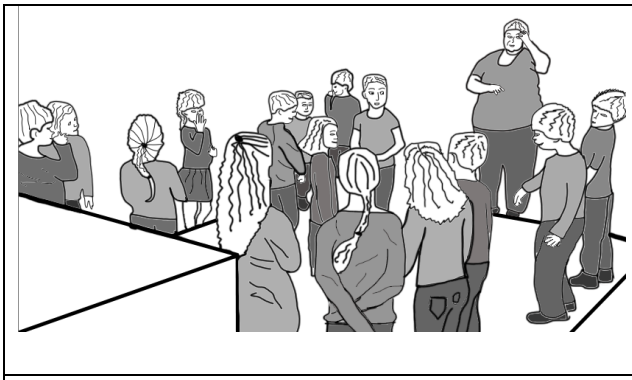
In general, two contemporary trends can be discerned about the role of preschool. One of them considers preschool as a space of socialization and play suitable for the intellectual and physical growth of the child. The other trend is not in opposition to the first one, but it considers preschool as a preparation for school. While the former is usually immersed in the romantic view of the child of the Enlightenment, the latter is more preoccupied with school readiness. While the former usually advocates free play, the latter usually advocates learning in settings that follow a similar—although simplified—structure to what children will find in Grade 1. Furthermore, the latter view gives special attention to literacy and numeracy. Without expecting that children acquire deep concepts of numbers and forms, preschools are considered as channels to ensure the children's first contact with mathematics. This is the case in the Canadian province of Ontario, where my example comes from. My example comes, indeed, from a preschool classroom of 4-6-year-old children and is about a mathematical game whose goal is to introduce children to counting.

The emphasized presence of mathematics at the preschool level is coherent with the purpose of Ontario's vision of the school: the preparation of the young for a highly technological society characterized by quick change and adaptability. Of course, this emphasis on mathematics (and language) is not something new. Since the dawn of the 20th century, mathematics came to occupy a privileged position in the school curriculum of those countries that saw in industrialization the path towards modern society. Mathematics became the ally and support of the new capitalist forms of production. To a large extent, the main problem of 20th-century educational reform was the problem of massive schooling to train the young in the participation and development of a technological society (Radford, 2004). One century later, things have not changed much. Capitalism has not vanished. It has become trans-national, diversified, and globalized. It is hence not surprising that the preschoolers I see entering the school every morning start the day with activities around counting. They start by singing, that is true. However, the content of the singing is about counting (see Figure 1, Picture 1):

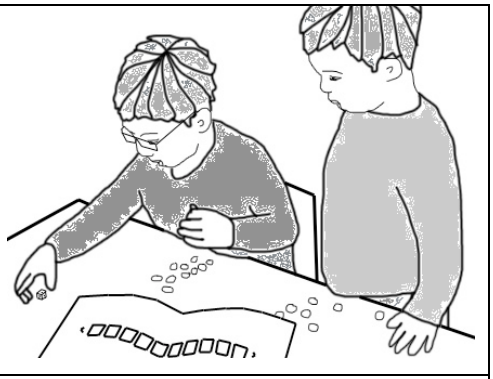
> One little lamb in my house that jumps and turns around.
>
> One, two, three, four, five.
>
> One, two, three, four, five.
>
> One, two, three, four, five.
>
> It helps me fall asleep.

If the school has to create producers, consumers, technologically oriented minds, and "entrepreneurs"—as an important official document in Ontario insists again and again (see Ontario Ministry of Education, 2014, pp. 1, 3, 4, and *passim*)—counting has to be the starting point.

The mathematical game that I discuss involved two players and concrete artefacts to play it: a plastic sheet that contained two rows made up of 10 squares with space enough for the children to place a small plastic bear in each, 10 bears of one color for one child, and 10 bears of another color for the other child, and one dice (see Figure 1, Picture 2 below).



Pic. 1



Pic. 2

Figure 1. Left, the children with their teacher singing an arithmetic song. Right, two players and the concrete material to play a mathematical game.

In the second part of the game, which is the focus of my discussion, the children started with empty rows. The rules were as follows:

(a) Taking turns, each child had to place on her/his row the number of bears that corresponded to the number shown by the dice after the child rolled the dice.

(b) The winner is the child who filled her/his row first.

(c) To fill the row, the child had to roll the dice and obtain the exact number of points on the dice as the number of spaces left on her/his row.

To demonstrate the rules, the teacher played a game with a child in front of the class. Then, the class was divided into groups of two.

There were several mathematical notions involved in the game, such as:

(a) producing a numerosity (the points shown by the dice);

(b) counting the numerosity (quantity) either perceptually or with their fingers and/or words;

(c) determining the number;

(d) choosing a quantity of bears that corresponds to the number; and

(e) placing the bears on the row and determining whether or not the game has been finished.

There were also some social dimensions involved in the game, such as:

(a) subjecting oneself to the rules of the game;

(b) articulating one's actions with those of the other child; and

(c) paying attention to the various phases of the game.

Here is an account of the game played between Carl and Jack.

Jack rolls the dice and gets 6. With a tone of satisfaction, he says "6!" and proceeds to place six bears on his row while counting aloud. Carl follows Jack's actions. He waits for Jack to finish putting the bears on the corresponding row. When Carl is done, he says, "OK. My turn, my turn!" Carl takes the dice, rolls it, and says "Oh! 2!" He takes one bear at a time and places them on his row while counting

aloud. Jack follows Carl's actions. Carl finishes placing his bears, moves the dice close to Jack's row and says, "OK, it's your go."

So far so good. The children have taken turns and moved the bears according to the game's rules. Unfortunately, things went badly right after. Here is the continuation of the game: Jack picks up the dice and rolls it. The upper face shows two points (see Figure 2).

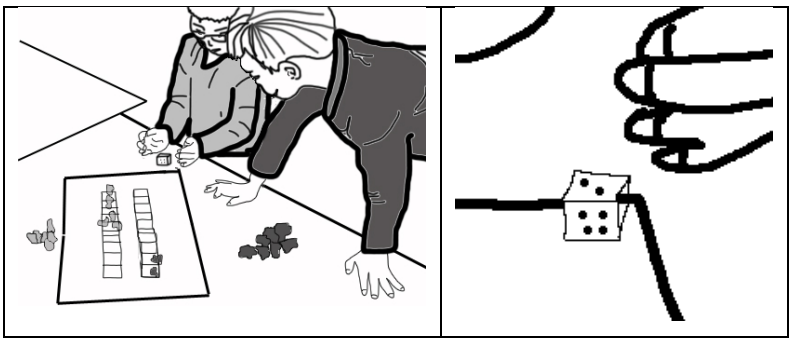


Figure 2. Left, Jack rolls the dice and gets 2 points. Right, a close-up of the dice.

Jack is not happy with the result, picks up the dice again, puts it in his hands, shakes his hands vigorously, and lets the dice fall. He utters, "5!" Satisfied with the result, he starts adding bears while counting "1, 2, 3, 4." He is puzzled as he realizes that he does not have enough bears. Carl has been looking at what Jack does, apparently without fully understanding Jack's actions. Carl does not seem perturbed by the fact that Jack has ignored the first result (the dice showing 2 points) and has rolled the dice again.

At this moment a child from another group calls the teacher and Carl's attention moves to that group. In the meantime, Jack is busy reordering his bears on his row. Thirteen seconds later, Carl's attention comes back to Jack. Jack is still reordering his bears on his row. Carl stretches his arm and tries to get the dice, which is in front of Jack. Jack prevents Carl from taking the dice (see Figure 3) and says, "So, it's … wait! OK, it's …."

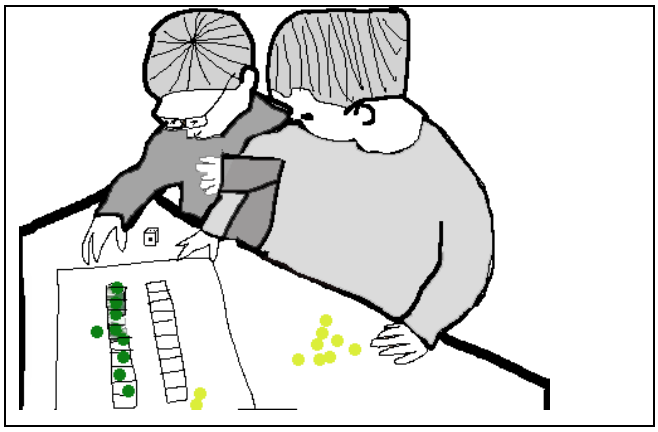


Figure 3. With his left arm, Jack (left) prevents Carl (right) from taking the dice.

Carl does not pay attention to Jack and says, "OK, my [turn], I …" Jack interrupts and says, "No, wait! Wait! Wait!" After some physical struggle, Carl succeeds in getting the dice. Jack continues, "So, it's 1, 2, 3, 4, 5, 6" and keeps on placing and counting bears: "1, 2, 3, 4." Carl is not paying attention to what Jack does. Carl rolls the dice twice. Jack finishes counting and puts his arms in a victory position. He utters, "I won! I won! I won! I won! I won! I won! Look!" Carl turns the dice in

his hand, and when he finds the 6-point face, he stops and starts counting the points: "1, 2, 3, 4, 5, 6 … 6!" He tries to start putting six bears on his row. Jack puts his arms on the page covering all the bears to prevent Carl from placing his bears. Jack says, "I won! … Me, I won!" Carl moves his body towards the page and in a very frustrated tone says, "Ughhhhhh!" (see Figure 4, Picture 1). Jack insists, "Me, I won!" Carl replies, "Me is getting mad at you!" Jack responds, "Me, I won! Won!" Jack takes the dice and shakes it vigorously as if to start a new game. Carl exclaims "No! JACK . . . Ughhhhhhh! No! This is enough!" He succeeds in getting the dice. "My was only when [I] have this" (he points to 6 on the dice) "So, my turn." Jack answers, "No, you didn't get that! . . . You did like (he pretends to hold a dice in his hand and to move it around) flip, flip, flip and then you found 6! Um, Carl cheated, he does like flip, flip, flip, flip! … (pointing at Carl) Cheater! Cheater! Cheater! Cheater!" Carl reacts with his body. He comes very close to Jack as if he is going to hit him (see Figure 4, Picture 2).

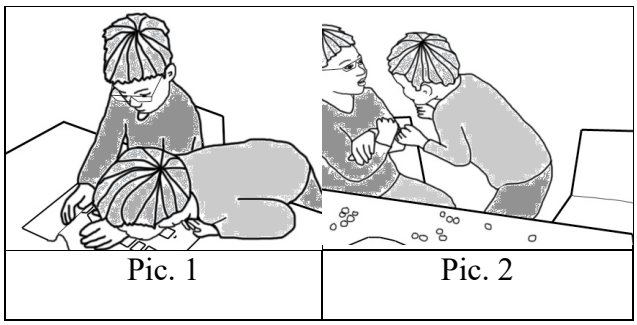


Figure 4. Carl showing his frustration to Jack

THE INSCRIPTION OF THE CHILDREN IN THE SOCIAL WORLD

The role of rules

Generally speaking, following social rules is a crucial step towards inscribing oneself in the social world. A rule, indeed, provides a normative dimension and an agentic space of action that, as far as the rule is followed, keeps in principle the individuals' interaction within the scope of the socially expected. The medieval example discussed above provides an example of transgression. Not all rules are explicit. And even when they are—depending on the complexity of the behavior, duties, responsibilities, etc. that they target— rules may become objects of *interpretation*, for a rule is, by nature, *general*: it applies not only to a specific case, but a range of potential (i.e., not yet produced concrete) cases. The rules of the mathematical game played by Carl and Jack were explicit. The rules do not make a distinction between players. In this sense, the mathematical game's rules introduced above have a homogenizing effect on the children.

In the first part of the episode we see how, drawing on the game's rules, the children come to position themselves in the game: they take turns, they wait for the other child to play, they even collaborate in sharing the dice; they seem to accept their responsibilities and the responsibilities towards the other player. Still, the inscription of the children in the social world is not an easy task. They have to pay attention to the evolution of the game; they have to wait for the other player to finish placing his bears. Moreover, to do so, they have to *control themselves*. As our data suggest, usually, in playing competitive games in preschool, it does not take long for the rules of the game to be broken. When Jack rolls the dice twice, he transgresses the social dimension of the rule. He seems to be aware of it. Figure 2, left, shows Jack rolling the dice and getting two points on the upper face of the dice.

Disappointed, he picks up the dice again, shakes his hands vigorously with a sneaky smile on his face, which may mean something like: "I know that I should not be doing this, but …" Since Carl does not react, he continues playing seriously as if nothing had happened. We saw above that, right after, Carl got distracted and his attention moved to another group. The result is a *rupture* in the children's collaboration that was present in the early part of this game. The collaboration includes a *coordination* of actions (e.g., taking turns) but also *paying attention* to what each player does. Part of collaboration is indeed to pay attention to others, even if it is not one's turn. To maintain his attention on the game is a tremendous task for Carl, who is one year younger than Jack. In turn, although Jack's attention is on the dice and his bears, he does not realize that Carl is not paying attention. Jack is focused on his own actions. When Carl's attention comes back to the game, it is focused on taking his turn, regardless of the position of the game. *The rules that hold the children together and oriented the processes of subjectification in the first part of the game are no longer there.* The rules, which provided the children with rights and duties before, have evaporated. As a result, the social and theoretical common ground embodied in the rules of the game disappeared. The positioning of the children in the social world no longer has a shared reference. Without a shared reference, the connection and mutual recognizance that the children achieved before are lost. The relationship to the other takes a different turn. Impulse, desire, and imposition now drive the children's processes of subjectification. It is in this context that Jack draws on the stock of cultural categories at his disposal (the category of "cheater") to disqualify Carl. Carl, who exhibits a lesser mastery of the language than Jack, does not like to be called a cheater and responds with unarticulated phrases and with frustrating emotions expressed verbally ("Ughhhhhhh!") and with threatening body language (Figure 4, Picture 2).

The role of the mathematical content

In addition to the rules, the mathematical content required in the game also offers the children an important support to inscribe themselves in the social world. Indeed, the mathematical content offers the children entrance into a *shared space* of counting. For to play the game, the children have to count following the *same* culturally and historically constituted way of counting—they have to follow a same arithmetic and its counting principles. It would be a mistake to think that counting, as the children do in this game, is something natural. As shown by anthropological and ethnomathematical research, not all cultures count in the same way and not all count the same things (see e.g., Lancy, 1983; Owens, 2001). Despite the presence of the bears, their colours, the plastic sheet with the rows, the dice, etc., the apparently concrete arithmetic these preschool children are playing targets an *abstract* form of arithmetic thinking that will be required in the abstract commercial exchange network that the children will find in society. The arithmetic that the children are encountering is, in fact, already economic and politically oriented towards a certain way of living and dealing with events in the world. The *Semiotic Systems of Cultural Signification* that ubiquitously operates in the school, the school system, and society as a whole *naturalize* this way of counting and its importance in children's education. It is only as a result of the effects of the *Semiotic Systems of Cultural Signification* that we end up assuming that counting things as the children do in this game is something obvious, necessary, and natural.

In short, the children's co-production as subjectivities and their inscription in the social world takes place in processes (the process of subjectification) that occur as children engage in classroom

activity—in this case, an activity around a mathematical game. Two important elements in these processes are: (a) the manner in which children do (or fail to) subject themselves to the social rules and (b) the necessarily ideological stance of the content that they are learning. By ideological I do not mean something that is purposely misleading (like a false consciousness). Following Voloshinov (1973), by ideological I mean that all theoretical content (like the arithmetical one conveyed by the game) is unavoidably the bearer of a *vision* or *idea* of the world—hence the term *ideo*logical. This is why the rules and the mathematical content are both also part of the very fabric of the children's subjectivity and their inscription in the social world.

There is still a third very important element in the children's inscription in the social world: the teacher.

The teacher as the embodiment of an ideal form

In a landmark paper, *The problem of the environment*, Vygotsky (1994) called attention to the fact that the settings in which children live are replete of "ideal forms" or "models" (p. 348) of behaving, thinking, speaking, doing, and so on, and argued that their greatest characteristic is not that these cultural and historically constituted ideal forms are already there in the environment or in society. Their greatest characteristic consists of how these ideal forms exert a real influence on the child. But how can this ideal form exert such an influence on the child? Vygotsky's (1994) answer is: under particular conditions of interaction between the ideal form and the child. Following Vygotsky's idea, I want to submit that the teacher is an *embodiment* of ideal forms—forms about knowledge, but also about *being*. In interacting with the children in classroom activity, teachers bring to the fore, and make available to the children, features of knowledge and *being* that are relevant in teaching and learning. To explain my point, let us come back to the classroom episode and continue with what happened in the children's game right after Carl expressed his unhappiness and frustration to Jack (Figure 4, Picture 2).

At this point, the teacher came to see Carl and Jack. She put herself close to Carl and, in a calm tone, asked him to sit down (see Figure 5).

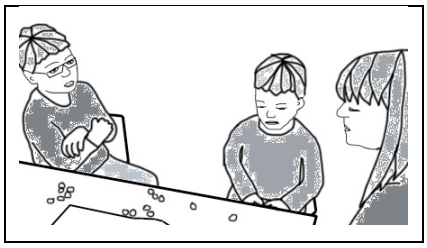


Figure 5. From left to right, Jack, Carl, and the teacher

Jack was furious; pointing to Carl, he said loudly, "Cheater!" Carl defended himself responding, "Me no cheater." Carl turned to the teacher and, in a complaining tone, told her, "He does not want to listen to me!" In a patient, supportive, and comforting tone, the teacher responded to Carl with a question: "He doesn't listen to you?" In a discouraged tone, Carl responded with a brief "No!" Taking him seriously, the teacher asked, "What are you trying to tell him?" In the meantime, Jack pointed to Carl and shouted, "He, he cheats!" The teacher turned to Jack, and in the same calm tone she talked to Carl said, "OK. Stop saying that." Jack explained, "He was doing like (making some gestures with his hands) … and found 6." Coming back to Carl, the teacher asked him in a calm manner "What …

what do you want to tell him?" Carl did not articulate a full answer and barely said, "Uh …" Then, the teacher invited the children to continue the game. Talking to both children, she said, "Whose turn is it?" Carl responded, "Me, me, me rolled like that but he didn't listen." In a comforting tone, the teacher said, "OK. Roll it [the dice] again. We'll restart [the game]."

At this point, the children started collaborating again. They started taking turns, paying attention to the other, putting the bears on their row and counting aloud. The teacher remained with them for 12 seconds and, having succeeded in calming both children, left to see another group.

What happened? In her interaction with the children, the teacher was able to calm them down. The teacher made available for the students forms of *being* (more specifically, forms of behaving and addressing the other) that were not within the children's reach. The teacher was able to show in a concrete way how to listen and how to care. She also showed empathy to the children. By showing empathy, she was able to connect with them and provide the reconstruction of a social, fluid, and dynamic structure where the children could reorganize their deeds around the rules of the game.

SYNTHESIS AND CONCLUDING REMARKS

In this paper, I attempted to explore the question of the production of subjectivities in preschool. The question is based on a conception according to which individuals are *affected* by their cultural-historical context. However, as I pointed out, this affection should not be understood in a mechanical or causal sense: it should be understood in a *reflexive* manner. What this conception means is that while individuals are living agentic entities in a continuous process of transformation, the scope and parameters of their agentic dimension can only be understood against the backdrop of culture and history. It is in this sense that I talk about individuals co-producing themselves and, at the same time, being produced by their cultural-historical context.

One of the fundamental manners in which individuals are reflexively affected by their cultural settings is by the manner in which cultures offer their individuals a range of traits about how to show to, and position oneself in, the world. This is the idea of the concept of *being* that I introduced above. *Being*, I suggested, is as a *generative capacity* constituted of *cultural conceptions* of living in the world. *Being* is an ontological category, subsumed in symbolic superstructures that I termed *Semiotic Systems of Cultural Signification.* These systems operate ubiquitously through a complex web of historical, political, legal, and economic relations. It should not come as a surprise that schools, as places of preparation to life in society, draw from those *Semiotic Systems*, implicitly as well as explicitly. The Ontario system of education, for instance, seeks to produce "graduates who are personally successful, economically productive and actively engaged citizens" (Ontario Ministry of Education, 2014, p. 1). These three traits or forms of being obey a historical tradition anchored around the Enlightened concept of the child mentioned in the Introduction of this paper, a specific Canadian conception of life around the individuals' community, and a contemporary economic urge to move successfully in the direction of the global economy, an urge that translates into the insistent inclination towards the formation of entrepreneurial minds.

Preschool is, in this context, the first step in the long journey on which children are about to embark. It is in preschool that they make the first contact with numbers, shapes, and social life. It is there that they start meeting, in a more or less structured way, the forms of *being* that society has to offer—for instance, to be an "actively engaged citizen," which includes knowing how to live by social rules.

But *being*, in any of its cultural forms, is always something latent, a kind of archetype, something that in order to be perceived or noticed by the children, have to be *materialized* in the concrete world. Its materialization is what I called *becoming*.

In the first part of the game, the children followed the rules of the game. They enacted a way of *being—being* a good citizen. And because the game was about introducing children to abstract counting (as required in the counting of merchandise, the calculation of their prices, etc.) we could argue that the game is also about introducing the children to the sphere of (to use the Ministry's expression) the "economically productive," for how could you be economically productive if you do not know how to count? Then, we saw that the game was disrupted. Carl and Jack stopped following the rules. We also saw the crucial role that the teacher came to play. The teacher actively participated *along with the children* in recreating a social context where Jack and Carl could resurface and find manners of becoming a presence in the world, manners of agentically positioning themselves again in socially accepted ways (e.g., politely addressing the other, waiting for their turn, etc.).

The teacher's and the students' success in recreating a fruitful social context to continue the game calls attention to the fact that such an enterprise would not be possible without the reciprocated willingness to repair what was lost and mutual trust. As an embodiment of culturally and historically constituted ideal forms, the teacher was able to make available for the children traits of *being* of an ethical nature, such as genuine listening ("What are you trying to tell him?"), caring, answerability, and empathy. In practising empathy, I would like to contend, the teacher is not just showing compassion. She is touching upon perhaps one of the most central features of what makes us human, namely the recognition of our fragility in the fragility of the other. I do not claim, though, that the children recognized those ethical traits of being as such. What I could claim is that, in playing the mathematical game, the children made the experience of those traits, that they sensed them, that those traits might have become objects of consciousness (not necessarily theoretical consciousness) and that, hopefully, those traits will become orienting parts of their subjectivity and their future deeds. Moreover, if this is so, I think that mathematics education can no longer ignore the centrality of the question of ethics in teaching and learning.

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