

Semiosis and Subjectification: The Classroom Constitution of Mathematical Subjects

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Abstract In this chapter, I consider semiosis as the continuous production of signs and significations. However, I do not limit the scope of signs to marks or inscriptions. I consider individuals as signs too. Like signs, individuals come to occupy positions in the social world and behave in ways that are not at all different from signs in a text. A crucial difference between inscriptions and individuals, though, is that individuals are not merely signified through well-defined syntaxes as inscriptions and traditional signs are. The cultural syntaxes through which individuals come to be positioned in the social world are less visible: they are part of a dynamic cultural symbolic superstructure. Another crucial difference is that, unlike inscriptions and marks, individuals co-produce themselves—even if it is within the limits of the aforementioned symbolic superstructure. Individuals co-produce themselves in what in this chapter I term processes of subjectification. This chapter is an attempt to study the processes of subjectification in the mathematics classroom. To do so, I analyze a classroom episode with pre-school children.

2.1 Introduction: Life as a Semiotic Zone

Traditionally speaking, signs have been considered as marks or inscriptions. The inscriptions on Mesopotamian clay tablets or footprints on the sand are examples of signs. In the first case, the signs are produced intentionally by an individual or group of individuals. In the second case, the production of signs may not be the result of an intentional act. Yet, by being noticed and accentuated someone may

interpret the footprint as the presence of human beings in the surroundings. In both cases, the signs signify something.

Now, we can also think of signs in a more dynamic way—as entities that unfold in time, as part of a semiotic process that generates and regenerates itself according to semiotic rules—e.g., implicit, explicit or partially explicit rules of syntax or rules of meaning production, such as politeness and social behavior more generally. The heroes or protagonists in a novel may be seen as signs in this sense. Heroes or protagonists evolve as they engage with others in various manners and activities. They evolve in accordance with semiotic rules that are historically and culturally situated—an idea that brought Bakhtin (1981) to talk about *literary genres*. Bakhtin considered literary genres as being framed by three driving vectors: ideology, differentiation, and polyglossia. I would like to go a step further and argue that life can also be seen as a semiotic zone: the confluence and interaction of various activities in which, through semiosis—i.e., through processes of signification—individuals come to agentially position themselves in differential, polyglossic, and ideological manners.

Polyglossia (the acknowledgment of a variety of ways of thinking/speaking) and differentiation (A6B4 as well as A 6A) refer to an always unique individual who, through her engagement in social activities, continuously positions herself through other individuals in the cultural-historical world as an unrepeatably entity always in flux—an entity in perpetual *be-coming*. Ideology refers to an already present system of ideas (scientific, ethical, aesthetic, legal, etc.) that subsumes the individuals and that transcends the individual *qua* individual.¹ Ideology ubiquitously operates in the agentic processes of differentiation and polyglossia so that, as different and unique as individuals are, their uniqueness is nonetheless shaped by the common cultural ground of ideology. Similarity and dissimilarity, likeness and unlikeness, difference and uniqueness are cultural concepts whose content does not come from the individual's own interior but from the situationally operating ideology, history, and culture.

This way of conceptualizing individuals is certainly at odds with the conception of the individual that we have inherited from the philosophy of the Enlightenment that has informed modern Western thought. We have become used to the idea that we are equipped with an interior from where our true Being emanates. It is in this interior that our deepest feelings and meanings are allegedly formed, so that what we need in order to grow as human beings is simply a stimulating environment. The transposition of this idea to children during the 19th and 20th centuries has allowed educators and psychologists to imagine and formulate education as a project that consists precisely in getting rid of the multitude of interferences that impede the natural growth and self-fulfillment of the student. The conceptualization of

¹Hence, my use of ideology has nothing to do with something such as false consciousness or with claims of mistaken outlooks of reality. I use ideology as Bakhtin (1981) and Voloshinov (1973) used it: as a system of ideas that operates in a culture at a certain historical moment and that unavoidably embodies and refracts the contradictions of the various voices and theoretical and practical dominant and non-dominant attitudes of the individuals.

individuals that I am suggesting here, as reflective signs in an unfinished process of becoming, subsumed within a dynamic symbolic ideological superstructure, goes in another direction. It is a direction that does not posit the student as the origin of knowing and becoming, as an essentially already given and already made entity. It rather conceives of the student as a continuously moving sign in the making.

This chapter is organized into three parts. The first part is a continuation of this introduction. In this part I continue to problematize the question of the subject and to show how the manner in which we come to conceive of ourselves is always culturally and historically situated. In the second part I suggest a definition of Being and subjectivity. Subjectivity refers to the idea of the subject as a dynamic sign, an unachievable project of life. The way subjectivity moves and how it is subjectively and societally produced, are accounted for in terms of a conception of life as the confluence and interaction of activities embedded in an always changing semiotic zone full of contradictions and agonies, hope and laughter, whose syntactic and semantic (explicit and implicit) rules (more often implicit than explicit) I explore through the concept of *semiotic systems of cultural signification*. In the third part I discuss a classroom episode in a pre-school setting that intends to show a concrete example of the processes through which subjectivities are societally co- and self-produced.

2.2 The Question of the Subject

To a very large extent, mathematics education research has drawn—rather implicitly—on the concept of the subject that philosophers of the Enlightenment articulated in the 18th century. As I argue elsewhere (Radford 2012), the Enlightened philosophers sought to build, in the overcoming of fear, tradition, and feudal hierarchies, their idea of the new subject. They found in freedom the subject's most fundamental trait (Adorno 2006). The idea of subject that they envisioned is someone who is not there to follow what others say or do, but one who has to think and reason by him or herself. Kant illustrates this idea perhaps better than anyone else: the Kantian subject is a subject of reason, the crafter of its own destiny, the architect of its own projects of life, the origin and source of meaning and knowledge. From the Kantian perspective, to be a subject is to be free. And to be free is not to be subjected to anything other than one's reasons. The result is a self-sufficient, humanist, and substantialist idea of the subject. In this chapter I explore the question of the subject from a different viewpoint. I suggest that the subject is a cultural-historical entity in perpetual transformation—i.e., a subjectivity-in-the-making. My starting point is the rather banal—although rarely considered—fact that mathematics classrooms are not only producers of knowledge but of subjectivities as well. Drawing on the theory of objectification (Radford 2008a), I am particularly interested in investigating the processes of subjectification out of which subjectivities produce themselves and are at the same time produced by the activities in which they engage.

2.3 Being and Subjectivity

To move beyond the Enlightened substantialist concept of the subject, I suggest distinguishing between Being and subjectivity. The concept of Being that the theory of objectification brings forward highlights what we may call the *being's cultural nature*. What this means is that our idea of what an individual and his/her power of action and will (i.e., agency) are, are relative to their historical moment. If we were born in ancient Greece or another historical period, we would have conceived of ourselves in a very different manner from the way we do today. In the Athens of Plato, for example, in the midst of a society articulated around the distinction between free citizens and slaves, with a negative valence to manual work and a positive valence to intellectual work, our sense of individuality would have been embedded in a political-geographical criterion of inclusion/exclusion—Athenians versus foreigners—and defined in terms of the opposition between passion and temperance and the struggle for “self-mastery” (Taylor 1989). Very different is the contemporary concept of the individual, defined as a private owner (Radford 2014)—an individualist subject, drawn by possessive, consumerist and instant gratification drives who is continuously urged to express herself “creatively and authentically” (Illouz 1997, p. 35).

Taylor's and Illouz's remarks reveal not only the political-cultural axis that structures the subject, but also the fact that the sense of ourselves is ineluctably embedded in an *ethical* axis: how we conduct ourselves in the social world, how we show ourselves to others, and how we are expected to behave socially and to be recognized by others. These examples show that culture provides the “raw material” from which subjects draw the ideas of what they are (their meaning, their identity, their power of action, etc.). This “raw material” is part of what in the theory of objectification is termed *semiotic systems of cultural significations* (SSCS). SSCS are dynamic symbolic superstructures (Radford 2008a) that include cultural conceptions about the world and the individuals. They comprise (a) ideas about things in the world (e.g., the nature of mathematical objects and their way of existing), (b) ideas about truth (e.g., how truth is and can be established), and (c) ideas about the individuals. SSCS are full of tensions, as are the activities from which they emanate, and have a normative function (which may be explicit, or implicit, or partially implicit).

Clearly, the relationship between the cultural “raw material” included in the SSCS and the concrete individuals cannot be seen as a logical, or causal, or mechanical relationship. This is so because the account that I am sketching here adopts a Spinozist ontological nature about humans according to which humans are unavoidably and profoundly affected by their context (Spinoza 1989). But they are affected in a *reflexive* manner. What emerges from this affection bears the imprint of the culture, but it is always an entity in flux impossible to anticipate and predict. The relationship between the cultural “raw material” included in the SSCS and the concrete individuals is in fact *dialectical*. It is a dialectical relationship between what I mean by “Being” and “subjectivities.”

The adjective “dialectical” does not refer here to a simple reciprocal influence between two given entities—in this case Being and subjectivity. Although, in usual parlance ‘reciprocal influence’ is a common meaning of dialectic and its adjective dialectical, I consider the relationship between Being and subjectivity along the lines of dialectical materialism. In the naïve use of dialectics, the entities in relationship appear as already formed. In Hegel’s dialectic, which is at the basis of Marx’s dialectical materialism, in contrast, the relation is one of transformation. The transformation of something dynamic, general, and virtual into something concrete that affirms its source and at the same time negates it. In his *Phenomenology of Spirit*, Hegel considers the example of a bud: “The bud disappears in the bursting-forth of the blossom, and one might say that the former is refuted by the latter” (Hegel 1977, p. 2). The bud is sublated in the blossom, and as such the blossom affirms the bud. But at the same time, the blossom (which is the transformation of the bud) refutes or negates the bud, as it does not coincide with it. It is something else, yet something that could only come to life through the movement and transformation of the bud. The key idea in the Hegelian conception of dialectic is *movement*. “It is of the highest importance,” says Hegel, “to interpret the dialectical [moment] properly, and to [re]cognise it. It is in general the principle of all motion, of all life” (Hegel 1991, p. 128).

In the Hegelian train of thought, metaphorically speaking, it might be useful to consider Being as something like the bud—something out of which subjectivities emerge. Each subjectivity is different from other subjectivities (as blossoms are never identical to each other), yet all subjectivities are the transformation of something (i.e., Being) that is never fully given, but always in the process of change, something indefinite, potential.

Being, I suggest, is a general, cultural, dynamic (that is, always changing) non-metaphysical, ontological category. It is constituted of historically coded forms of conceptions about the individuals and the ways in which individuals are called to present themselves to the world and to interact with other individuals. More precisely, Being is constituted of cultural ways of living (i.e., *be-ing*) in the world: ways of conceiving of oneself and of being conceived, ways of positioning oneself and of being positioned, and forms of self and otherness (i.e., relationships with oneself and others). In this account, Being is *potentiality* (what Aristotle called *dūmalī*, *dunamis*); that is, something whose mode of existence is not *actual* but *potential*. What is *actual* is subjectivity.

Subjectivity is a process: the always ongoing instantiation or materialization of Being. This unachievable and always ongoing instantiation is a unique, *concrete* subject (a *subjectivity*), whose specificity results from the fact that it is a reflexive sentient entity always in a process of *be-com-ing*: an unfinished and unending project of life. Or to say it in terms of the ideas presented in the introduction, a subjectivity is a sign perpetually coming into life, a sign that agentically appears and co-produces in the social world through the *materialization* of the cultural-historical possibilities available to it (Radford 2008a). In this coproduction, the individual as subjectivity becomes aware and conscious of itself and from there can project and present herself to herself and others in new ways.

Subjectivities coproduce themselves not in contemplation but in the course of a process whose name is *human activity*. Following dialectical materialism, human activity is not a mere set of actions: activity is a *system* (Leont'ev 2009, p. 84) in constant development, incessantly affecting the individuals participating in the activity and at the same time affected by those individuals. We can ask: What is the nature of the human activity that, at school, produces teachers and students? What is its specificity? These questions orient the discussion of the classroom example that I discuss in the next section.

2.4 The Classroom Constitution of Mathematical Subjectivities

The example that I discuss comes from a pre-school classroom of combined pre-kindergarten and kindergarten children (4–6 years old). The content of my example is mathematics, which occupies an extremely important part of the Ontario pre-school curriculum. Naturally, the emphasized presence of mathematics at the pre-school level is coherent with what the children will find later on in primary and junior and senior high school: a curriculum formed around the axis of mathematics and language. Since the dawn of the 20th century, mathematics came to occupy a privileged position in the school curriculum of those countries that saw in industrialization the path towards modern society. Mathematics became the ally and support of the new capitalist forms of production. Many early 20th century pedagogues understood civilization as synonymous with industry (Radford 2004). To a large extent, the main problem of 20th century educational reform was the problem of massive schooling to train the young in the participation and development of a technological society. One century later, things have not changed much. Capitalism has not vanished. It has become trans-national, diversified, and globalized. It is hence not surprising that the preschoolers I see entering the school every morning start the day with activities around counting. If the school has to produce consumers and technologically oriented minds, counting has to be the starting point.

At first sight, counting may seem to be a *natural* activity: the same activity regardless of place and time. On a closer look, however, as anthropological and ethnomathematical research shows, not all cultures have counted in the same way and not all cultures have counted the same things (e.g., Lancy 1983). Counting can be better conceptualized as a culturally codified numerosity-oriented way of thinking and acting to make sense of the world (Radford 2008b). Classroom activity is an attempt to provoke the encounter of children with a specific culturally constituted form of thinking about numbers, figures, chance, information, etc. Rather than natural and conceptually neutral, the form of thinking favored in the Ontario and in other curricula around the world conveys a specific *worldview*. In the case of arithmetic and numerical literacy (or numeratie, as it is sometimes called), this worldview entails positions about what counts as counted, and operates within a

particular *rationality* (e.g., how things *should* be counted) thereby creating a specific *regime of truth*. The worldview, its rationality, and concomitant regime of truth are all central elements of the semiotic systems of cultural significations that organize the school as a social institution and sanction the kind of knowledge to which students are exposed. To avoid believing that cultural mathematical forms of thinking are purely conceptual and to better grasp their economical-political substratum, it is worth recalling the political struggle between merchants and the feudal aristocracy in 13th century Florence about the legitimate way to count—with Arabian numerals or counters—and the prohibition issued by the Guild of the Money Changers against the use of Arabian numerals (Struik 1968). And all this would have very little bearing on our discussion were it not for the formidable fact that subjectivities are not merely produced as a by-product of learning: on the contrary, *what* is learned and *how* it is learned are the threads out of which subjectivities are made. From the theoretical perspective that I am sketching here, it would be a mistake to conceive of the child as already equipped with her proclivities, tastes, and personality—as idealist and rationalist pedagogies do. The child is an individual in an unending process of becoming dialectically entangled with *what* she learns and *how* she learns it.

To sustain the previous ideas, I discuss a classroom activity in the rest of the chapter. The activity is about an arithmetic game played between two children. The object of the activity, as discussed with the teacher and our research team, was to offer the children an occasion to become acquainted with cultural forms of counting and thinking about numbers as targeted by the curriculum. A plastic sheet contained two rows made up of 10 squares with space enough for the children to place a small plastic bear in each (see Fig. 2.1, pic 2 below). One child received 10 bears of one colour, and the other child received 10 bears of another colour. They received one dice. I focus here on the second part of the game. In the second part of the game, the children started with empty rows. The rules were as follows: taking turns, each child had to place on her/his row the number of bears that corresponded to the number shown by the dice after the child rolled the dice. The winner is the child who fills her/his row first. To fill the row, the child has to roll the dice and obtain the exact number of points on the dice as the number of spaces left on her/his row. To demonstrate the rules, the teacher played a game with a child in front of the class. Then, the class was divided into groups of two.

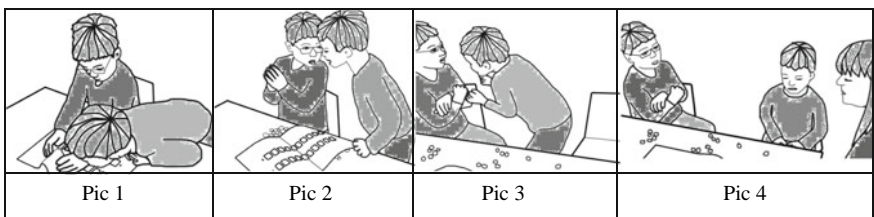


Fig. 2.1 Pic 1–3: Jack and Carl discuss the game. Pic 4: The teacher and the children

In terms of the mathematical notions involved, the students had to deal with: (a) producing a numerosity (the points shown by the dice); (b) counting the numerosity (quantity) either perceptually or with their fingers and/or words; (c) determining the number; (d) choosing a quantity of bears that corresponds to the number; (e) and placing the bears on the row and determining whether or not the game has been finished. In terms of the social dimension of the game, the game required the students to subject themselves to the rules of the game, to articulate their actions with those of the other child, and to pay attention to the various phases of the game. Here is an account of the game played between Carl and Jack.

Jack rolls the dice and gets 6. He says "six!" and places six bears on his row, one bear at a time while counting aloud "1, 2, 3, 4, 5, 6." Carl closely follows Jack's actions; he counts and says "six!" He waits for Jack to finish. Then he says "OK. My turn, my turn!" Jack responds "OK. I'll just put this [the dice] there for a, for now" and places the dice close to Carl's row. Carl takes the dice, rolls it, and says "Oh! 2!" He takes one bear at a time and places them on his row, while counting aloud "1, 2." Jack follows Carl's actions, counting closely. Jack seems to have forgotten how many bears are already on his row. He takes three bears from his row and counts those that remain, pointing to them successively, and says "1, 2, 3." Then, he proceeds to put back those that he just removed. As he puts them back one after the other, he says "4, 5, 6." The page moves a bit; the bears move from their square and now appear not properly placed. Several are on the same square. In the meantime, Carl moves the dice close to Jack's row and says "Ok, it's your go."

Up to this point, the children follow the rules of the game. Following the rules, I want to suggest, is an important moment in the children's process of subjectification; that is, in the process through which they co-produce themselves as subjects of mathematics and subjects of education, more generally. The children enact the game's rules, which means that they have to subject themselves to the same *regime of truth*: they have to count following a *same* culturally and historically constituted way of counting that, despite the presence of the bears, the dice, etc., targets an abstract form of arithmetic thinking that will be required in the abstract commercial exchange network that the children will find in society. The children also enact a culturally and historically constituted form of living in the world that expresses itself in the forms of collaboration (e.g., turn taking) and social behavior.

However, by following the rules the children are not merely materializing a human form of living in the world or practicing a theoretical way of counting. They are doing much more than that. They are positioning themselves in a social world where their actions, regardless of their *difference* and their *polyglossic* nature, are recognized as *legitimate*. The apparently unimportant attention that Carl pays to Jack's actions when Jack rolls the dice, obtains 6, picks up 6 bears and places them on his row serves indeed to legitimize Jack's deeds. Legitimation is a joint endeavor, not a solitary one. It is a social concept. It requires cooperation. The children's cooperation, as it will turn out later, is still very fragile.

To enter the social world, the children also have to *control themselves*. In our video, we see that they wait impatiently for the other player to finish placing his bears. As Vygotsky noted, "A very young child tends to gratify his desires

immediately. Any delay in fulfilling them is hard for him and is acceptable only within certain narrow limits" (1967, p. 7). This is why, Vygotsky contends, it is a mistake to conceive of the child "as a theoretical being" (p. 7) moving emotionless from one cognitive stage to another.

Let me continue with my account of the game. Jack rolls the dice again. This time the upper face shows two points. Jack is not happy with the result, picks up the dice again, puts it in his hands, shakes his hands vigorously, and lets the dice fall. He utters "5!" Satisfied with the result, he starts adding bears while counting "1, 2, 3, 4." He runs out of bears. Carl has been looking at what Jack does, apparently without fully understanding Jack's actions. Carl does not seem perturbed by the fact that Jack has ignored the first result (the dice showing 2 points). At this moment a child from another group calls the teacher and Carl's attention moves to that group. In the meantime, Jack is busy reordering his bears on his row. Thirteen seconds later, Carl's attention comes back to Jack. Jack is still reordering his bears on his row. Carl stretches his arm and tries to get the dice, which is in front of Jack. Jack prevents Carl from taking the dice, and says "So, it's ... wait! Ok, it's" Carl does not pay attention to Jack and says "Ok my [turn], I ..." Jack interrupts and says "No, wait! Wait! Wait!" After some physical struggle Carl succeeds in getting the dice. Jack continues "So, it's 1, 2, 3, 4, 5, 6" and keeps on placing and counting bears: "1, 2, 3, 4." Carl is not paying attention to what Jack does. Carl rolls the dice twice. Jack finishes counting and puts his arms in a victory position. He utters "I won! I won! I won! I won! I won! I won! Look!" Carl turns the dice in his hand, and when he finds the 6-point face, he stops and starts counting the points: "1, 2, 3, 4, 5, 6 ... 6!" He tries to start putting six bears on his row. Jack puts his arms on the page covering all the bears to impede Carl from placing his bears. Jack says "I won! ... Me, I won!" Carl moves his body towards the page and in a very frustrated tone says "Ughhhhhh!" (see Fig. 2.1, pic 1). Jack insists "Me, I won!" Carl replies "Me is getting mad at you!" Jack responds "Me, I won! Won!" Jack takes the dice and shakes it vigorously as if to start a new game (see Fig. 2.1, pic 2). Carl exclaims "No! JACK ... Ughhhhhh! No! This is enough!" He succeeds in getting the dice. "My was only when [I] have this" (he points to 6 on the dice) "So my turn." Jack answers: "No, you didn't get that! ... You did like (he pretends to hold a dice in his hand and to move it around) flip, flip, flip and then you found 6! Um, Carl cheated, he does like flip, flip, flip, flip! ... (pointing at Carl) Cheater! Cheater! Cheater! Cheater!" Carl reacts with his body. He comes very close to Jack as if he is going to hit him (see Fig. 2.1, pic 3).

The game turned very bad. At the beginning of this episode, Carl did not react to Jack's rolling the dice again after Jack got the discouraging 2 points. By rolling the dice twice, Jack transgresses the social dimension of the rule. To some extent, he is aware of it: when he picks up the dice the second time and shakes his hands vigorously, there is a sneaky smile on his face, which may mean something like: "You know, I know that I should not be doing this, but ..." Maybe he interprets Carl's silence as a kind of complicity and continues playing seriously as if nothing had happened. Right after, Carl got distracted and his attention moved to another group. The result is a rupture in the children's collaboration that was present in the

early part of this game. The collaboration includes a *coordination* of actions (e.g., taking turns) but also *paying attention* to what each player does. Part of collaboration is indeed to pay attention to others, even if it is not one's turn. To maintain his attention on the game is a tremendous task for Carl who is one year younger than Jack. In turn, although Jack's attention is on the dice and his bears, he does not realize that Carl is not paying attention. Jack is focused on his own actions. When Carl's attention comes back to the game, it is focused on taking his turn, regardless of the position of the game. The *regime of truth* that holds the children together in the first part of the game is no longer there. The social and theoretical common ground embodied in the rules of the game has disappeared. Without a common ground, the connection between the children is lost. Impulse and desire seem to drive the children's deeds. The other has become an impediment to one's own actions. Jack disqualifies Carl by treating him as a cheater. Carl, who exhibits a lesser mastery of the language than Jack, responds with unarticulated phrases and with frustrating emotions expressed verbally ("Ughhhhhhh!") and with threatening body language.

At this point, the teacher (T) comes to see Carl (C) and Jack (J):

- T: (She positions herself close to C and talks to him in a calm tone.) Sit down.
 J: (Furiously, points at C) Cheater!
 C: Me no cheater. (Turns to the teacher.) He does not want to listen to me!
 T: (Talks to C in a patient, supportive and comforting tone.) He doesn't listen to you? (See Fig. 2.1, Pic 4)
 C: No!
 T: What are you trying to tell him?
 J: (Points at C) He, he cheats!
 T: (Talks to J in the same calm tone she talked to C.). OK. Stop saying that.
 J: He was doing like (Makes some gestures with his hands.) ... and found 6.
 T: (Talks to C in a calm tone.) What ... what do you want to tell him?
 C: Uh...
 T: (Talks to both children.) Whose turn is it?
 C: Me, me, me rolled like that but he didn't listen.
 T: OK. Roll it [the dice] again. We'll restart.

At this point, the children started collaborating again. They started taking turns, paying attention to the other, showing solidarity, putting the bears on their row and counting aloud. The teacher remained with them for 12 s and left to see another group. The teacher succeeded in calming both children. Through her second and third utterance the teacher shows empathy; that is, as the Greek term *pátheia* intimates, the acknowledgment of the suffering of the other. Carl responds positively to the teacher's empathic attitude. The teacher also politely asks Jack to stop calling Carl a cheater. Not without effort, Jack acquiesces to the teacher's request, controls himself and calms down. The teacher is now in a position to restore the children's attention. The children can now move beyond accusatorial body, hand, and verbal actions and can focus on the game and its rules.

This episode shows the tensions that underpin the processes of subjectification out of which subjectivities are being produced. These tensions are not defects of a task design. Nor do they derive from a pedagogy that has gone wrong. They are part and parcel of the processes of subjectification and the disclosing of Being. Through these processes the children encounter forms of Being that have been culturally and historically constituted. These forms of Being are coded forms about the ways in which individuals are called to present themselves to the world and to interact with other individuals. These coded forms are what Hegel (2009) calls *generals* (as opposed to *singulars*). They are archetypes of living in the world and, as archetypes, they cannot be perceived or sensed directly. For instance, we cannot perceive or sense fairness as such—as we cannot perceive or sense directly, responsibility as such. To be sensed, to become objects of consciousness and reflection, to be idiosyncratically incorporated in the individuals' own repertoire, these coded forms of Being (e.g., the fair player and all the social, conceptual, cognitive, and ethical attributes that come with it, like, in our case, self-control, collaboration, solidarity, responsibility, sustained attention, theoretical perception, cultural-historical methods of arithmetic counting) need to *appear* in the concrete material world of action, sensation, language, feeling, and thought, where then they can be *cognized*. These coded forms of Being appear, in our case, in trying to follow the rules of a game. They appear first punctually, then in a more generalized way, as the children co-position themselves in the social world and become conscious of what entails behaving socially and intellectually in a given culture, and succeed progressively in incorporating these attributes in their own actions, always in unique and idiosyncratic ways and with unique and idiosyncratic results.

Talking about the development of self-control and voluntary direction of one's own actions, Vygotsky (1998) pointed out that they

develop in the process of children's group games with rules. The child who learns to conform and coordinate his [sic] actions with the actions of others, who learns to modify direct impulse and to subordinate his [sic] activity to one rule or another of the game, does this initially as a member of a small group within the whole group of playing children. Subordination to the rule, modification of direct impulses, coordination of personal and group actions initially ... is a form of behavior that appears among children and only later becomes an individual form of behavior of the child himself. (p. 169)

This transformation of self-control and interpersonal coordinated action, however, does not occur naturally. The classroom activity accomplishes that. It is the classroom activity that moves Being from its state of generality or potentiality to actuality through semiosis—i.e., semiotic collective processes of meaning-making. In this sense, the activity embeds the children and the teacher. But at the same time, the activity is produced by the deeds of the children and the teacher. This is why, methodologically speaking, we cannot attend to the participants without attending to the activity in which they are immersed, and reciprocally, we cannot attend to the activity without attending to the participants that produce the activity. This is the dialectic nature of activity and participants through which Being is disclosed. In its disclosing, the children and the teacher feel and socially experience anger, frustration, empathy, collaboration, responsibility, solidarity, etc.

These phylogenetically constituted attributes of Being become crucial elements of the always evolving Semiotic Systems of Cultural Significations that subsume the classroom, the school, and the educational system. These phylogenetically constituted attributes of Being become pointers of action and *be-com-ing*. Of course, such attributes of Being are not encountered, sensed, and experienced equally by the children. In the course of activity, they are understood (not necessarily at the conceptual level) in varied ways. They occur in an emerging interpersonal ethical attitude that, in previous work, we have termed *togetherness*; that is to say, a relational attitude based on a not necessarily explicit “ethical commitment participants make to engage in and produce activity” (Radford and Roth 2011, p. 227). And as we have seen, the encounter with the aforementioned attributes of Being is deeply entangled with the manner in which the teacher interacts with the students. The teacher draws on developed features of Being that, in the course of classroom activity, come to interact with the children’s emerging conceptual and emotional understanding of the situation. As Vygotsky (1989) once noticed, cultural forms of knowing and Being (voluntary attention, arithmetic thinking, forms of human collaboration and ethical dispositions) do not result from mere interaction. Contrary to other living species, humans are not pure biotypes. Instead, human cultural forms of knowing and Being result from the interaction between phylogeny (that is, the evolutionary development of a cultural group) and ontogeny (that is, the life-span development of individuals) in human activity (Moretti and Radford 2016).

2.5 Concluding Remarks

This chapter has been an attempt at investigating the production of subjectivities in the classroom. To do so, I resorted to semiotics, which is usually understood as a discipline dealing with the production of signs and their meanings. In the introduction I suggested considering signs in a more dynamic way and submitted that individuals can also be considered as signs. As a result, from this perspective, semiotics also deals with the manner in which individuals are signified and signify themselves. The key idea behind this theoretical stance is to conceive of the individuals as entities in the making who come to agentically position themselves—and are positioned by their actions and discourse—in differential, polyglossic, and ideological manners. While difference and polyglossia operate as centrifugal forces, ideology operates centripetally. I called these entities in the making *subjectivities*. They are co-produced in activity-bound processes of subjectification dialectically entangled with semiotic systems of cultural significations. These systems offer an ubiquitous symbolic superstructure to the individuals’ actions and reflections. They include the ontological category of Being, which I formulated as *potential* ways of living in the world and encompassing a sense of ourselves—how we conduct ourselves in the social world, how we show ourselves to others, and how we are expected to behave socially and to be recognized by others. Since this sense of

ourselves is ineluctably embedded in an ethical view, the production of subjectivities is always an ethical act.

These ideas were illustrated through a classroom activity game around counting. Although games and play in young children have been recently the object of research in mathematics education (see, e.g., Meaney et al. 2016; van Oers 2013), my interest focused on the manner in which a counting game opens up space for children to encounter historically and culturally constituted ways of knowing and Being—in this case, theoretical counting and living in the world, respectively. More specifically, the children were exposed to numerosity recognition (arithmetically interpreting the points on the dice), numbering (naming the numerosity), numerosity production (choosing a quantity of bears in accordance to the named numerosity), counting actions (placing the bears on the row), as well as sophisticated forms of cooperation, action coordination, but also empathy, respect, solidarity, and responsibility.

The classroom activity game allows us to see some of the dynamics and tensions that underpin classroom processes of subjectification. The game was based on a rule that, as all rules, is much more than conceptually following a sequence of instructions. The rule appears indeed as a social and conceptual structuring element through which the children become endowed with social *generality*: they recognize themselves and are socially recognized as players with the ensuing collective entailment of actions and expectations. In short, the children become endowed with a generic element that applies to them, as well as to others or to any player of the game for that matter. Rules in play (as in life), as well as other more implicit regulative mechanisms of behaviour, extricate the individual's experience from its pure subjective and private significance. They bring experience to a new realm. They position individuals into a cultural, historical, and social world. This is why it is a mistake to consider games from an educational viewpoint as mere promoters of conceptualizations. This is a reductive cognitive view of games.

In the game analysis we saw Carl, despite all his frustration, positively *responding* to the call of the teacher. To answer to the call of the other is part of *responsibility*. In his book *Éthique et infini*, Lévinas (1982) notes that responsibility is “the essential, primary and fundamental structure of subjectivity ... It is in ethics, understood as responsibility, that the very node of the subjective is knotted” (p. 91). He goes on to say that “[r]esponsibility in fact is not a simple attribute of subjectivity, as if the latter already existed in itself, before the ethical relationship. Subjectivity is not for itself; it is, once again, initially for another” (pp. 92–93).

In short, in the example here discussed, the classroom activity opened up a space for the children in which existential and ethical areas of human life appeared. The teacher's intervention brought to the fore, in a decisive manner, forms of living in the world that restored the flow of the interaction of the children. Her intervention emphasized an all-encompassing ethical disposition to the other oriented towards a reciprocated listening that is much more than conceptual and that is also much more than a mere negotiation of social positions. It was oriented towards a truly human listening, based on a pre-conceptual, emotional understanding of the misery and

agony of the other, an emotional understanding through which we recognize ourselves and the fragility of our human nature. These considerations may lead us to envision classroom activity in new non-individualist, communitarian based, aesthetic forms of human collaboration (Radford 2014) out of which we may be able to move towards what has been called “the liberation of both the senses and reason from their present servitude” (Marcuse 2007, p. 227).

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