

Alienation in mathematics education: a problem considered from neo-Vygotskian approaches

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Abstract In a recent article published in this journal, Williams (*Educational Studies in Mathematics*, 92, 59–72, 2016) offers a critique of neo-Vygotskian perspectives exemplified in recent work on the *funds of knowledge* and on *cultural-historical activity theoretic* perspectives. The critique has great value in that it asks interesting questions that often go unnoticed and fail to be discussed in mathematics education research—e.g., the question of alienation or the role of agency in Vygotsky’s notion of the *zone of proximal development*. Despite the great value of the questions raised, the critique fails because its idealist theoretical epistemological underpinning is at odds with Vygotsky’s thought and cultural-historical activity research in general. The critique falls short especially on the point of realizing the materialist dialectical method Vygotsky employed. The goal of this article is to address Williams’ critique by (a) revisiting the main argument of the critique, (b) articulating the departures of the critique from the materialist dialectical method, (c) conceptualizing *development* along the theoretical lines of the late (Spinozist-Marxian) Vygotsky, and (d) formulating a materialist dialectical approach to *alienation*. We conclude by summarizing three main problems of the critique.

Keywords Cultural-historical theory · Alienation · Development · Activity · Materialist dialectical method · Vygotsky

1 Introduction

Over the past several decades, sociocultural perspectives have gained considerable prominence in the field of mathematics education. Gradually, they have come to constitute what may be termed a “sociocultural turn.” Such a turn has been neither an easy nor direct venture. For one, at the time of its emergence (i.e., during the 1980s and 1990s), mathematics education was taking inspiration in positivist, rationalist, and cognitive psychological research. To a large extent, the sociocultural turn emerged in response to (a) the individualist learning approaches that have dominated the field (see, Lerman, 1996) and (b) a Eurocentric conception of mathematics (cf., Bishop, 1988; D’Ambrosio, 1985). To offer non-individualist conceptions of learning and non-Eurocentric conceptions of mathematics, sociocultural mathematics education research has drawn on (a) socially, historically, and culturally oriented conceptions of the mind and the individual, and (b) a broader conception of psychology than the narrow idealist context-free, subject-centered one inspired by the natural sciences. One of the results has been a new chart of research questions and problems—e.g., the role of language, discourse, and material culture and their effects on teaching, learning, and cognition; the social, cultural, and political context of mathematics teaching and learning; power, gender, and the production of subjectivities; and what counts as doing mathematics and thinking mathematically. These questions have been tackled from different theoretical angles, resorting to diverse spheres of sociocultural research in general (e.g., Bourdieu, Foucault, Freire, and Vygotsky).

Because of its different research foci and theoretical underpinnings, sociocultural research in mathematics education is not a monolithic paradigm. Certainly, there is a need to better understand the limits, differences, and similarities of sociocultural approaches and perspectives. There is also a need to better understand the differences between the sociocultural paradigm and other paradigms in mathematics education. Both endeavors rest on a better and deeper understanding of the theoretical assumptions that the various approaches make. It is exactly here that we find of an exceptional value a recent article entitled “Alienation in Mathematics Education” (Williams, 2016) as it constitutes an attempt to critique and further develop neo-Vygotskian perspectives. The critique is directed at two contemporary Vygotskian approaches: the funds of knowledge (Moll, Amanti, Neff, & Gonzalez, 1992) and the cultural-historical activity perspective (Roth & Radford, 2011). We believe that the critique has the great value of asking interesting questions that often go unnoticed or remain undiscussed in mathematics education. For instance, Williams brings to the discussion some questions that we need to keep examining—including the role of the student in learning and the exact difference between students’ behavioral actions and genuine understanding. Williams asks whether or not Vygotsky’s concept of zone of proximal development simply is a moment in the process of alienation. The goal of this article is to address the critique. To do so, we begin by revisiting the main arguments on which the critique is based. We move to the methodological and conceptual problems in the critique. We respond to Williams by exemplifying the conceptual differences that exist between the individualistic approach inherent in the critique and Vygotsky’s primacy of the social and historical when conceiving the phenomena of development and alienation. We conclude by stating three main shortcomings of the theoretical basis of the critique.

2 On the primacy of individual intention and action in learning and development

The critique has the learners' alienation from mathematics as its main topic. This alienation is attributed to the failure of mathematics educators, researchers, and practitioners to recognize the primacy of the students' intentions and actions in their learning and development. Rightly positioning Roth and Radford (2011) ("R&R" in the critique) within the field of cultural-historical activity theory (ChAT), the critique argues that a genuinely Vygotskian approach should feature the learner as an active, sense-making individual. Of course, we could not agree more with the article on this point. But the way in which learners are conceived is apparently at odds with what we, following the literature, termed the learning paradox:

Fundamental to the question about learning is how learners can intend what there is to learn and how to learn it, because that which is to be learned is precisely unknowable. If they already knew what they had to learn, then students did not have to engage in learning it. (Roth & Radford, 2011, p.107)

The critique suggests that there is something fundamentally disturbing in our conceptualization of learning. According to the critique, we provide an "erroneous account of Vygotsky's theory" (Williams, 2016, p. 61) by failing to note that "the learner has to be an active participant in making sense of everyday activities with mathematical concepts, and that, without this, classroom activity descends through 'pure verbalism' into alienation" (p. 61). The critique charges that our "adoption of the learning paradox . . . implicates alienation and is a quite unnecessary and unwelcome addition to the theory" (p. 61).

In our book, we analyze a classroom episode where fourth-grade students engage (or fail to engage) in a small group activity where they are dealing with a problem on pattern generalization involving the sequence $3 + 6$, $3 + 3 + 6$, $3 + 3 + 3 + 6$, etc. In his summary of the episode, Williams (2016) refers to a crucial passage in which Mario (one of the students on which the R&R episode is focused)

draws the teacher's attention, who then engages with Mario, leading him through the task (e.g., asking him to read the sheet, prompting "how many 3s?," pointing to spaces he must fill in on the worksheet, etc.) and apparently hinting in various ways. Communicating with pointing gestures to required actions, and vocal intonation feeding back if Mario's action is leading in the right direction, she leads Mario to what the task demands. (p. 62)

Acknowledging that our original analysis of Mario's emotions in which a change of frustration into satisfaction in his actions reveals a fulfilling mathematical understanding, Williams still argues that alienation is present. To make his point, Williams suggests a distinction between subjective and objective alienation. Subjective alienation refers to the student's sense of not feeling at home when learning mathematics. Objective alienation, by contrast, occurs when the student "is 'forced' into producing mathematics, e.g., to earn rewards rather than willingly to enjoy the expansion of their personality or capabilities" (p. 60). In Williams' interpretation of R&R's classroom episode, the student's "initial frustration realizes

his need to make sense, while his quiet calm invokes his satisfaction that he has finally understood" (p. 62). The critique "admit[s] that this does not feel like a subjective experience

of alienation” and yet “suggest[s] that [Mario] may be objectively alienated even so” (p. 62). In concluding, the critique notes:

the task and worksheet activity does not invite the children to mathematise algebraically, it arbitrarily imposes it; when this does not make sense to a learner, the pedagogy obliges the student to conform behaviourally to the demands of the worksheet, without any “sense” of a need for algebraic formulation arising. This is what Vygotsky means by “pure verbalism,” and this type of senseless, arbitrary “imposition” and “obligation” is what Marx means by alienation. (p. 63)

The critique goes on to argue that Mario’s learning “might not be mathematically developmental and in fact might be seen as compliance with the teacher’s evaluation that he has understood when in fact, objectively, he may have understood little or nothing about algebraic reasoning or reflection as such” (p. 63). The aforementioned conclusion is followed by an interesting section entitled “What is wrong with ‘internalisation’ in R&R’s interpretations of Vygotsky?” The discussion focuses on Vygotsky’s famous example of the pointing gesture. In our book, we cite a passage from the fragmentary text entitled “Concrete Human Psychology” (Vygotsky, 1989). There, Vygotsky notes the three-stage character of cultural development: “development in itself, for others, and for oneself” (p. 56). He then provides the example of “a demonstrative gesture—at first it is simply a failed grasping movement aimed at an object and designating an action; then the mother understands it as an instruction; and, finally, the child begins to point” (p. 56). Commenting on this passage, we elaborate in the book:

In [Vygotsky’s] description of cultural development, a movement receives the sense of an action of a particular kind first by the culturally competent individual before this sense comes to be actualized by the child. In the example Vygotsky provides, there first is a random movement. The child does not know its cultural signification; it does not (yet) know to point. Rather, the parent who sees the child move understands it as a pointing gesture and, in acting toward the child, transforms it as such. It is in and through the social interactions with the parent that the child comes to understand that by means of such movements things are being pointed out. (Roth & Radford, 2011, p. 69)

What bothers Williams is our analysis of the developmental nature of the child’s arm’s movement. He claims that it is not a random movement. In his interpretation of the cited passage, the child already has a motive: to grasp the intended object. And this is, indeed, exactly what Vygotsky seems to be saying: “at first it [the demonstrative gesture] is simply a failed grasping movement aimed at an object” (Vygotsky, 1989, p. 56). On first impression, the disagreement may seem pointless and to discuss it could be considered a waste of time. After all, what difference does it make that the movement occurs randomly or not? This is not the problem. As Williams writes, “R&R misunderstand Vygotsky in an apparently small but vital way” (p. 64). It is in this “small but vital” difference in the understanding of the child’s role in learning and development that the tremendous value of Williams’ article resides.

According to Williams, the interpretation of “internalization”—a word that is never used in the interpretation of the classroom episode, and the two times it is actually used it appears in quotation marks—is wrong because it appears to minimize the child’s role. This is so because Vygotsky’s mother and our fourth-grade teacher each appear to be in command of the respective activity. As a result, the pointing infant and the fourth-grade student Mario are

merely taught something. Discounting or missing the double active role of the infant and Mario—acting and then “recognize[ing] the object/motive in their own actions” (Roth & Radford, 2011, p. 16)—Williams suggests that both children are alienated from learning and development in the first example, and from mathematics, in the second one. For even if Mario ends up enjoying understanding the generalizing problem, as William admits, he still considers Mario to be “objectively alienated” (p. 64). Because the initiative is not with the infant/student but with the mother/teacher, the transformation of a not-yet culturally significant arm movement (the random gesture) into a culturally recognized pointing gesture, and the transformation of an arithmetic method into an algebraic one to tackle a generalization problem, cannot amount to anything more than behavioral conformism. As the critique puts it in the fourth-grade example, “the pedagogy obliges the student to conform behaviourally to the demands of the worksheet, without any ‘sense’ of a need for algebraic formulation” (p. 64).

The critique articulates a reading of Vygotsky’s concept of learning and development that not only shows where we have erred but also explains why mathematics education in general has gone wrong: mathematics education has forgotten to take into account that non-alienating learning and development activity starts with the student’s initiative. Failing to review the distinction we make between expansive learning and defensive learning (Roth & Radford, 2011, p. 97), concepts from critical psychology that distinguish between actively taking up a collective motive versus working around it (Holzkamp, 1993), Williams concludes:

This is a key issue for mathematics education, as it explains why it is so difficult to design and implement developmental activity within the confines of school: the initiative is all with the teacher who has to engage the student in a task that requires a relevant mathematical development. The danger is that the teacher successfully seduces the student to feel engaged in the arbitrary of schooling per se, while disengaging from anything that looks like critical mathematics. (p. 65)

Indeed, in Williams’ understanding of Vygotsky’s idea of learning and development, the infant and the student must be in the driver’s seat. The first move must be theirs. Then, the adult intervenes, for her role is, in Williams’ interpretation, one of a culturally competent empathizing helper. He explains:

In my reading, the infant began this interaction with a goal in reaching out to grasp some desired object . . . it is the desire to grasp an object which is interpreted by a culturally competent helper, one who empathises with the infant’s frustration when it fails to grasp it . . . The developing infant then has to notice the carer’s action, recognise its association with their own grasping movement, and probably practise this on a number of occasions; one imagines the infant perhaps dropping the toy and reaching/pointing to see the effect of these on the adult. (p. 64)

Williams can now summarize the difference between his and our reading of Vygotsky. The differences reside in the different roles that we ascribe to the child’s agency, goal, and initiative. He expresses differences between our text and his reading of Leont’ev: “the agency, goal and initiative of the child is an essential moment, and the intervention of the adult/teacher in joint activity only makes sense to the learner on that basis” (Williams, 2016, p. 65). We here see that the crucial point under discussion is not really whether or not the infant’s arm movement is a random body movement. The point under discussion, and where the differences

rest, is not even the infant's or the student's role, but the whole concept of the developing and learning activity—a concept that includes and re-theorizes the role of the infant/student, the mother/teacher, their embodied and ethical actions, and the objects of learning. This is why learning and development do not occur in Williams' dualist interpretation of the "joint activity" of infant/student and his mother/teacher. In Williams' view, joint activity remains reduced to a reciprocated ongoing interpretation of actors' actions, which was exactly the way in which Piaget understood interaction and collaboration (Piaget, 1967; for a critique see Radford & Roth, 2011). It does not come as a surprise then that Williams' account of the pointing gesture comes close to the interpretation that Thompson (2014) offers in the entry "Constructivism" in the *Encyclopedia of Mathematics Education*. In the constructivist interpretation, the child has the intention of grasping the object and makes himself the connections "between his attempted grasping action and someone fetching what he wanted" realizing through what he "can make others do with [his] action" (p. 98).

By contrast, we see learning and development occurring in an unfolding joint activity understood as a single social and historical whole. Thus, the action of an individual (self) is always social because the other is involved or implied, not merely when another person is present (Roth, 2016). An action is for (the purpose of) another, having been initiated by an action of the other, and thereby returning to the other. This is why Vygotsky (1989) notes that any higher psychological function, any higher specifically human behavior "was the social relation with another person" (p. 56, emphasis added). The action of the individual is also historical, as it is embedded in an activity whose conceptual context has been built and refined historically. In addition to being social and historical, we do not conceive of the infant's/student's and mother's/teacher's actions as merely occurring sequentially one after the other after their rational evaluation/interpretation, as Williams' and Thompson's constructivist interpretations do: "The developing infant then has to notice the carer's action" (Williams, 2016, p. 64) and "Look at what I can make others do with this action" (Thompson, 2014, p. 98). To say that we understand joint activity as a social and historical *whole* means that we see the infant's/student's and the mother's/teacher's actions as intertwined with each other and as being of a relational nature (as seen from the italicized prepositions "for," "by," and "to" above—two crucial ideas in our book that Williams entirely misses in his critique). It is in the non-dualist relational constitution of activity that the affective, emotional, and ethical dimensions truly come to the fore. They are not cosmetic or trivial epiphenomena but something consubstantial with learning and development, knowing and becoming. It is not the reciprocated actions of actors that make an activity a truly joint endeavor, not at least in the dialectical materialist stance that we take in our work, a stance that really makes futile the discussion of whether the gesture was initiated by the mother or the infant, or if it was made randomly or not; or whether the origin of intention lies with the child or not. Relevant to our discussion is a commentary that Vygotsky makes about the child's first drawings:

At the first stage, the child draws, then names what he drew. He does not even know what he is drawing when he is asked. He still does not have a plan, *he does not have an intention with respect to his own action*. (Vygotsky, 1998, p. 115, emphasis added)

To sum up, we believe that to cast non-alienated learning as William does, that is, as something that starts with the infant's/student's agency, goal, and initiative is a reductive choice that is plainly against the materialist dialectical stance of Vygotsky's view of learning and development. It posits the student as the origin of meaning and intention. As

the work with congenitally deaf-blind children shows, even intention (innate for Piaget) is first a social relation with another person before it appears in the individual—e.g., a child never wanting to stand up on her own (Meshcheryakov, 1974). In these children, nothing that we associate with the human mind initially exists (Il'enkov, 2007). In the constructivist view, however, Mario appears as a rational being capable of intending and aiming at what is to be learned. All Mario needs is just a shiny object, something susceptible to catch his attention. According to the critique, what our discipline needs—and here we find the remedy for the problems that torment mathematics education—is a classroom activity that would seduce Mario's interests so that he engages with the object to learn: “if there had been a meaningful goal that Mario had been seduced into trying to ‘grasp’ . . . maybe he would have reached out and tried to grasp or engage with a goal (before he had completed it)” (Williams, 2016, p. 66). Such reaching out grounded in Mario's own intention could have led “to the emergence in reflection after the event of a new mathematical point—‘to multiplicatively model patterns,’ say” (p. 66). From the viewpoint of dialectical materialism, the critique remains trapped in the dualism of “teacher actions” versus “students' actions” that has been so detrimental to mathematics education. The critique ends up (a) picturing the student as a rational being “that already knows its business, one that requires only a facilitative grooming to become more fully socialized and intellectually engaged” (Martin, 2004, p. 197); (b) portraying the teacher as a dispenser of knowledge (a “helper” to use Williams' exact term—an alienated helper, indeed) and (c) considering knowledge as something that is simply there, ready to be grasped. This is why the learning paradox that we formulated in our book appears to Williams not only misleading but unnecessary.

3 On materialist dialectics methodology

In his critique, Williams (2016) appeals to dialectical thinkers such as Vygotsky and Marx (and Bourdieu). Yet, the method he uses to sustain his argument is entirely contrary to the methodological tenets of dialectical materialism. In the following, we provide the historical context of the intrinsic problem in Williams' method; in the sections that follow, we exemplify how this methodological problem is reproduced in the critique.

Friedrich Engels' text on dialectics begins with a parenthetical note to the author containing its purpose: “to develop the general nature of dialectics as the science of the connections” (Marx & Engels, 1975, p. 348). The text then states that the three basic laws of dialectics are derived from the history of nature and human society. The three laws include: “the law of transformation of quantity into quality and vice versa; the law of the interpenetration of opposites; the law of the negation of negation” (p. 348). It was Hegel who, according to Engels, had developed these three laws. The problem with Hegel's framing is that these were “mere laws of *thought* [bloße *Denkgesetze*]” (p. 348). Engels then makes a statement that Vygotsky would take up in his recommendations for a psychology to be developed: “The error consists that these laws as laws of thought are *forced upon* rather than *developed out of* nature and history” (p. 348, emphasis added). Vygotsky (1997) directly refers to this quotation when he chastises the “Marxist” psychology of his day for failing to “create a general psychology” (p. 330). For him, a genuine general psychology should use concepts “formulated in direct dependence upon general dialectics” (p. 330). Vygotsky thought that the concepts have to be developed out

of the situation, in strict analysis of the real life of human beings. Doing psychology in any other way would “inevitably lead to scholastic, verbal constructions . . . to judgment about things according to their external, accidental, secondary features” (p. 330). All of this would end up to be “a gross distortion of both Marxism and psychology” (p. 330).

Concerning the specific concept at issue, Marx and Engels (1978) exhort their readers to “describe the real individuals in their real alienation and the empirical relations/conditions of this alienation” (p. 262). Unfortunately, Williams does the opposite, despite the fact that Marx and Engels urge their readers to refrain from replacing “development of all purely empirical relations/conditions by the mere idea of alienation” (p. 262). Thus, the critique parachutes the statement that the students were obliged “to conform behaviourally to the demands of the worksheet, without any ‘sense’ of a need for algebraic formulation arising” (Williams, 2016, p. 63). Williams concludes: “this is what Vygotsky means by ‘pure verbalism,’ and this type of senseless, arbitrary ‘imposition’ and ‘obligation’ is what Marx means by alienation” (p. 63). None of the words in this compilation has been developed from the analysis of the empirical relations and conditions. Instead, they are words taken from or attributed to Vygotsky and Marx that lack direct connection with the available empirical materials. As Vygotsky writes, these are manifestations of the weakness of scientific concepts, a weakness that stems from an “insufficient saturation with the concrete.” In other words, the words in the critique are empty, just abstract and abstracted words—“pure verbalism.” In a remarkable text, Hegel (1979) attributes such verbalism to commoners. He concludes that abstract thinking is to consider nothing but an abstraction and to annul all other empirical facts. We observe precisely the same method at work in the text of the critique, where everything that the original article meticulously develops from the empirical material is subsumed to empty concepts abstractly imposed from the outside in a speculative manner.

There actually is a parallel but in the reverse direction in the description Roth and Radford provide. Mario does not impose abstract concepts onto a concrete situation. He does engage in the task until, eventually, the abstraction first exists not merely *in* but *as* the relation with the teacher. That is, when the abstraction—goblet 3’s content $\rightarrow 3 + 3 + 3 + 6 \rightarrow 3 \times 3 + 6$ —emerges into Mario’s consciousness while doing the same for weeks 5 and 6, he realizes the object/motive of the activity in his own actions. That is, his own actions, initially an integral part of the joint labor with the teacher, are the concrete, sensual things that then obtain a supersensual quality—which they can obtain only in and as part of the (verbal) exchange relation with another (Marx & Engels, 1962b). In this instance, the concept (generalization) is neither an empty nor arbitrary thing, as Williams suggests, but is a supersensual reflection of the student’s concrete, sensual labor initially jointly accomplished with the teacher before he accomplished it on his own.

4 Investigating development dialectically—thinking with the late (Spinozist-Marxian) Vygotsky

The critique speculates that Roth and Radford’s history of the emergence of a generalization in the life of Mario amounted to “pure verbalism” (Williams, 2016, p. 63), words directly attributed to Vygotsky as per their enclosure by quotation marks. A word search reveals that none of the Vygotsky texts in Williams’ references actually contains the expression. When Vygotsky uses the term “verbalism” (three times in *Thinking and*

Speech, 1987), what Williams refers to constitutes only one part of a dialectical situation. The original text reads:

Thus, at a single stage in the development of a single child, we find differing strengths and weaknesses in scientific and everyday concepts.

Our data indicate that the weakness of the *everyday* concept lies in its *incapacity for abstraction*, in the child's incapacity to operate on it in a voluntary manner. Where volition is required, the everyday concept is generally used incorrectly. In contrast, the weakness of the scientific concept lies in its *verbalism*, in its insufficient saturation with the concrete. This is the basic danger in the development of the scientific concept. The strength of the scientific concept lies in the child's capacity to use it in a voluntary manner, in its "readiness for action." (Vygotsky, 1987, p.169)

As we see from the quotation, everyday and scientific concepts are related in a double bind. Whereas the everyday concept may be concrete (familiar), its problem is (a) its incapacity for abstraction, (b) the child's incapacity to use it in a voluntary manner (the will being a key concept in the works of the Spinozist Vygotsky), and (c) its incorrect deployment. Against this, Vygotsky articulates the strengths and weaknesses of the scientific concept, which, as any higher psychological function, will have been the relation with another before becoming a function (Vygotsky, 1989). It is in this sense that Roth & Radford (2011) describe how "in this activity, Mario takes up previous parts of the experience of humanity" (p. 92); that is, the authors describe how culture is continuously born in the actions even of those who are not yet members of a cultural practice. That is, the scientific concept, to become available to the child at all, presupposes a relation with another—not any other, but a person who already uses the scientific concept. The scientific concept is no different in this from the everyday concept word. In that situation, "first, the word must acquire sense (a relation to things) in itself (an objective connection; and if it is not there, nothing is there); then the child's mother uses it functionally as a word, and, finally, the child does so" (Vygotsky, 1989, p. 57). Thus, "the functions of a word . . . were first divided and distributed among people, and then became part of the person" (p. 61). Development, for Vygotsky, is not socialization, as he ascribes it to Piaget, but is an "individualization of social functions" (p. 61). But, for any individualization to occur, a social function first has to exist; and this existence of a social function presupposes a social relation and a conceptual culturally shared content. When the natural history of a (scientific) word in the life of a child is established, it will have had a function in a social relation; and the sociogenetic method lies in identifying where and in which relation with another person the use of a word first emerges. In Vygotsky's earlier work, the psychological function of the word first is social, an action for or (up-) on another person, before the word becomes an action (stimulus) for the brain (Vygotsky, 1989). Vygotsky captures this situation, sociogenesis of the word (sign), in two triangular diagrams, where the sign first establishes a connection between two human subjects (S₁-sign-S₂) before establishing the connection between the subject and her brain (S₁-sign-brain).

The critique states, without providing any form of evidence: "the task and worksheet activity does not invite the children to mathematise algebraically, it arbitrarily imposes it" (Williams, 2016, p. 63). There are at least two problems with characterizing a concrete cultural-historical activity to be arbitrary. First, the task evidently was not arbitrary, as the critique states, but arose from, and consistent with, the curriculum document of the Canadian province where the curriculum was developed and where the empirical materials were

collected. Thus, for example, the curriculum states under the heading of “patterning and algebra”: “By the end of Grade 4, students will: extend, describe, and create repeating, growing, and shrinking number patterns ... connect each term in a growing or shrinking pattern with its term number” (Ontario Ministry of Education, 2005, p. 73). From the individualistic perspective that characterizes the critique, the task appears as arbitrary, as it does not find its origin and motivation in the subject (the student) itself.

Second, the activity did indeed invite the students, as we know from their acceptance of the invitation. A particular phrase is social phenomenon, not in itself such that we can say it is an invitation but because the next phrase or action treats it as such: an invitation is an invitation because it is accepted or rejected. That is, there is an indivisible couplet {invitation | acceptance/rejection} each of which presupposes the other. This couplet is an indivisible social fact. In the same way, {question | answer}, {agreement | disagreement}, or {affirmation | supplementation} are irreducible social-sociological facts (Bakhtin, 1984; Vološinov, 1930). Roth and Radford provide a moment-to-moment analysis of the dialectics of this invitation in terms of the concept of object/motive, which can be known to the student only after the fact. Mario and his peers are invited to do something without yet knowing what it is and for what reason. They are therefore in the same situation in which any person finds herself who creates or is to create (a poet in the wider sense) something really new: such a person “is typically unable to make clear exactly what it is that he wants to do before developing the language in which he succeeds in doing it” (Rorty, 1989, p. 13). This is precisely what we do: we show how Mario eventually is able to describe what he wants to do, which he could not prior to arriving at that language. Moreover, we develop our account by showing that what will be a form of behavior that is attributable to Mario first “*was* the social relation between two people” (Vygotzky, 1989, p. 56). His teacher, who now exhibits the generalization in her own behavior, at some point in her life, was in a similar situation: the generalization first was a relation with another person. Now she finds herself in a situation where there is a “*renewed division into two of what had been fused into one*” (p. 58).

5 Investigating alienation (non-/dialectically)

We note above that the critique of Williams imposes concepts on situations and fails to analyze the real historical conditions and relations available in the empirical data. Instead of developing a materialist dialectical account, the critique speculates that Mario “may be objectively alienated” (p. 62). The critique charges R&R with interpreting the data “without evidence or insights from [the] wider context” (p. 63), completely omitting the fact that the episode is indeed contextualized—both in the text and in a diagram (Fig. 7.1, Roth & Radford, 2011, p. 130)—as part of Mario’s and his teacher’s whole-life trajectories. That is, we explicitly state that the episode needs to be understood not just in the immediate context of the history of this particular classroom and research project, as the critique wants to have it, but in the context of the protagonists’ entire lives. The diagram in the book includes Mario’s family, his relationships with peers, and a possible participation on a hockey team as an overall frame within which his school mathematics has to be understood and theorized. The problem of William’s treatment of alienation is both methodological and conceptual.

Methodologically, losing sight of the dialectical materialist concept of alienation, the text simply posits that the worksheet activity did not invite the children to reason mathematically but “arbitrarily imposes” it (Williams, 2016, p. 63). Conceptually, the critique then ignores the

Marxian insight that new needs do not just arise “naturally,” and particularly, new needs do not just arise for the individual outside of the societal context—any intimation of this nature is pure “Robinsonade,” whereby different needs of inherently social nature are said to emerge for the individual (Marx & Engels, 1962b, p. 90). Marx’s analyses hinge upon the societal and not merely the social nature of production, so that there is “no production without need” (Marx & Engels, 1983, p. 27), which is the part that Williams acknowledges, all the while forgetting the second part, whereby “consumption reproduces the need” (p. 27). For a child, the societal, generalized need (e.g., mathematical generalization) cannot yet exist. It is precisely in social relations with others that it first comes to participate in “consumption,” where the need is reproduced in collective, joint labor, prior to existing for the societal object/motive in consciousness towards the production of what it can voluntarily orient. The need for a scientific concept does not mysteriously and on its own arise in the child—notwithstanding rationalism and subjective accounts of knowledge production. The need for a scientific concept becomes available in the “joint production,” which also is “consumption” (Marx & Engels, 1983), where both the product and the need emerge. It therefore is “production [that] produces the consumer” (p. 27); and “the production delivers material to the need, but also delivers a need to the material” (p. 27).

As noted, we investigate in our book an apparent contradiction of education (the paradox of learning), which requires the students to orient towards learning something that they do not already know and, in many instances, cannot be deduced from the already known. This is the case of the object of the labor in which students participate as part of their schooling experience (i.e., as captured in “instructional objectives”). This object *cannot be* apparent to them precisely because students *will be* able to do and know this after the curriculum has ended. Being a student, thus, means being confronted with the alien, an experience any traveler to or anthropologist of a foreign culture also has experienced. The encounter with the alien, thus, is something fundamental to being human.

The conceptual problems characterizing the critique are apparent when we turn to German versions of Marx’s works, where the frequency of the noun, verb, and adjective corresponding to alienation (i.e., *Entfremdung*, *entfremden*, *entfremdet*) is actually low compared to the use of the root “fremd” (foreign, strange, alien, other) in its various forms. Attending to the root word makes it possible to reorient the understanding and investigation of alienation as an encounter with the other and the alien (e.g., Waldenfels, 2006). This is what we find in Vygotsky’s work. Indeed, Vygotsky (1989) articulates development in terms of the individualization of the social. Ontogenetically, everything that the child does not know and that is not already contained as a possibility in current knowing is alien. It is in the relation with the other that the alien object first exists, all the while not existing because the child already participates in its production. It is from the encounter with the alien that the qualitatively new forms of behavior not yet contained in the already existing behavior of the child actually emerge (Roth, 2012; Roth & Maheux, 2015). Learning (development) is the equivalent of the process in which the alien (culture) is transformed and recognized, opening up a new space of action, reflection, and transformation (Radford, 2012, 2016). That is, as learning and developing individuals, we find ourselves in the paradoxical situation: “it is qua alienus—foreigner and other—that man is not alienated” (Levinas, 1978, p. 99).

Methodologically speaking, the critique fails to provide such an account of the encounter with the alien, and, therefore, of any process or thing that might be alienating. Throughout the critique, the link between the activity of this mathematics classroom, including the children and their teacher, is speculative. (For a critique of such a

speculative method, see Marx & Engels, 1978.) Nowhere does the critique actually show any form of alienation occurring, concretely, developed from the data, rather than merely imposing an everyday, abstract use of the concept alienation on an equally abstract account of the classroom activity. From a historical point of view, the origin of alienation does not lie in labor; instead, its origin lies in “the fundamental alienation of market economy” (Henry, 1976, p. 480). In wages, taking the money form, humans are confronted with their work and their being in alienated form, and this alien thing dominates them, and they worship it (Marx & Engels, 1981). In which way does Mario or Aurélie encounter the fruits of their labor in alien form? What is the equivalent of the wages in the original story that Roth and Radford provide? In William’s analysis we find arbitrary classifications: some relations are characterized as alienating whereas others are characterized as non-alienating. The critique asserts (without evidence) that schooling, being an integral part of the child’s life, is alienated from everyday life. There is a gap between the concrete data provided in the account of Roth and Radford, on the one hand, and the theoretical discourse of the critique, on the other. In this gap lies a form of alienation: the theory offered is alienated from the real praxis of mathematics in school that Roth and Radford describe. With Marx we might say that “the two sides [the critique and R&R’s account of the mathematics classroom practice] are so alienated from one another that they do no longer resemble each other” (Marx & Engels, 1981, p. 577).

From a methodological point of view, alienation may be used metaphorically. This, however, forces the investigator to show how one situation is equivalent to another; that is, that there is an equivalence of structure and content between two domains. The critique fails to show any equivalence between students and laborers. The latter exchange their physical labor for wages. They are thus producing goods that are not their own, precisely because their labor is the good that they exchange for money. What do students produce that is not their own, which they give away in return the equivalence of a wage? When the critique suggests that students are alienated from mathematics, in which way can this be given that they have not yet been one with mathematics?

The children relate to the culture of mathematics as the first-time Western visitor to China relates to Chinese culture. In *A Cultural-Historical Perspective*, we provide an account of how a future behavior (the abstracting movement that Mario conducts on his own) is realized in the present (Mario participates in the joint labor of the abstraction). Because the behavior is not yet his own, it is the alien in the familiar (after all, he does participate in a relation). We describe and theorize this phenomenon in terms of the object/motive of the activity that will reveal itself to Mario, after the fact, in his own preceding actions. A slightly different but consistent way of accounting for the phenomenon exists in cultural psychology in the form of the concept prolepsis (Cole, 1996). Prolepsis is defined as “the representation of a future act or development as being presently existing” (p. 183). That future is an alien in the present. Mario participates in an activity, which has an object/motive, but it is only after the fact, when he has completed the activity, that he can realize its object/motive. That is, after the fact he recognizes that what he knows already has been present in the past without him recognizing it as such. The process is not unlike any other learning, such as language, which requires students to know how to speak (write) their mother tongue before they can know the grammar of the new language. These relations are in fact only a form of the Marxist realization of the primacy of praxis over theory: “It is not consciousness that determines life, but life determines consciousness” (Marx & Engels, 1978, p. 27).

6 Conclusion

In this paper, we respond to the critique Williams (2016) articulates concerning neo-Vygotskian approaches, which are said to fail to deal with the alienation of students from mathematics, alienation of schooling from everyday life, or objective alienation in curriculum tasks. Our study shows that the critique fails (and must fail) because despite its dialectical jargon and its alleged Marxist and Vygotskian orientation, it turns out, in fact, to be based on an individualistic conception of learning and development. Despite Marx & Engels's (1978) warning that autopoiesis (self-construction) is a "speculative-idealist" conception and that "individuals . . . do not make themselves" (p. 37), Williams' critique is based on an individualistic approach. In particular, the critique

- fails taking into account a crucial theoretical tenet of dialectical materialism which provides the primacy to the social and historical in learning and development. This is exactly the dialectical tenet in Vygotsky's (1998) first, second, and third laws of the development of higher mental functions;
- does not follow a materialist dialectical method; and
- falls short on appropriately theorizing topics such as development and alienation.

Vygotsky might have oriented Williams to a place where Marx and Engels (1962a) write, "In Germany, *real humanism* has no more-dangerous enemy than *spiritualism* or *speculative idealism*, which replaces the *real individual human being* by the '*self-consciousness*' or the '*mind*'" (p. 7). By positing the primacy of the social, requiring a sociogenetic method, Vygotsky eschews the individualism and the idealism that underpin Williams' critique. Even though the critique describes itself as following the works of Marx, we show here how its method and its articulations are non-Marxian.

We do acknowledge that Williams is right in pointing out that there are irreducible differences between his (individualistic) understanding of learning and development, and alienation, and ours, which is grounded in the primacy of the social as Vygotsky (1989) and other dialectical thinkers articulated it. But the differences are not where he thinks they are. Throughout this article, we show that the differences are much deeper. Nevertheless, Williams' article provides a marvelous and rare opportunity to discuss these matters, which we consider fundamental in contemporary discussions in the field of mathematics education. To keep discussing these matters may help all of us to better understand crucial nuances of how to theorize the students, teachers, teaching and learning, mathematics, and mathematics classroom activity.

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