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On inferentialism

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Abstract This article is a critical commentary on inferentialism in mathematics education. In the first part, I comment on some of the major shortcomings that inferentialists see in the theoretical underpinnings of representationalist, empiricist, and socioconstructivist mathematics education theories. I discuss in particular the criticism that inferentialism makes of the social dimension as conceptualized by socioconstructivism and the question related to the objectivity of knowledge. In the second part, I discuss some of the theoretical foundations of inferentialism in mathematics education and try to answer the question of whether or not inferentialism overcomes the individual-social divide. In the third part, I speculate on what I think inferentialism accomplishes and what I think it does not.

Keywords Socioconstructivism · Reason · Objectivity · Concept · Theories in mathematics education

Introduction

The decades of the 1970s, 1980s, and 1990s witnessed the emergence of the foundational theories in Mathematics Education Research—for example, constructivist theories in North America (Cobb and Yackel 1996; Steffe and Gale 1995), the theory of didactic situations in France (Brousseau 1997), and Davydov's (1991) program in Russia. The foundational theories in mathematics education were part of an effort to move away from the paradigm of direct teaching. The theoretical orientations of the foundational theories did not appear out of the blue. They capitalized directly or indirectly, explicitly or implicitly, on an educational movement that during the nineteenth and twentieth centuries, appeared in response to a redefinition of education as a

societal problem and that led to what educators and historians of education call the Reform Movement (see, e.g., Holmes 1991).

The Reform was based on two main ideas. First, that education was essential to create the industrial society that many countries envisioned at the end of the nineteenth century and beginning of the twentieth century as both the model of society and the chief mark of civilization. Second, that the industrial society required a new kind of individual—a technologically minded one. The Reform Movement was not a homogeneous endeavor. It was comprised of different paradigms, one of its most distinctive ones being what educators and historians of education call "progressivism" (see, e.g., Labaree 2005). Progressivism was erected against the idea of learning as passive acquisition of knowledge. Although progressivism evolved differently in Germany, England, USA, Canada, and other countries (Röhrs and Lenhart 1995), it was based on a new conception of the child—in particular as a human being in need of an adequate environment in which to unfold its own intellectual capacities. Starting from this conception of the child—a humanist conception developed by Rousseau, Pestalozzi, and others, and whose theoretical articulation goes back to the European philosophy of the Enlightenment—progressivism stressed the role of investigation and play in learning, emphasized the learner's autonomy (see, e.g., Neill 1992), and devised a child-centered pedagogy. It made it possible to offer something different from the transmissive pedagogy and to think of the students as deeply involved in their learning. Knowledge was no longer considered as something to transmit but rather as a "personal acquisition, obtained by learning from experience" (Darling and Nordenbo 2002, p. 298).

It is against this general cultural and historical background that the foundational theories in Mathematics Education Research emerged. Constructivism turned to Piaget's genetic epistemology and more specifically to von Glasersfeld's (1995) adaptation of Piaget's principles. The theory of didactic situations turned to Piaget as well but was also influenced by Bachelard's (1986) concept of knowledge and knowing. Davydov, by contrast, drew on dialectical materialism and the understanding of mathematics brought to the fore by Bourbaki and the modern mathematics movement (see, e.g., Davydov 1962).

Since the dawn of the twenty-first century, however, an intense effort has been directed towards re-orienting our field and overcoming what seem to be the limits of the foundational theories. Two short examples are as follows: a panel held at the 23rd annual conference of the International Group for the Psychology of Mathematics Education conference (PME) in 1999, which had the suggestive title "Doing research in mathematics education in time of paradigm wars" (Sfard 1999), and the general topic of the annual 2009 PME conference: "In search for theories in Mathematics Education" (Tzekaki et al. 2009). We can add to these examples the regular working groups on theories in mathematics education at CERME (see, e.g., Kidron et al. 2012) and the various attempts at understanding theories and their relationships (see, e.g., Bikner-Ahsbahs and Prediger 2006, 2014).

What is it that motivates this clear increased interest in the development of new theoretical approaches to teaching and learning in Mathematics Education Research? There may not be a straight and direct answer. It will be up to the

historians of our research field to come up with accounts and explanations. There is, nonetheless, something easy to spot that we can mention already: at the dawn of the twenty-first century, many Mathematics Education researchers became more and more interested in understanding the role of culture, history, and society in conceptions about the students' learning. There has been a feeling that the various brands of child-centered pedagogy are no longer responding to the new societal sensibilities, problems, and tensions—including political and multicultural ones. Efforts have been made to articulate socially, historically, and culturally oriented conceptions of the mind and the learner, and a broader conception of psychology than the narrow idealist subject-centered one inspired by the natural sciences. Following in the footsteps of pioneer work by Bishop (1985) and D'Ambrosio (1985), there has also been an interest in moving away from the Eurocentric conception of mathematics. These new sensibilities seem to have led to a need to redefine the theoretical bases in our field. Inferentialism, as we shall see, presents itself as an alternative to one of the foundational theories: the North American socioconstructivism that, in the words of the inferentialist theoreticians, fails to provide (among other things) a satisfactory account of the social in knowing and learning.

This article is a critical commentary on inferentialism. In the first section, I discuss some of the major shortcomings that inferentialists see in the theoretical underpinnings of representationalist and empiricist mathematics education theories. In doing so, inferentialists trace the borders of what inferentialism is not. I think that this strategic move is an essential part of defining inferentialism and perhaps theories in general. That is, it might be the case that a theory always (or very often) emerges *against* other theories (I mentioned before that progressivism emerged against direct teaching). A theory is first defined as what it is not. In the second part, I discuss the theoretical underpinnings of inferentialism. In the third part, I speculate on what I think inferentialism accomplishes and what I think it does not. This article is based on the five articles that comprise this Special Issue only.

What inferentialism is not

Inferentalism is presented as a theoretical approach opposed to representationalism. It makes an effort to overcome the underpinning mind-body or mind-world split that is behind representationalism, which conceives of the human mind as constructing internal representations that mirror the external world.

For inferentialism, the mind-world split is not merely an unnecessary assumption but is, over all, a misconception of the true nature of the human mind whose characteristic is to be found elsewhere. In the inferentialist account, in the realm of pedagogy, the representationalist idea of inner (or mental) representations has introduced a range of problems in the way we understand learning. Mackrell and Pratt (2017) expressed them as follows: "This idea [of mental representations] has created huge problems for understanding learning, as it has led to the tacit assumption that all knowledge must involve representation and hence that to learn is to construct such representations."

Inferentialism is also presented as a theoretical approach opposed to empiricism. In empiricism, the world is already out there, "Given," ready to be experienced by a sentient subject. Derry argues that

> The idea of the Given is that we have immediate awareness of the world from the very start, without concepts being in play, and that knowledge results from impressions that, having impinged on our senses, are interpreted by human construction. According to this view of things, concepts come into play late in the day after the world has already been taken in. This common and unexamined assumption of a bare Given upon which we make constructions pervades much pedagogic practice. (Derry, 2017)

The main problem that inferentialists find with empiricism is not merely its emphasis on impressions as the chief source of knowledge but the fact that it distracts us from what they take to be the true nature of the human mind. Indeed, sensations are not specific to humans; apes and other species have sensations too. Derry (2017) notes that "the need to appreciate what distinguishes us from other life forms or inanimate objects is crucial." The distinction is not to be found in the studies of "chimpanzees and pigeons that conflate the actions of other animals with what appear to be similar actions of humans." The key to unravel the mystery of the human mind has to be found not in the dividing line between humans and other species but elsewhere. The key is not located in the creation and use of tools, as Vygotsky thought, but way down the road of evolution. And what is it? The inferentialist answer is that what is really distinctive about human beings is our *responsiveness to reasons*.

I shall come back to this concept later. For the time being, let me note that inferentialists explain the concept of responsiveness to reasons as the human disposition that leads individuals to produce inferentially articulated responses. This view on the truly human, on what distinguishes humans from other species, is embedded in an epistemological and ontological view of humans and the world and on a concept of language that I will discuss after I make some comments on the inferentialist critique of socioconstructivism.

Inferentialism as a critique of socioconstructivism

The inferentialist critique of socioconstructivism is interesting on several counts. It provides us with a view of what, through inferentialist lenses, socioconstructivism failed to offer. But in doing so, inferentialism also provides us with a view of how it intends to overcome the socioconstructivist shortcomings, enlightening us thereby about what inferentialism as a new theory in Mathematics Education *is*.

The inferentialist critique of socioconstructivism is clearly articulated in the article by Noorloos et al. (2017). They pinpoint three problems of socioconstructivism:

(i) its problematic conception of the relation between the social and the individual,

(ii) its lack of an adequate account of the objectivity or world dependence of knowledge, and
(iii) the vagueness of the constructivist metaphor.

In this section I deal with the first two points only.

The social

In dealing with the first point, Noorloos et al. remind us that there was an intention within the constructivist camp to include in an explicit way a social dimension in their account of knowledge production. This intention has to be understood against the historical background that I mentioned in the introduction and in particular the critique that was made in the 1990s to child-centered pedagogies (see, e.g., Lerman 1996). In order to remain coherent, constructivists were very well aware that the addition of the social dimension should conform to the constructivist tenets and in particular to the manner in which constructivism conceives of knowledge production. The addition of the social was accomplished by emphasizing the manner in which individuals construct knowledge while participating in social practices. The theoretical move is extremely interesting in that it shows an example of how an educational theory can be expanded without contradicting its conceptual premises, in this case, the central premise that it is the individual who constructs her/his own knowledge. The ensuing theory—the North American socioconstructivism—then appeared including a social dimension whose cornerstone was the *interaction* between the individuals and the *personal understanding* that each one of the individuals could gather from such an interaction. Noorloos et al. refer to a passage in which Cobb and collaborators explain the matter: the idea was

> to treat people in general and mathematics teachers and students in particular as active constructors of their ways of knowing and as participants in social practices [...]. Knowing would then be seen as a matter of being able to participate in mathematical practices in the course of which one can appropriately explain and justifying [sic] one's actions. (Cobb et al. 1992, p. 15) (Noorloos et al. 2017)

Now, for Noorloos et al., this constructivist move is not satisfactory. The reason is that, in their eyes, the constructivist pragmatic move tried to reconcile the cognitive approach (where individuals are considered to be building their own meanings and concepts) and a participationist sociocultural approach (where individuals are considered to be learning by participating in social practices) without really resolving the dichotomy between individual-centric and social-centric approaches to learning:

> The dichotomy is kept in place by the large differences in the underlying epistemological and metaphorical orientations of the cognitive and sociocultural approaches . . . Because of this, the question has arisen whether these approaches are simply too disjoint to be reconciled. Will any modification of them be sufficient to adequately theorize both the social and individual aspects of learning (as we should surely attempt to do)? (Noorloos et al., 2017)

Noorloos et al. go on to suggest a criterion in order to decide whether or not a theory overcomes the social-individual divide:

> *a theory overcomes the social-individual dichotomy if it can describe the learning activity of the student simultaneously and essentially in both cognitive and social terms*. By "simultaneously and essentially," we mean that the theory should be able to describe the activity in such a way that both its social and its cognitive aspects are inseparably connected to it — in other words, the activity would not be what it was unless it had *both* social and cognitive aspects. To use philosophical terminology, these aspects must be "internally related": If either aspect were to be removed, the activity would cease to be. It is not enough, then, for the activity to have both a social and an individual aspect, if it is constituted such that these aspects are not internally related (so that, conceivably, the social or the individual aspect could exist in isolation from the other). (Noorloos et al., 2017; emphasis in the original)

To what extent is the inferentialist criterion useful to deal with the problem at hand? It seems to me that socioconstructivists could still argue that their manner of conceiving of learning involves both the individual and the social and that, even if there is a hierarchy between them, learning is something that always happens in a social context, as we live immersed in society. As a result, in learning (in the socioconstructivist sense of the term), both the social and the individual are simultaneous and essential. And I think that this is true even of direct teaching, which always includes a social component and an individual one. Simultaneity and essentiality remain concepts too vague to make the criterion useful. Maybe the problem is not just to give equal emphasis to both the social and the individual but to *re*formulate, in explicit and theoretically manageable ways, the individual and the social.

Objectivity

The central inferentialist qualm with socioconstructivism seems to be placed elsewhere. Where the qualm really is, it seems to me, is in the epistemological relativism that socioconstructivism and other educational constructivist theories adopt. Thus, after asking whether or not Cobb et al. succeeded in describing learning activity as simultaneously and essentially both social and cognitive, Noorloos et al. answer negatively:

> We believe they do not. For Cobb et al., intersubjectivity turns upon the fact that interpretations of mathematical objects are taken as shared — that is to say, they are not correct in an absolutely objective (realist) manner, but their correctness is based only upon the fact that these interpretations are agreed to be shared by the different participants in the practice, with no way of verifying in an absolute manner whether they are really correct. (Noorloos et al., 2017)

Elsewhere in their article, Noorloos et al. argue that

> the central concern is that if all knowledge derives from individual interpretations, we seem to have sacrificed the possibility of objectivity and risk running into the specter of relativism . . . the worry is that constructivists cannot give a role in their theories to reality itself. (Noorloos et al., 2017)

Indeed, from an inferentialist viewpoint, the accounts that we provide of things and events around us are not mere personal constructions or concepts, but accounts of reality, of how things and events really *are*. Our accounts are certainly subjected to social agreements and intersubjective norms, but they are not merely consensual adopted manners to keep the conversation unfolding. They are social agreements that keep us in touch with the world as it really is. Thus,

> when talking about a triangle, we are purporting to talk about this actual triangle. Even though this claim depends on intersubjective norms both for its correctness and even for its fundamental intelligibility, this does not change the fact that when it is spoken, it has to be assessed depending on whether it correctly speaks of *the* triangle. (Noorloos et al., 2017; my emphasis)

Socioconstructivists and constructivists in general may still retort that, in fact, in referring to the triangle, the inferentialist is not referring to *the* triangle as such, but to the personal beliefs of what this individual holds. The conversation would not be about reality itself but about our beliefs concerning reality. However, inferentialists insist that this is not the case: "it is not enough that what the person says about the triangle corresponds with the *beliefs* other people have about this triangle; instead, it has to correspond with the actual properties of the triangle." (Noorloos et al., 2017; my emphasis)

I think that we have touched here upon one of the most central differences between inferentialism and constructivism. The difference is epistemological. While constructivism adopted a relativist position about what we know about the world, inferentialism, as it appears in the articles in this Special Issue, moves away from this epistemological relativism, and adopts a kind of realist epistemology: the inferentialists' conversation is about the real state of the world. Interestingly enough, both constructivism and inferentialism adopt a pragmatic philosophy. However, with inferentialism, pragmatism evolves in a sort of realism and, after a long detour, we go back to the magnificent adequacy between world and mind that characterized the epistemological theories of the Enlightenment.

What inferentialism is

Inferentialism comes from a contemporary branch of semantics. It focuses on how we respond to things around us, more specifically how we respond in a reasonable manner to what we say. Although in principle there are many ways in which we may reason about what we do and say, inferentialism focuses on inferences; that is, how we deduce things from other things. It is in the inferentialist apparatus of reason that inferentialists find "what is distinctively human" (Derry, 2017). Brandom explains the point as follows:

> Picking us out by our capacity for reason and understanding expresses a commitment to take *sapience*, rather than *sentience* as the constellation of characteristics that distinguishes us. Sentience is what we share with nonverbal animals such as cats-the capacity to be *aware* in the sense of being *awake*. Sentience, which so far as our understanding yet reaches is an exclusively biological phenomenon, is in turn to be distinguished from the mere reliable differential responsiveness we sentients share with artifacts such as thermostats and land mines. Sapience concerns understanding or intelligence, rather than irritability or arousal. One is treating something as sapient insofar as one explains its behavior by attributing to it intentional states such as belief and desire as constituting reasons for that behavior. (Brandom 1994, p. 5; emphasis in the original)

In the inferentialist jargon, what makes us human is our *responsiveness to reason*.

Now, the individual making inferences is not merely an inference-maker. The individual of inferentialism is portrayed as a committed claimer: a true seeker who endorses her/his assertions and those that follow inferentially, as these derived assertions express the individual's commitments. Brandom explains that

> The idea is to show what kind of understanding and explanatory power one gets from talking this way [i.e., the inferentialist way — LR], rather than to argue that one is somehow rationally *obliged* to talk this way . . . One of the central tenets of the account of linguistic practice put forward here is that the characteristic *authority* on which the role of assertions in communication depends is intelligible only against the background of a correlative *responsibility* to vindicate one's entitlement to the commitments such speech acts express. (Brandom 1994, xii; emphasis in the original)

We see, hence, that the question of agency enters the inferentialist landscape through the idea of *commitment*. This point deserves attention for, as we shall see, inferentialism portrays humans as individuals participating in the world of reason. In the inferentialist account, individuals participate in a social practice that they call the game of giving and asking for reasons. In participating in this game, the question for the individual is not just of being obliged to reason according to a normative dimension that inference-makers follow but to position oneself within a world ruled by reason through the subjective idea of commitment. Is commitment to one's claims in the inferentialist sense broad enough to provide a cogent account of agency? I will return to this question later. For the time being, against this short background, let us see how inferentialist educators present inferentialism. Derry starts by acknowledging the difficulties in locating inferentialism within theories that inform mathematics education research:

> Locating inferentialism among the various theories relevant to mathematics education is not at all straightforward. Simply put it is a theory of meaning but this fails to do full justice to its theoretical reach. It is a theory which forms part of a move in thought which sees mind as inseparable from world and language "not as formal structure but as a feature of the natural history of being like us." (Williams & Brandom, 2013, p. 372) (Derry, 2017)

Noorloos et al. introduce inferentialism in the following terms:

> Inferentialism is a semantic theory that explains concept formation in terms of the inferences individuals make in the context of an intersubjective practice of acknowledging, attributing, and challenging one another's commitments. For inferentialism, inferences cannot be understood apart from the norms that exist in this intersubjective practice, the game of giving and asking for reasons, with the consequence that individual reasoning cannot be understood apart from this social, norm-laden game. Inferentialism provides an alternative characterization to constructivism's conception of social-individual interaction that replaces the latter's emphasis on construction with a focus on the role of reasoning in learning. (Noorloos et al., 2017)

Inferentialist pedagogy

Inferentialism offers a holistic idea of concepts. That is, concepts are interrelated through inferential connections; they do not exist in isolation. As Noorloos et al. put it,

> The key idea of inferentialism is that concepts should be understood in terms of their inferential connections (Bakker and Derry 2011). For example, the meaning of "triangle," for Brandom, depends on the fact that one can, inter alia, derive "p's three angles are equal to two right angles" or "p is not a circle" from "p is a triangle," but not "p has four sides." The inferences one can and cannot make on the basis of a given claim articulate what that claim means. So, in learning new inferences that can be made on the basis of "p is a triangle," one is learning more about the meaning of triangle. (Noorloos et al., 2017)

How do inferentialists define understanding? Bakker, Ben-Zvi, and Makar refer to Brandom:

> To grasp or understand (…) a concept is to have practical mastery over the inferences it is involved in — to know, in the practical sense of being able to distinguish, what follows from the applicability of a concept, and what it follows from. (Brandom 2000, p. 48) (Bakker et al., 2017)

Schindler, Hußmann, Nilsson, and Bakker explain the content of concepts as related to the place of judgements in inferences: "Content of concepts is conceptualized according to the inferential structure and role of judgements" (Schindler et al., 2017). From there, they provide the following definition of understanding: "in an inferentialist perspective understanding can be conceptualized as the 'mastery of properties of theoretical and practical inference' (Brandom 1994, p. 5), where inferential relationships exist between sentences, which have a propositional content (Brandom 2000)" (Schindler et al., 2017).

Following in the footsteps of sociocultural participationist theories, learning is conceptualized as the *mastering* of a practice:

> Mastering is the process whereby learners come to grasp practices, including reasoning practices, and hence concepts. By learning to make the correct inferences and exhibit the correct patterns of behavior, they are mastering the reasons and norms that are implicit in these reasoning practices. Mastering, therefore, combines aspects from both the acquisition and the participation metaphors of learning into a new metaphorical framework. (Noorloos et al., 2017)

Unfortunately, the articles in this Special Issue do not present a detailed example of learning in order for us to have a clear idea of the way in which learning is specifically investigated both theoretically and empirically. The article by Bakker et al. (2017) deals with a case study of a research project carried out by an individual (Sam) as part of his internship in a hospital laboratory. The investigation is about how knowledge and actions are required to reduce uncertainty in the context of valid statistical inference. Bakker et al. focus on the following research question: in "the process of making a statistical inference, how are reasons and actions coordinated to reduce uncertainty?" As we can see, the question is not about Sam's processes of learning; the research question is rather of an epistemological nature. In the article, Bakker et al. do not seem interested in investigating the particular subject's reasons. The subject intervenes as an example—a window to explore the space of reason and to see its mechanisms at work.

The article by Schindler et al. sets out to investigate "how students draw on out-of-school and school-based experiences when reasoning on the order relation for integers and how they infer from these experiences" (2017). In an interview context, much in the vein of psychological research, the authors found "different aspects of students' reasoning, especially asking for the reasons students have, for their origins, and related prior experiences" (Schindler et al., 2017).

From the studies of Schindler et al. and Bakker et al., it is difficult to come up with an idea of the kind of pedagogy that inferentialism has in mind. However, Derry provides us with a hint of what such a pedagogy would look like. She starts first by picturing a teacher whose pedagogy is inspired by a representationalist account of knowledge (i.e., someone who considers knowledge as a matter of representation):

> A [representationalist] teacher will attempt to convey meaning by attending to the relation between representations and what they represent. They will be aware that they are faced with a group of learners who are each making a different sense of the representations that they have before them and so they will attempt to provide additional clarification and explanation to expand what lies behind the initial thought of what is that is represented. (Derry, 2017)

The inferentialist teacher, by contrast, would attend to

> the inferential articulation of concept use, namely, making meaning explicit. On the teacher's part, this involves taking seriously what learners say and unpacking their attempt to articulate what they understand. This may involve showing the learners what they have committed themselves too and what is entailed by their commitments and as a result attending closely to the learner's reasoning, in effect teaching the learner to think. This is something that does not occur when teacher questioning fails to probe what lies behind learner responses. (Derry, 2017)

The teacher's understanding of her/his own role rests on understanding that teaching is a matter of immersing the students in the practice of giving and asking for reasons; teaching is about helping students master "that part of the space of reasons in which the concept is embedded" (Noorloos et al., 2017), for learning is in the end portrayed as the "initiation into language and tradition" (Derry, 2017), or as "becoming inculcated into the practice of mathematics" (Mackrell and Pratt, 2017).

Summing up

Derry complains that

> the move in recent years to attend to pedagogic issues has led to a neglect of knowledge and it is here that inferentialism offers insights. At the very least, it offers a way of reconfiguring how we think about these issues, it also opens the way to a more fine-grained account of pedagogic practice and of subject knowledge. (Derry, 2017)

I concur with Derry that in recent years much of the mathematics education research field has moved towards questions of pedagogy—more precisely, towards questions surrounding the teacher, and what the teacher has to know both at the content level and the pedagogical level. To some extent, such a move may have shifted the conversation away from the epistemological domain of mathematics. However, I think that focusing on knowledge is what educational approaches such as the French didactic theories excel at—for example, the theory of didactic situations (Brousseau 1997) or its young sibling theory "the mathematical working space" (see Kuzniak et al. 2016; for a commentary, see Radford 2016a).

However, despite their common interest in knowledge, inferentialism and the French didactic theories are not looking at knowledge in the same way. Inferentialism, as we have seen, privileges a view of knowledge and concepts where things are connected inferentially. It stresses the role of the context (see Bakker et al., 2017). The French didactic theories also privilege the contexts, but instead of assuming inference as an organizing epistemological and didactical category, they focus on the problems (the "situations") that summon mathematical knowledge. This central difference can be understood, I believe, by considering the intellectual traditions in which both approaches are embedded. The theory of didactic situations and its young sibling draw on the epistemological tradition of Bachelard (1986) and his idea of the scientific spirit. Inferentialism comes from an altogether different tradition—a semantic one. Within this tradition, the starting point is what makes us really human. And what is it? What is it that really distinguishes us from other sentient species? It is not just reasoning, as we know very well from primatologists and animal cognition research that chimpanzees, birds, chickens, dogs, etc. do reason too. What other animals do not do, according to the inferentialists, is to be as aware of their reasons as we humans are. In contrast to humans, animals do not articulate their reasons explicitly in inferential manners. This is why inferentialists see that the chief characteristic of the human species is to be responsive to reasons. As Brandom puts it "We are makers and takers of reasons, seekers and speakers of truth" (1994, p. 5).

Mathematics education has evolved along the lines of different ontologies. One that has enjoyed a prominent place in this research field is the one that resorts to a realist ontology where truth transcends the doings and actions of the individuals. Truth is not something that humans construct. Truth is something that pertains to an extra-human ontological layer—*reality*—something that humans discover. Another line of mathematics education research resorts to a relativist ontology. Here, we find socioconstructivism whose relativist position derives from the subjectivist approach it adopts to knowledge and knowing. In general, socioculturally and politically oriented mathematics educators also adopt a relativist position, but for completely different reasons: they make a case for the historical-cultural-political *situatedness* of knowledge and knowing (see, e.g., D'Ambrosio 2006). They try to move away from the universalist views of reality and the world that were the trademark of the accounts of knowledge of modernity (Lyotard 1979). In this line of thought, there is not an all-encompassing world, but cultural-historical *worlds*. Where does inferentialism locate itself in the ontological arena? In "Objectivity" section, I summarized the critique that mathematics education inferentialists make of the relativist ontology of socioconstructivism. Does it mean that inferentialism adopts a realist position? The question is difficult, if not impossible, to answer in light of the contents of the papers in this Special Issue. There would be grounds, at least to some extent, it seems to me, to think that inferentialism comes to mathematics education armed with a kind of realist ontology. But the point is not clearly articulated in the articles. And turning to Brandom— who is, as noted above, one of the main scholars to inspire the inferentialist movement in mathematics education—might be an inconclusive effort. You may read, for instance, Chapter 8 of Brandom's book *Making it explicit*, where the question of objectivity is addressed through the question of the objectivity of the norms that govern our inferences and their conceptual content. But the result is very disappointing—and unexpected in a book whose theme is precisely about our capacity as human beings to make our theoretical positions and their consequences *explicit*. The ontological commitment of Brandom's "conceptual realism" is never spelled out clearly.

As Danielle Macbeth notes,

> Brandom's problem is to understand the interface between a merely causal world and what, following Sellars, he thinks of as the space of reasons, and that problem arises because the causal realm is not normatively characterizable and the space of reasons is inherently normative. His solution is to say that although our responses to environing circumstances are merely causally elicited, they are nonetheless describable in normative terms — as the expression of our commitments, as game moves — in virtue of the context of social practices within which they are embedded. (Macbeth, n.d.)

There remains an ambiguous gap in Brandom's account between reality on the one hand, and the inferential norms and their conceptual content, on the other—a gap between the "non-inferentially warranted content" (Voller 2012) and the normative sphere of reason. We will let the philosophers of language to try to solve the puzzle in Brandom's work. By contrast, it will be up to our mathematics education inferentialist theoreticians to be explicit in their future work about the kind of

ontology that they are ready to hold. Notice that I do not take this point as an irrelevant and pedantic academic inquiry. I have two reasons for encouraging mathematics educator inferentialists to be explicit about their ontology. First, it is precisely against the socioconstructivist ontological relativism that inferentialism defines itself (at least in part; see Bakker et al., 2017). It is hence only fair to the socioconstructivists and the mathematics education community at large to be clear about where they stand on this point. Second, I think that mathematics education research has evolved to a point where it is no longer possible to claim that our ontological assumptions are without relevance to the teaching and learning of mathematics. Through our teaching, unavoidably, we convey visions and interpretations of the world. I think that we are now beyond the innocent attitude according to which our mathematical practices are neutral and without bearing on the issues of how we conceptually and politically understand and should act in the world.

Be that as it may, inferentialism comes into mathematics education research with a sophisticated arsenal of epistemological ideas. It offers a fresh perspective on knowledge, concept formation, and learning that privileges inferential thinking. It comes with plenty of possibilities to investigate the manner in which students enter the space of reasons. It paves the way to think in new ways the question of task design and pedagogical action.

Inferentialism arrives in mathematics education with an explicit intention to replace the North American socioconstructivism that, as we saw, inferentialists think not to have resolved the social-individual divide. In a previous section, I suggested that the participationist view adopted by inferentialism (individuals participating in the social game of giving and asking for reasons) might not be enough to dissolve the divide. I have several reasons in mind.

First, I think that participationist theories (which, by the way, come from anthropological research that features learning as a kind of apprenticeship) are not strong enough. They fail to provide a cogent account of agency (Radford 2016b). The individual is reduced to *mastering* a given practice. In my view, a proper account of agency should include a truly dialectical understanding of the subject and her culture, so that "both the individual and culture are [seen as] coterminous entities in perpetual flux, one continuously becoming the other and the other the one" (Presmeg et al, 2017).

Second, the social in inferentialism focuses on a very peculiar practice: the individuals' participation in the social game of giving and asking for reasons. The social is reduced to the underlying normativity of the game where interlocutors are pictured as kind of calculating agents ("scorekeepers" in the inferentialist jargon); that is, as individuals keeping two sets of books to track what one interlocutor says (commits) and the other interlocutor attributes to her so that one can "move back and forth between the point of view of the speaker and the audience" (Brandom 1994, p. 590). The ensuing structure of intersubjectivity is a rationalist-logical one that seems to me to be not broad enough.

But perhaps the deepest difficulties with inferentialism rest, for me, on its view of (a) knowledge and (b) the human.

(a) On the one hand, knowledge and conceptual content are thematized along the lines of what is *expressible* through language and discourse. "Conceptual content,"

Brandom (1994, p. 586), tells us, "is understood in this work as what can be made explicit in discursive practice" through "propositional" and "sentential" locutions. "Making anything explicit, *saying* it, requires using one linguistic expression rather than another" (p. 586; emphasis in the original).

There is a clear rationalist view of expression and the expressible in inferentialism that gives the approach the basis for a specific understanding of knowledge and knowing. This point is clearly made by Bransen in an article review of Brandom's (2000) *Articulating Reasons*. Bransen notes that

> rather than trying to explain the use of concepts in terms of their content, he [Brandom] will try to explain the content of concepts in terms of their use; rather than thinking of conceptual activity as basically a matter of representation, he will think of conceptual activity as basically a matter of expression . . . rather than thinking of expression along Romantic, traditional lines emphasizing creativity, depth, and spontaneity, he prefers to think of expression along a rationalist line: expression is articulation, is a process of making content inferentially significant, is first and foremost a contribution to the game of giving and asking for reasons. (Bransen 2002, p. 375)

And even if some room could be made to account for the role of context, history, and experience in our knowing about the world, the commitment to a linguistic and rationalist conception of expression ends up, I think, in a too logo-centric conception of knowledge. Although I do not want to minimize the role of language and discursive practices, it seems to me that what we express through language is a thin layer of what we know and are aware of. Research such as the one of Sheets-Johnstone (2009) provides insight into embodied forms and expressions of knowledge (see also Radford et al. 2017).

(b) On the other hand, humans are pictured in a too rationalistic and logical manner moving into the realm of the expressive and the explicit: "We discursive creatures — rational, logical, concept-using ones — are construed here in expressive terms; we are the ones who can make it explicit" (Brandom 1994, p. 587). Although Mackrell and Pratt, and Bakker et al. (2017) make a continuous effort to include what they call the nonlogical within the space of reason (something that involves the affective domain and the sentient body), the connection to the basic tenets of inferentialism still remains to be clearly articulated.

Overall, the most controversial assumption of inferentialism might rest on its answer to the questions of what makes us really human. As I noted above, inferentialism's answer is in our capacity of being makers of reason, seekers and speakers of truth. Historically speaking, the seekers and speakers of truth arose in a long process of secularization of truth that goes from the warriors of the Greek archaic period, where language became "an instrument of social relations and as a means of knowing reality" (Detienne 1996, p. 106), to the aristocratic intellectuals of Ancient Greece who moved from the tangible and the empirical to deductive (or inferential) reasoning (Szabó 1978). In trying to find an answer to the questions of what makes us really human, I would look rather on the side of our cultural and historical evolved capacities to live in solidarity with each other, to ethically respond to the call of the other (not rationally but pre-conceptually); I would look

on the side of our quotidian practice of empathy and the care of the other, in our capacity to dream and hope together for a better world, for us and for those to come. But this may be my bias as a cultural-historical theorist.

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