

Mathematics and Mathematics classroom activity through the lens of a metaphor

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¿Será posible saber sin ser?
(D'Amore, 2015, p. 153)

Abstract. *This essay is my contribution to the celebration of Bruno D'Amore's 70th birthday, In it I suggest a metaphor for mathematics and mathematics classroom activity. I suggest to conceive of mathematics classroom activity as a joint endeavour that is carried out together by teachers and students, much like the joint endeavour that is carried out by an orchestra that performs, say, a symphony in a music hall. What the orchestra produces through its activity is a sensible aural phenomenon: music. In the same way, I submit, mathematics is something sensible, something that is produced by the joint endeavour of the teachers and the students and that is simultaneously visual, tactile, olfactory, aural, material, artefactual, gestural, and kinesthetic.*

1. Introduction

One of the themes that surfaces again and again in the work of Bruno D'Amore is that of practice. In his recent work, D'Amore draws on the sociological idea of practice to offer an understanding of mathematics classroom and a typology of practices. He suggests that we consider the mathematics classroom “as a community of shared practices having as its goal the construction of knowledge” (D'Amore & Radford, in press).

In this short essay, with which I would like to contribute to the celebration of Bruno D'Amore's 70th birthday, I would like to reflect on the idea of the mathematics classroom as a community of practices. But I will dare to reverse the pieces somewhat and argue that what is usually termed practice can be better rendered through what Russian dialectical thinkers such as Vygotsky and Leont'ev have called *deyatel'nost'* and that is usually translated as *activity*. But, as we shall see in a moment, the translation is misleading.

In the first part of this essay I will summarize the idea behind *deyatel'nost'*. I will then resort to a metaphor to argue that mathematics classroom practice (or mathematics classroom activity) can be considered very much like the musical activity of an orchestra or a musical ensemble in a music hall. First, I would like to start with a culinary anecdote because it was while enjoying a piece of Santa Fe chicken and a glass of wine that I was confronted with the difference between activity and practice.

2. The Santa Fe Chicken

Sometime in the early 2000s Bruno sent me an email to see whether or not one of his PhD students could come to spend some time in my Research Laboratory at Laurentian University. The student, I was told, was working on semiotics. After some email exchanges, we fixed the date for the student's visit to be in late spring. The student was interested in understanding the differences between Raymond Duval's (1995, 1998) approach to signs and human learning, Juan Godino and collaborators's famous onto-semiotic approach (see, e.g., Font, Godino, & Gallardo, 2013), and the Vygotskian semiotic cultural approach I was trying to articulate (Radford, 2006; for recent formulations, see Radford, 2008, 2014).

After some weeks of intense discussions around mathematics classroom videos and the analysis of students' productions, we made some progress. The difference between the approaches started to emerge with more clarity. But, of course, the student had to return to his country. So, on the eve of his departure I made a reservation at a restaurant not far from the university.

That evening, with his kind smile and manners, working on a robust and colourful salad, Giorgio Santi was sitting across from me. With a glass of wine in his hand, he mentioned that differences between the onto-semiotic approach and the cultural one cannot be found in the concept of activity. Although the onto-semiotic approach may not have "activity" among its main theoretical constructs, it resorts systematically to the concept of practice. "In the onto-semiotic approach, they talk about the mathematician's practice," Giorgio contended. Giorgio was right (Santi, 2011).

I remember that I tried to counter by saying that these were not the same thing. But I had to surrender. I could not explain why. The differences were not clear. What, indeed, do we mean by practice and by activity? The discussion ended up without conclusion. It ended with a sense that, to make the differences visible, I had to keep thinking about the meaning of activity. To argue, as I did, that I was using it in Leont'ev's (1978) sense was not enough.

3. *Deyatel'nost'*

The Merriam-Webster online dictionary gives the following definition of activity: "something that is done as work or for a particular purpose" (<http://www.merriam-webster.com/dictionary/activity>). This definition highlights two things: First, activity is about doing; second, it is about doing something purposely. It is from this sense of activity that the usual conceptions of activity derive. For instance, activity is conceived of as a series of actions that an individual performs in the attainment of his or her goal. The German and Russian languages have a specific term for this conception of activity as being simply busy with something: *Aktivität*, and *aktivnost'*, respectively (Roth & Radford 2011).

There is, however, another sense for activity, which is the one emphasized in dialectical materialism, where activity does not merely mean “to do, and be busy with, something.” The German and Russian languages have a specific term that better conveys the idea of activity of dialectical materialism: *Tätigkeit* (in German) and *deyatel’nost’* (in Russian), one that puts at its heart the idea of life.

When Leont’ev tries to define activity, this is what he does, and to do so he talks about a *unit of life*. Leont’ev says that activity

is a molar, not an additive unit of the life of the physical, material subject . . . activity is not a reaction and not a totality of reactions but a system that has structure, its own internal transitions and transformations, its own development. (1978, p. 50)

Let me insist: activity as *Tätigkeit* or *deyatel’nost’* is a *form of life*. More precisely, it is a social form of joint endeavour that comprises self-expression, intellectual and social development, and aesthetic enjoyment.

In this line of thought, activity refers to what Aristotle describes in his *Metaphysics* as a process – an unfolding energy – , something that in modern terms we may call a *dynamic system* that, instead of being simply object- or goal-oriented, is geared to the satisfaction of collective needs and the self-expression of the individuals.

When I was finishing my Santa Fe chicken that memorable evening with Santi, I had not realized yet that behind this dialectical idea of activity rests a specific anthropological conception of the human. I had not realized yet that the dialectical idea of activity can only be understood if we think of it along with the corresponding anthropological conception of the human. It took me years to realize it. It is perhaps the French philosopher Frank Fischbach that helped me the most to understand it during the lengthy conversations that he had when he was visiting Laurentian University and my research lab. The interested reader can watch Fischbach’s brilliant lecture on subjectivity in our site: <http://penseeculture.ca/2015-16-conferences/>

The deep relationship between activity and the concomitant concept of the human can be stated as follows (of course, I am simplifying, but the account is sufficient to make my point, I hope. The sceptical readers will have to dig into Marx’s Parisian manuscripts, i.e., the *1844 economic and philosophical manuscripts* and *The German Ideology*). Humans, following Marx’s (1998) Spinozist stance, are considered to be part of nature: they are considered *natural beings*. That humans are natural beings means that they are sensible beings. And to say that humans are sensible beings means that humans are unavoidably *affected* by the other parts of nature: by things and people, and by what other people do and think. In this context, sensations and passions are conceptualized as ontological affirmations of the individual’s nature as a natural being (Fischbach, 2004).

One important consequence of this theoretical stance is that the individual's existence cannot be conceived of as a substantial entity, produced from within, as articulated by the humanist trend of the Enlightenment. The individual's existence is *relational* through and through. It appears to be profoundly linked to an ensemble of relationships with other parts of nature – including social relationships – and is based on culturally and historically constituted conditions of life. This is what Marx says in the famous sixth thesis on Feuerbach, when he defines that which makes us human: “But the essence of man [sic] is no abstraction inherent in each single individual. In its reality it is the ensemble of the social relations” (1998, p. 570). From this idea of humans, Marx can assert that “Essence, therefore, can be regarded only as ‘species’, as an inner, mute, general character which unites the many individuals in a *natural way*” (p. 570; emphasis in the original). In this line of thought, to be a natural being means also that, like other natural living beings, humans are *beings of need* who find their satisfaction in objects *outside* of themselves.

But the story does not stop there. To meet their needs (needs of survival and also artistic, spiritual, intellectual, and other needs created by/in society), humans engage actively in the world. They *produce*. What they produce to fulfil their needs occurs in a social process that is, at the same time, the process of the *individuals' inscription in the social world and the production of their own existence*. In dialectical materialism, the name of this process is *deyatel'nost'*, that is, *activity*. This is why, from this perspective, sensuous, material activity is considered the ultimate field of aesthetic experience and cognition and that discourse-oriented, and practice-oriented, and *deyatel'nost'*-oriented ways of theorizing are not the same (Radford, in press).

In *A cultural historical perspective on teaching and learning* (Roth & Radford, 2011) my colleague Michael Roth and I tried to look at classroom activity through the lens of *deyatel'nost'*. To do so, we resorted to Leont'ev's (1978) seminal work. In articulating a psychological approach based on the idea of activity as *deyatel'nost'*, Leont'ev (1978) highlighted some of activity's basic components: an activity for him is characterized by its object and its motive. The object and motive of an activity are the engines that keep activity in motion. In practice, in the pursuit of the activity's object, individuals break down the object into a sequence of goals with which actions are associated. He referred to the material conditions through which the actions occur as operations. In the Supplement to his important 1978 book – a supplement dedicated to educational matters – Leont'ev discusses the conditions under which a certain theoretical learning content can be meaningfully perceived or attended to by the student. He contends that

in order that the perceived content be recognized, it is necessary that it occupy the structural place of a direct goal of action in the subject's activity, and thus that it appear in a corresponding relation to the motive of this activity. (Leont'ev, 1978, p. 153)

It is hence through activity and the structural interconnection between motive, object, goals, and actions that the learning content becomes disclosed to the student's consciousness.

Activity Theory, as this approach has come to be known, has had an important impact on education in general and mathematics education in particular. Yet, in focusing on the *procedural* aspect of activity, activity is reduced to its operational and functional dimension, eradicating the aesthetic and political dimensions of action and creation, culminating unfortunately in a dull technological account of what was originally thought of as the sensible experience of *life* – human life.

How, then, could we recover the idea of activity in the sense of deyatel'nost'? It is here where I need to turn to the metaphor of music.

4. Mathematics as a sensible, material phenomenon

In a Colloquium on Symbolic Cognition that Stephen Hegedus organized in a secluded and remote hotel in Vermont a few years after Giorgio's visit to my laboratory, I was having breakfast with some mathematicians and mathematics educators at a table by a window. We were there together for one week. We could see the beautiful, totally white landscape. It had been snowing without interruption for days. It was January and it was cold. I do not remember what brought us to discuss the nature of mathematics. Maybe it was a good night's rest, or an interesting discussion on symbols the day before, or both. I ventured to mention that mathematics could not be equated to symbols. Mathematics, I argued, is not comprised of the symbols on the pages of a book. The symbols on the pages of a book are exactly that – *symbols*, or *marks*, if you want me to put it more bluntly. To sustain my claim, I resorted to music. In the same way that there is no music in a score sheet, there is no mathematics in the pages of a textbook. Music is what we *hear* when people play instruments. Mathematics is ... Well, you see, I do not have an exact equivalent term with which to refer to the sensible phenomenon that *appears* when I talk about mathematics as I have when I say that something appears when an orchestra plays a symphony. But it does not mean that we cannot think of mathematics as something that *appears* as students and teachers engage in a certain classroom activity. What appears in the mathematics classroom is not exactly an *aural* phenomenon or a *visual* or *tactile* or an *olfactory* one. Yet, *something* appears (and perhaps is something that is all of that: visual, tactile, olfactory, aural, material, artefactual, gestural, and kinesthetic) and that, being all of that, becomes the object of consciousness and thought. Mathematics, in this materialist and phenomenological line of thought, is what is made sensible through the teachers' and students' activity.

To continue with the Vermont metaphor, we need to make some distinctions. I am not saying that mathematics and the activity that produces it are the same. Yet, both are *deeply* intertwined. We cannot extract one from the other, as we

cannot extract the orchestra's activity from what we hear. Mathematics has a deep influence on the kind of activity that will bring it into sensible existence, and vice versa. This is why we can say that there are good mathematical activities and that there are bad ones.

Now, activity is not a static thing that happens all of a sudden, nor is mathematics. As the activity unfolds, mathematics appears – much as, for example, Beethoven's 7th symphony appears as the orchestra activity unfolds. Unfolding and appearing have to be understood here in a dialectical relationship. The unfolding affects, moves, and transforms the appearing, and the appearing affects, moves, and transforms the unfolding.

But things do not merely happen or appear out of the blue. The sound that is produced by a violin, for example, has its source in the instrument. The instrument is bearer of a what Aristotle called *potentiality*. The sound may or may not be produced. And if it is produced, it can be produced in countless many ways. It is both: contingent and historically bounded. In being produced, the sound materializes or actualizes that which was potentiality or pure possibility. Hegel talks about the *general*. In his terminology, mathematics or music are sensuous evolved forms of something that before being materialized and coming into sensible existence was general.

The general is *formless*. It belongs to the realm of the potential or the virtual. Yet, it is not a Platonic Form. The realm of the potential or the virtual belongs to an always changing immaterial sphere of culture that is intertwined with the material world of objects and human actions. This immaterial sphere of culture is part of what Marx (1998) called the “non-organic” realm of nature and it is also part of the conditions out of which human existence is crafted. This sphere cannot be sensed by us humans through our culturally and historically evolved senses and sensations. Can we sense or perceive or touch the Pythagorean theorem *as such*? We cannot. Can we hear Beethoven's 7th symphony *as such*? We cannot. To become the object of consciousness, feeling, and thought, the general has to be set into motion to transform it into something sensible, and *appear*. Its appearance is the *singular*. The singular is the appearance of the general through the mediation of human activity.

To make the previous ideas clearer, let me turn to Beethoven's 7th symphony. As we know, Beethoven's 7th symphony has four movements: Poco sostenuto – Vivace, Allegretto, Scherzo, and Allegro. Table 1 presents the duration of the symphony as conducted by two orchestra directors who are considered to be among the best 20th century Beethoven specialists: Herbert von Karajan and Leonard Bernstein.

Table 1

Total time of two famous recordings of Beethoven's 7th symphony

	Herbert von Karajan (Berlin Philharmonic Orchestra, 1963 recording)	Leonard Bernstein (New York Philharmonic Orchestra, 1958 recording)
Poco sostenuto – Vivace	11:25	12:27
Allegretto	8:02	9:44
Scherzo	7:50	8:23
Allegro	6:37	7:27
Total time	33 min 54 s	38 min 01 s

Bernstein's recording is 12% longer than Karajan's. And I think that the reader would agree with me that 12% is a lot. It is not an insignificant difference. Which one is the true 7th symphony? Neither of them. The 7th symphony as such is a *general*. Bernstein's and Karajan's recordings are materializations of this general; that is, they are singulars, or, in other words, they are appearances of the general. I would like to insist that this general has nothing to do with a metaphysical or Platonic concept. The 7th symphony as a general belongs to a musical tradition of leisurely symphony prologues, a romantic paradigm, an increasing focus on rhythm and the smart use of available musical artifacts (e.g., metronomes for measuring tempo), among others. In other words, rather than existing in itself and by itself, the general is to be found in culture and history.

We can summarize these ideas by saying that the singular is the unending appearing of the general. In other words, the singular is the coming into existence of the general as an evolved ontological form transformed under the force of an activity (deyatel'nost'). That this activity is not merely an Aktivität or aktivnost' is shown by the fact that the Aktivität or aktivnost' is what would appear if the 7th symphony were to be interpreted by programmed artifacts and mechanical devices only. Such appearing would in fact lack exactly that which makes deyatel'nost' what it is, namely human, natural life.

Let us come back now to mathematics and mathematics classroom activity. What produces the activity is mathematics as a *sensible phenomenon* – a *singular evolved form* of something *general* that before being set into motion by the classroom activity was pure potentiality. The activity that happens in the classroom, as I mentioned previously, can be good or bad. The bad activity is precisely that which looks pretty much like the mechanical one of my example, where people do not really connect, where they do not work *together*. They simply do things, as in Aktivität or aktivnost'. It is *lifeless* activity, like in traditional teaching and learning. It is something like knowing without being. Yet, as D'Amore asks: ¿Será posible saber sin ser? [Would it be possible to know without being?]. Certainly not. The good activities would be, by contrast, those where students and teachers engage, debate, agree and disagree, where they object and find a place for subversion, and where

students could show respect, responsibility, and care for each other. In short, the good activities would be those in which knowing and being (or becoming) are there, simultaneously. How to get there? To answer this question I might need to have a few more dinners with Giorgio Santi, to digest the ensuing ideas, and to wait until Bruno D'Amore's 80th birthday celebration for the ideas to be ripen.

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