

Mathematics Education as a Matter of Labor

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Introduction

During the twentieth century, mathematics education was predominantly conceptualized either as the diffusion of mathematical contents or as the facilitation of the students' development of mathematical cognitive structures. In the first case, the emphasis was generally put on the organization of the mathematics curriculum and the efficient management of the learning environment. In the second case, the emphasis was often put on mental structures and the understanding of students' mathematical conceptualizations. In the first case, the underpinning theoretical orientation was essentially *epistemological*. In the second case, the theoretical orientation was *psychological*. Although the aforementioned conceptualizations of mathematics education have shown their merits, in the past few years, there has been an increasing awareness that to come to grips with the complexity of contemporary societal demands, mathematics education can no longer be fruitfully formulated either as an epistemological or as a psychological matter – not even as epistemological *and* psychological.

Sociocultural theories developed in the fields of sociology and anthropology (from Émile Durkheim to Pierre Bourdieu and beyond) have provided new perspectives by which to consider mathematics education. In particular, sociocultural theories have provided mathematics educators with new possibilities to conceptualize the students, the teachers, and the school and to better understand the political, economic, social, and cultural dimensions that shape mathematics as a scientific discipline and mathematics education as a social-political-pedagogical project.

Sociocultural theories differ categorically from the individualist idealist approaches to the mind and the rationalist epistemologies that have informed mathematics education since the early twentieth century. The individualist approaches to the mind understand the production of meaning and ideas as a mere subjective endeavor. Rationalist epistemologies understand it as an abstract, nonhistorical, a-cultural process. Sociocultural theories, by contrast, understand the production of human beings and the ideas and meanings that humans produce as embedded in the individuals' cultures. The common denominator of sociocultural theories is the claim that human beings are *consubstantial* with the culture in which they live their lives. In other words, cultures are not merely a constant source of stimuli to which humans adapt. On the contrary, the way in which human beings think, take action, feel, imagine, hope, and dream is deeply entangled in the historically constituted forms of thinking, sensing, feeling, and

interacting that they find in their culture. To a great extent, differences between sociocultural theories appear according to the manner in which the aforementioned consubstantiality is understood and theoretically thematized—and so is the case of sociocultural approaches to mathematics education.

Historically speaking, the differences between sociocultural theories did not appear all of a sudden. Nor did they appear clearly formulated. They turned around the problem of the individual and the social, and the subjective and the objective. It is in this context that, in the works of Wilhelm von Humboldt, Ernst Cassirer, and Valentin Voloshinov (or Vološinov), language came to be considered the link between cultures and their individuals. Voloshinov (1973), for example, turned to the *word*. Noticing that the word is implicated in every act and contact between people, Voloshinov found in the word the ontological connection between the individuals. Drawing on this conception of the word, Voloshinov, as well as Bakhtin, came to see literature not just as one of the fields of aesthetic experience and cognition but the central field through which the other cultural fields are refracted. It is ultimately through language and literature that reality is produced and interpreted. The problem of the individual and the social, and the subjective and the objective, is resolved, in Voloshinov's account, in the dialectical tension between the relatively stable centripetal forms of culture (epitomized by the *novel*) and the centrifugal forms of resistance and novelty (epitomized in Bakhtin's idea of *carnival*).

Language-oriented sociocultural research (e.g., research based on Bakhtin's and Vygotsky's work) has had an important influence on current mathematics education. For instance, inspired by the work of Vygotsky (1987) and discursive psychology, Lerman (1996) has studied the role of language in the constitution of intersubjectivity. Barwell (2014) has turned to Voloshinov's and Bakhtin's work to understand mathematics classroom discourse, while Sfard (2008) has drawn on Vygotsky's ideas to develop an educational discursive approach to mathematics teaching and learning. This research area has recently led to

questions of ideology, agency, gender, and power in the mathematics classroom (Radford and Barwell 2016).

In the following another sociocultural way of theorizing mathematics teaching and learning is described, where the primacy is not given to language but to human activity.

Mathematics Education as a Matter of Activity

To consider mathematics education – and in particular its teaching and learning – as a matter of *activity* means to place oneself within a different perspective from the one in which language, discourse, and literature appear as the ultimate field of aesthetic experience and cognition. To think of mathematics education as a matter of activity is not to dismiss the role of language in the processes of knowing and becoming but to assert the fundamental ontological and epistemological role of matter, body, movement, action, rhythm, passion, and sensation. To think of mathematics education as a matter of activity is an invitation to consider teaching and learning mathematics in accordance with the way in which teachers and students engage in classroom activity. It is an invitation to attend to the sensuous manners in which teachers and students bring mathematical ideas to the fore and produce mathematical meanings. Those sensuous manners include perceptual activity, gestures, kinesthetic actions, posture, language, and the use of artifacts, symbols, graphs, and diagrams (Radford 2009).

Behind the idea of mathematics education as a matter of activity rests a specific anthropological conception of the human. Humans, following Marx's (1998) Spinozist stance, are considered to be part of nature: they are *natural beings*. That humans are natural beings means that they are sensible beings, unavoidably *affected* by the other parts of nature. In this context, sensations and passions are conceptualized as ontological affirmations of the individual's nature as a natural being. One important consequence of this theoretical stance is that the individual's existence cannot be conceived of as a substantial entity, produced

from within, as articulated by the humanist trend of the Enlightenment. The individual's existence is *relational* through and through. It appears to be profoundly linked to an ensemble of relationships with other parts of nature – including social relationships – and is based on culturally and historically constituted conditions of life. In this line of thought, to be a natural being means also that, like other natural living beings, humans are *beings of need* who find their satisfaction in objects *outside* of themselves.

To meet their needs (needs of survival and also artistic, spiritual, intellectual, and other needs created by/in society), humans engage themselves actively in the world. They *produce*. What they produce to fulfill their needs occurs in a social process that is at the same time the process of the *individuals' inscription in the social world* and *the production of their own existence*. In dialectic materialism, the name of this process is *activity*. Sensuous, material activity is considered the ultimate field of aesthetic experience and cognition.

This conception of activity is very different from usual conceptions that understand activity as a series of actions performed by an individual in the attainment of his or her goal. In dialectical materialism, activity is something else. It is precisely the specific form in which the individuals *express* their life. "As individuals express their life, so they are. What they are, therefore, coincides with their production, both with *what* they produce and with *how* they produce" (Marx 1998, p. 37). Activity, in short, is a social form of joint endeavor that comprises self-expression, intellectual and social development, and aesthetic enjoyment. It is a process in a system of social relations that realizes the societal nature of human beings (Roth and Radford 2011).

In articulating a psychological approach based on the dialectic materialist idea of activity, Leont'ev (1978) suggested a basic structure of activity. An activity for him is characterized by its object and its motive. The object and motive of an activity are the engines that keep activity in motion. In practice, in the pursuit of the activity's object, individuals break down the object into a sequence of goals to which actions are associated.

In the "Supplement" to his important 1978 book – a supplement dedicated to educational matters – Leont'ev discusses the conditions under which a certain theoretical learning content can be meaningfully perceived or attended to by the student. He contends that

in order that the perceived content be recognized, it is necessary that it occupy the structural place of a direct goal of action in the subject's activity, and thus that it appear in a corresponding relation to the motive of this activity. (Leont'ev 1978, p. 153)

It is hence through activity and the structural interconnection between motive, object, goals, and actions that the learning content becomes disclosed to the student's consciousness.

Activity theory, as this sociocultural approach has come to be known, has had an important impact on education in general and mathematics education in particular (see, e.g., Jaworski et al. 2012; Roth and Radford 2011). Yet, in focusing on the *procedural* aspect of activity, activity is reduced to its *operational* and *functional* dimension, eradicating from it the aesthetic and political dimensions of action and creation. The account of activity culminates, unfortunately, in a technological dull analysis of what was originally thought of as the sensible experience of human *life*.

The idea of mathematics education as a matter of joint labor is discussed below. The idea of joint labor seeks to restore to activity its most precious ontological force, namely, the dynamic locus where human existence creates and recreates itself against the backdrop of culture and history. Yet, with its utilitarian and consumerist orientation, contemporary mathematics classroom activity tends to produce and reproduce alienated students. It is argued that the search for non-alienating classroom activity requires a reconceptualization of the classroom's forms of human collaboration and its modes of knowledge production. The section ends with a view of classroom activity as joint labor, that is, a collective, critical endeavor of mutual and self-fulfillment, and a discussion of the communitarian ethic that supports it.

Mathematics Education as a Matter of Joint Labor

In dialectic materialism (see, e.g., Ilyenkov 1977), knowledge (mathematical, scientific, artistic, legal, etc.) is considered to be constituted of forms of human action that have become historically and culturally synthesized. They are synthesized forms of action and reflection bearing, in sedimented ways, the political tensions and contradictions of human life. They are always in the process of continuous movement, constantly born and reborn, incessantly transformed in practice. Knowledge belongs to an immaterial sphere of culture that is intertwined with the material world of objects and human actions. This immaterial sphere of culture is part of what Marx (1998) called the nonorganic realm of nature and is also part of the conditions out of which human existence is crafted.

Instead of being conceptually neutral, knowledge already conveys a specific *ideology*. That is to say, unavoidably, knowledge allows one to always see the world in a *certain* way. The symbolic algebra of the Renaissance, for instance, conveys the theoretical stance of the instrumental reason of the Western sixteenth century and the social abstractions brought forward by the emerging mercantilist capitalism. And it is under the theoretical stance of practical reason and calculation that contemporary school mathematics conveys, through the curriculum and other institutional mechanisms, an instrumental and technical view of the world of objects and humans.

The instrumental and technical view of the world of objects and humans is produced and reproduced through a utilitarian orientation of classroom activity. This is what traditional (or direct) learning does. In it, mathematics appears as a disembodied realm of truths, and the students' work is reduced to passively receiving information, repeating and memorizing it (Freire 2004). The students cannot *express* themselves in the products of their learning. In traditional learning classroom activity is the expression, not of a fulfilling life but of an alienated one. The so-called reformed learning and its

student-centered Piagetian pedagogy has sought to find in the student's work an escape to the technical view of the world of objects and humans. To do so, it has promoted an individualist and romantic pedagogy that emphasizes the student's freedom and autonomy. In this pedagogy, the students are left to their own cogitations, interacting among themselves, yet moved by their own interest. In this approach, the students do express themselves but remain imprisoned within the confines of their subjective universe, living a one-sided existence in a chimerical taken-as-shared world, cut off from cultural and historical perspectives at large, and, hence, alienated from them. As a result, classroom activity is again the expression, not of a fulfilling life but of an alienated one (Radford 2016).

Mathematics education as a matter of joint labor is an attempt at restoring the idea of activity in general and classroom activity in particular as a non-alienating form of life. It is inscribed within an understanding of mathematics education as a political, societal, historical, and cultural endeavor. Such an endeavor aims at the dialectic creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical practices and ponder and deliberate on new possibilities of action and thinking. To avoid confusions with other meanings, and to emphasize the idea of activity as a historically produced aesthetic form of life where matter, body, movement, action, rhythm, passion, and sensation come to the fore, activity, in this approach, is termed *joint labor*.

The concept of joint labor, which plays a central role in the theory of objectification (Radford 2008), offers a reconceptualization of teaching and learning. In joint labor, the students are not reduced to a role as simple cognitive subjects. They do not appear as passive subjects receiving knowledge or as self-contained subjects constructing their own knowledge. In the same vein, teachers are not reduced to a role as technological and bureaucratic agents – guardians and implementers of the curriculum. They do not appear as possessors of knowledge who deliver or transmit knowledge to the students either directly or through scaffolding strategies. The

concept of joint labor suggests an educational perspective in which to envision teaching and learning not as two separate activities but as a single and same activity: one where teachers and the students, although without doing the same things, *engage together*, intellectually and emotionally, toward the production of a *common work*. *Common work* is the *sensuous appearance* of knowledge (e.g., the sensuous appearance of a covariational algebraic or statistical way of thinking through collective problem posing and solving and discussion and debate in the classroom). *Common work* is the bearer of dialectic tensions because of the emotional and conceptual contradictions of which it is made. Through it, knowledge appears sensuously in the classroom (through action, perception, symbols, artifacts, gestures, language), much in the same way and, with similar aesthetic force, that music appears aurally in a concert hall through the common work of the members of the orchestra.

The joint labor-bounded encounters with historically constituted mathematical knowledge materialized in the classroom *common work* are termed *processes of objectification*. Through these social, material, embodied, and semiotic processes, the students and teachers not only create and re-create knowledge but they also coproduce themselves as subjects in general and as subjects of education, in particular. More precisely, they produce subjectivities, that is to say, singular individuals in the making. This is why, from this perspective, processes of objectification are at the same time processes of *subjectification*.

The concept of joint labor resorts to (a) specific collective forms of classroom knowledge production and (b) definite modes of human collaboration that rest on critical community *ethics*. The ethical forms of human collaboration are driven by a general attitude toward the world and serve to configure the teachers' and students' *joint labor* in the classroom. These critical and community ethical forms blur the borders that separate the teachers from the students. Teachers and students labor *in concert as one*. The classroom appears as a public space of debates in which the students are encouraged to show openness toward others, responsibility, solidarity, care, and critical

awareness. The classroom indeed appears as a space of encounters where teachers and students become *presences in the world* (Freire 2004). That is to say, the classroom appears as a space of encounters, dissidence, and subversion, where teachers and students become individuals who are more than in the world – they are individuals with a vested interest in one another and in their joint enterprise; individuals who intervene, transform, dream, apprehend, suffer, and hope *together*.

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