ON TEACHERS *AND* STUDENTS: AN ETHICAL CULTURAL-HISTORICAL PERSPECTIVE

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My purpose in this paper is to try to understand the ideas that underpin our conceptions of mathematics teachers and students. The path that I follow rests on the thesis that our conceptions of teachers and students are related to, and derive from, conceptions of classroom forms of mathematical knowledge production and forms of human cooperation. In the first part of the paper I focus on two influential educational paradigms of Western modernity and late modernity—the transmissive program and the "progressive" educational program. In the second part of the article, I discuss a historical and cultural conception of teachers and students that is based on a non-utilitarian ethical dialectical materialist logic of production of knowledge and subjectivities.

INTRODUCTION

In *Politics*, Aristotle remarks that when it comes to determining what should be the aim of education "The existing practice is perplexing; no one knows on what principle we should proceed." Should we teach children what is "useful in life," or "higher knowledge," or what promotes "excellence" (that is, what contributes to virtue)? (Aristotle, 1984, p. 2121; 1337a33-1337b22)

Were Aristotle able to come back from his grave, he would certainly be disappointed in the utilitarian outlook of our 21st century school mathematics. He would be surprised to see school board officials, principals, and teachers meticulously absorbed in the implementation of abstract consumerist skills and competencies through business-like management techniques. And he would definitely be confused by our tremendous fixation on subjecting students to regional, provincial, national, and international evaluations.

We may guess that, as a good Athenian, Aristotle would regret the disappearance of curriculum principles concerning the common good and truly social communitarian life. He would be appalled by the instrumental conception of teachers and students. And maybe he would be very curious about how we ended up where we are today.

My purpose here is not to suggest how we should tell Aristotle our history. My purpose is rather to try to understand the ideas that underpin our conceptions of teachers and students. For instance, what is it that has moved us to conceive of the students in an instrumental manner? That is, how did we end up conceiving of students as rational- and instrumental-oriented cognitive problem solvers denuded, as it were, of all subjectivity?

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There are, of course, different ways in which to carry out this investigation. The path that I shall follow rests on the thesis that our conceptions of students and teachers are related to, and derive from, conceptions of classroom forms of knowledge production and forms of human cooperation. I will be arguing that the classroom forms of knowledge production and human cooperation are not created *in situ*, on the spot. On the contrary, they are related to culturally and historically constituted political and economical forms of production of human existence. Within this context, to conduct my investigation, I need to pay attention to the cultural forms of production of material, social, and spiritual life. Hence I will be trying to scrutinize what we can term *cultural logics of production* out of which knowledge and subjectivities are co-produced in the classroom. I am interested here in discussing the conceptions of teachers and students conveyed by two influential educational paradigms of Western modernity and late modernity. These are the transmissive educational program of the 20th century, and the "progressive" educational program that sought to replace the previous one during the last third of the 20th century and that continues to serve to a large extent as a general model for mathematics education today. In the second part of the article, I focus on conceptualizations of mathematics teachers and students from an ethic cultural-historical perspective.

ON TEACHERS AND STUDENTS

A few months ago, our research team was videotaping in one of the schools we regularly work with. At lunchtime we went to the staff room and joined a discussion about the forthcoming provincial assessments. Quickly, the discussion moved to questions about best teaching practices and, unavoidably, to the expectations we set about our students. These expectations do not merely refer to the reasonableness of having the students learn this or that mathematical content in a certain span of time. More than anything else, they refer to the very idea we adopt about what it means to be a student. Let us pause for a second and think what we take a student to be beyond, of course, its legal definition—that is, an individual subjected to a learning institution.

The idea of the student that we articulate in education in general and in mathematics education research in particular derives from conceptions about how knowledge is produced and the role of students and teachers therein. Conceptions of knowledge production are in turn embedded in cultural conceptions of the production of existence more generally and in their ensuing conceptions of the individual. This is why our conceptions of students are not insulated from general conceptions of the individual. The latter serves, indeed, as the basis to create expectations about the students —e.g., how students should behave, what they should or should not do, what they should learn and how. What are these conceptions?

The transmissive educational program

In modern times, one predominant conception was developed during the first half of the 20th century. It was underpinned by a bureaucratic idea of agents as implementers of solutions that were required in the business production context. Individuals were

expected to acquire skills and use them in order to cope with well-defined problems. This conception of the individual nourished the idea of teachers and students conveyed in Paulo Freire's "banking concept" of education. In *The pedagogy of the oppressed*, Freire (2005) pointed out that students were receiving knowledge in a passive manner by a knowledgeable teacher who treated the students as deficient beings—empty containers or depositories that, as education progressed, were gradually filled with static, monotonous and irrelevant knowledge. Education, Freire argued,

> becomes an act of depositing, in which the students are the depositories and the teacher is the depositor. Instead of communicating, the teacher issues communiqués and makes deposits which the students patiently receive, memorize, and repeat. This is the "banking" concept of education, in which the scope of action allowed to the students extends only as far as receiving, filing, and storing the deposits. (p. 72)

What is the *logic of production* on which the transmissive educational program rests? The *logic of production* provides an educational program not only with a general framework to operate (e.g., by indicating how knowledge is produced and reproduced), but also with the parameters within which teachers and students are conceptualized.

The transmissive educational program, which was supported by behaviourist psychology, resorts to a *logic of production* that conceives of individuals as "adaptable, manageable things" (Freire, 2005, p. 73) and of knowledge as possession. Knowledge is treated as a commodity or merchandise that can be moved from one place to another and that can be passed from one individual (the teacher) to another (the student). Within this logic of production, the teacher *owns* knowledge and gives it to the student. In turn, the student comes to *own* knowledge through unreflective drill and repetition. Knowledge delivery is sanctioned and ensured by an institutional process where commodified knowledge is endowed with capital value.

What is the ultimate characteristic of this educational model? As we can see, the ultimate characteristic of this model is to consider knowledge as a commodity and to conceive of teachers and students as related to each other through processes of transmission of goods. Knowledge is considered a commodity that teachers *possess* and students *acquire*. Within this educational model, teachers and students are conceived of as *private owners*. The logic of production that underpins this conception is the *private owner logic of production*.

Although this conception of teachers and students has not completely vanished from education—at least not in practice—there are other conceptions nowadays. There is in particular a much more elaborated and sophisticated conception of the student that, roughly speaking, emerged in the second half of the second part of the 20th century and that Canadian psychologist Jack Martin (2004) describes in detail in a famous article—"The educational inadequacy of conceptions of self in educational psychology."

The "progressive" educational program

In the aforementioned article, Martin refers to a concept of the individual that has influenced contemporary educational and psychological research. The concept is a self-regulated adaptive

> individual labouring in relative solitude, constituted of componential mechanisms, processes, parts, and strategies . . . an individual actor capable of simultaneous action and reflection on this action, much like a stereotypic scientist in close scrutiny and judgment of experimental phenomena of interest . . . [An individual] whose most vital resources are apparently available within its detached internality . . . a self that already knows its business, one that requires only a facilitative grooming to become more fully socialized and intellectually engaged. (pp. 193-194, 197)

Drawing on this cultural conception of the modern individual, the modern conception of the student is largely based on the ideas of rational self-regulation, autonomy, and self-sufficiency. It assumes that the origin of meaning, knowledge, and intentionality is located *within*, and must come *from*, the individual.

This rational conception of the modern individual and its concomitant idea of the student do not come out of the blue. Both are a historical invention. Morris (1972) locates the first steps of this invention in the late Middle Ages. Traces of this historical invention are also found in the Renaissance—when some merchants and bankers emancipated from traditional societal structures and started conceiving of themselves as owners and crafters of their own destiny. However, it was only in the 17th and 18th centuries that Descartes, Kant and other philosophers articulated, in its clearest form, the modern idea of the individual as a sovereign, rational autonomous subject. During the nineteenth and twentieth centuries, the idea was gradually translated to the educational context, leading to the educational movement that has been called "progressivism" and its chief idea that "knowledge is … [a] personal acquisition, obtained by learning from experience" (Darling & Nordenbo, 2002, p. 298). Although "progressivism" evolved differently in German, England, and other countries, stressing with various nuances the learner's autonomy, the role of investigation and play (see, e.g., Neill, 1960/1992), bit by bit, from the aforementioned idea of knowledge as personal acquisition emerged the idea of the student as someone who is not there to be taught but rather someone expected to think and learn through his/her own deeds. For instance, drawing on Kant's ideas, Piaget (1973) asserted towards the end of his life that "The goal of intellectual education is in learning to master the truth *by oneself*" (p. 106; emphasis added). Piaget and educators of that time (e.g. Dearden, 1972) were instantiating the general view of the modern student, already advocated by Jean Jacques Rousseau in his *Émile*, written in 1762, that resulted in a Piagetian inspired child-centred "progressive" educational reform in the last third of the 20th century. The reform was based on the idea that knowledge is something that each student has to construct *by him/herself*—as opposed to something that can be passed on or learned from others. Within this context, leading the students towards an idea that did not come from them was often understood as constraining the students' freedom

and autonomy: it was seen as coercing the students' own solutions and imposing the teachers' meanings upon them (Lerman, 1996; Radford, 2012).

What is the logic of production on which the Piagetian inspired "progressive" educational reform rests? As we can see, it rests on a logic of knowledge production that equates *doing* and *belonging*: *what belongs to me is what I do by myself. What I do not do by myself, does not belong to me*.

From the previous discussion we clearly see that, as in the case of the traditional educational program, the student of the educational "progressive" reform of modernity and late modernity is still conceived of as a *private owner*. It is hence not surprising that we treat them as private owners and talk about them as if what they do and should do is to produce their own wealth (in our case, mathematical knowledge). In their interaction with others we come to see them as *negotiating* meanings regulated by didactic contracts—as entrepreneurs negotiate goods in their own transactions bound by commercial agreements. We organize the curriculum around the idea of credits. And as a speaker did during the 2013 Ontario Educational Research Symposium, we talk about the students as *assets* of society. Within this context, as can be expected teachers are put at a difficult juncture. If students are to build their own knowledge through their own initiative and autonomy, what is then left to the teachers? There is no much left. What teachers can do is to give students their freedom (Darling & Nordenbo, 2002). An unending series of terms have been used to try to come up with the teacher's job description: coach, aide, helper, stimulator, consultant, guide, and so on. Whatever the term, in the end, within the logic of private ownership, they work indirectly, as *financial advisors*, helping the students secure and increase the knowledge they are supposed to create and grow by themselves.

Naturally, the private owner conception of the student is problematic on several counts. For one thing, it makes it difficult to understand the role of culture, history and society in the formation of the student. "There certainly is little here," Martin (2004) argues, "that might speak to the possible socio-culture, political, and moral constitution of personhood" (pp. 193-94). In the last few years—as a result of a flux of human migration and immigration, and the generalization of national capitalisms to a global capitalist economy and in tune with the condescending attitude of neoliberalism—cultural elements such as sensitivity to multiculturalism, as well as tolerance towards cultural and social differences have been added to the educational picture. However, in the end, social, cultural, historical and political factors remain considered as peripheral or partially understood in our constitution as individuals. Within the given logic of cultural production, social justice amounts to a mere redistribution of, and access to, the material.

Let us recapitulate. The transmissive educational program is based on the private owner production logic. It can only offer an alienating and oppressing structure where the teacher acts as a "bank-teller" that pays out wealth to the student. The student acts as someone who acquires wealth from the teacher through drill and repetition. The Piagetian inspired "progressive" educational program is based on the exact same logic,

but reverses the roles of the agents. In the transmissive educational program the teacher assumes power while the students are relegated to a passive role. In the Piagetian "progressive" educational program the students assume power while teachers are relegated to an ancillary role. There is a nuance, though. In the first case, students receive knowledge. In the second case, the students no longer receive knowledge; they are expected to produce (and hence *own*) it. If a piece of knowledge has not been produced by the student, then it is not his/hers, and learning has not occurred. In the first case, the teacher's agency is emphasized to the detriment of the student's agency. In the second case, we have the exact opposite situation. Regardless of the nuance, in one case as in the other, education as a social practice is left structurally *the same*: the forms of knowledge production are those of private ownership. Although there is a displacement in the distribution of power and agency, both practices are equally alienating, since in both cases the student and the teacher remain alienated from each other, and from the broad historical and cultural context, without a truly possible connection. The teacher and the student are like screws placed in different parts of a machine, connected as it were by a formal link–a piece of metal that holds them together.

Now, since the structures of educational praxis remain the same, oppression is not removed: the autonomous student of the "progressive" educational program remains as oppressed as the student of the transmissive program. Freire (2005) saw this problem coming: "The truth is . . . that the . . . solution is not to 'integrate' them [the oppressed] into the structure of oppression, but to *transform* that structure so that they can become 'beings for themselves'" [that is, authentic critical beings- LR] (p. 74; emphasis added)

Late modernity expands and generalizes in more sophisticated and global terms the forms of production of modernity. But the forms of production of human existence remain basically the same. They are not *transformed*. It is hence not surprising that the corresponding educational projects—i.e., the transmissive and Piagetian "progressive" programs— remain unable to overcome alienation as well as their most striking contradiction, namely their conception of the individual as an acultural and ahistorical subject. What is at stake in this contradiction is the understanding of the relationship between the individual and society, and the student and knowledge. And the chances are that we will remain in the impasse we are in today, if we are not able to conceive of students other than as private owners. Indeed, to move beyond this predicament we seem to be in need of new conceptions of classroom knowledge production providing non-alienating roles for students and teachers. What could these roles be? Freire (2005) gives us a hint: "Education," he notes, "must begin with the solution of the teacher-student contradiction, by reconciling the poles of the contradiction so that both are simultaneously teachers *and* students (p. 76; emphasis in the original).

In the rest of the paper I sketch a different logic of production that provides room for exploring the aforementioned "reconciling contradiction" of teachers and students on the basis of a different conception of knowledge and its production.

A CULTURAL-HISTORICAL UNDERSTANDING OF TEACHERS AND STUDENTS

The cultural logic of production that I outline here draws on a Hegelian-Marxist dialectical materialist conception of knowledge. Within this logic of production, knowledge is not something that individuals, possess, acquire or construct. As a result, the relationship between students and teachers is not predicated in terms of individuals who possess knowledge and individuals who lack it.

Now, if knowledge is not possession, what is it? In dialectical materialism, knowledge is not a psychological or mental entity. The dialectical materialist idea of knowledge rests on the distinction between the *Potential* (something that may happen, i.e., possibility) and the *Actual* (its happening). Knowledge and its individual kinds, that is, *concepts*, are social-historical-cultural entities: a "complete totality of possible interpretations—those already known, and those yet to be invented" (Ilyenkov, 2012, p. 150). Knowledge includes possibilities of making calculations, or thinking and classifying spatial forms in certain "geometric" manners; possibilities of taking courses of action or imagining new ways of doing things, etc. This is what school knowledge is when the student crosses for the first time the school door—*pure open possibility*.

Let me note that knowledge as possibility is not something eternal, static, or independent of all human experience (as in Kant's concept of things-in-themselves or as in Plato's forms). In fact knowledge results from, and is produced through, human social labour. Knowledge is a *cultural synthesis* of people's doings. More precisely, knowledge is a dynamic and evolving implicit or explicit culturally codified way of doing, thinking, and relating to others and the world.

Knowledge as possibility means that knowledge is indeterminate, general (Hegel, 2009). Knowledge is not representable. In order for it to become an object of thought and consciousness, knowledge has to be set into motion. That is to say, it has to acquire cultural determinations. And the only manner in which knowledge can acquire cultural determinations is through specific *activities*. Let us take the example of algebraic knowledge—an example that I will develop with more detail below. Algebraic knowledge is not the sequence of signs we see on a paper. Algebraic knowledge is pure possibility—possibilities of thinking about indeterminate and known numbers in manners that are opened up by certain historically constituted analytical ways of thinking. Algebraic knowledge can only become an object of thought and interpretation by being put into motion and being made into an object of senses and consciousness through sensuous and sign-mediated specific problem-solving and problem-posing activities.

In more general terms, through activity knowledge moves from an indeterminate form of possibilities to a determinate singularized form filled with concrete determinations (e.g., the singularized knowledge-form that results from dealing with some *specific* equations). In this context the general/singular (or abstract/concrete) are not two

opposed, disjoint kinds. They are two entangled ontological categories—two moments in the becoming of knowledge. This is why, in its becoming through activity, knowledge and concepts are simultaneously abstract and concrete.

The characterization of knowledge as movement from indeterminate possibilities to their determinate actualization or concretion through the mediation of activity offers room to envision in new ways the relationship between teachers and students. In engaging in activity, knowledge is something that teachers and students *produce*. That is, knowledge is something that they "bring forward." This is what 'to produce' means etymologically: to bring something forward (in this case, possibilities of mathematical action and reflection). In the transmissive and the "progressive" programs, the production of knowledge appears impoverished as a result of the structure of the knowledge-mediating-activity. In the transmissive program, it is the teacher who is essentially in charge of the activity. In the "progressive" program, it is the student who is essentially in charge of the activity. Within the dialectical materialist logic of production that I am outlining here, teachers and students carry out the knowledge-mediating-activity *together*. Knowledge is produced *collectively*. The collective nature of knowledge production means that students and teachers *work together* in order to bring forward possible mathematical interpretations and courses of action. Knowledge production refers to emergent classroom collective and dynamic ways of thinking and doing arising against the backdrop of culture and history. They include modes of mathematical inquiry, conceptions of truth, evidence, mathematical argumentation, symbol use, and meaning making.

Forms of human collaboration

There is something missing in the previous account. In engaging in classroom activities teachers and students do not merely produce knowledge. They co-produce themselves too. They co-produce themselves in accordance not only to the forms of knowledge production but also in accordance to the activity's *forms of human collaboration*. In the "progressive" educational program, forms of human collaboration are usually reduced to utilitarian tools. Thus, interaction is considered a form of reciprocity where agents mutually trade services driven by instrumental self-interest (see, e.g., Piaget's (1967) reckonable reciprocal interactionism; for a critique, see Radford & Roth, 2011). Within this context, students end up considered as purely rational- and instrumental-oriented cognitive problem solvers (Valero, 2004) denuded, as it were, of all subjectivity and unconcerned by questions of social existence.

As Mészáros (2010) put it, within

> the contemporary liberal orientation, we see that in society and in our schools, the legitimately feasible objectives of human activity must be conceptualized in terms of material advancement . . . remaining blind to the social dimension of human existence in other than essentially functional/ operative and manipulative terms. (p. 29; italics in the original)

Human collaboration, Mészáros concluded, is reduced to a "material factor of production" (p. 29).

The dialectical materialist cultural logic of production of knowledge and subjectivities that I am outlining here—what we have termed the 'theory of objectification' (Radford, 2008)—puts forth a *communitarian ethic* that promotes forms of human interaction driven by *solidarity*, *commitment*, *responsibility*, and *caring*. This communitarian ethic is based on a non-essentialist conception of the individual. It is a conception of the individual as constituted in a relation to *alterity*, that is to the *Other*—that which is not *I*.

Individuals, indeed, are seen as subjects ontologically constituted with others (Nancy, 2000): individuals who co-produce themselves as they engage in historically, economically, and politically configured social activities to produce and reproduce their material, interpersonal, and spiritual life. The individual that we portray is an individual whose "interiority," to borrow an Augustinian metaphor, is not *inside* the individual, but *outside*, in the exterior. More precisely, the individual's "interiority" is located in the ensemble of culturally, historically constituted social relations that the individual finds in front of him/her—an ensemble of social relations that reflexively shape each one of us through the activities in which we participate and through which we come to think, feel, hope and grow as cultural beings. Hegel remarked once that, to find their foundational support, plants use their roots, which reach outside themselves. Humans are not different: our constitutive foundational ontological support lies in our exterior, in material and spiritual culture, and in its individuals.

Although there seems to be a pre-reflective sensibility or pre-conceptual proclivity to attend to, and to attune with, others (Roth, 2013; Tomasello, 2009), more sophisticated ethical forms of relations between subjects are of a cultural and historical nature. They are embedded in culturally evolved forms of *being*. Like knowledge, being appears as *pure possibility*. But instead of presenting possibilities for knowing they are pure possibilities for *becoming*.

The ethical forms of human collaboration that we emphasize drive a general attitude towards the world and serve to configure the teachers' and students' participation in the classroom. The classroom appears as a public space of debates in which the students are encouraged to show openness towards others, solidarity, and critical awareness. The classroom indeed appears as a space of encounters where teachers and students become what Freire called "presences in the world" (Freire, 2004, p. 98). That is to say, the classroom appears as a space of encounters where teachers and students become individuals who are *more* than *in* the world, individuals with a vested interest in one another and the joint enterprise; individuals who intervene, transform, dream, apprehend, and hope.

To recapitulate, the cultural-historical perspective previously outlined rests on a dialectical materialist logic of production that offers an alternative by which to conceptualize teachers and students. Teachers and students are not conceptualized as

private owners. They are conceptualized as individuals that engage in joint activity in the production of knowledge and subjectivities. Such a production rests on the idea that knowledge and being are pure possibilities—possibilities of knowing and becoming in collective emergent and unending processes configured by history and culture.

Within this cultural-historical perspective, the forms of knowledge production and human collaboration that mediate classroom activity are based on collaborative modes of action and interaction that seek: (1) to foster deep and varied mathematical understandings and interpretations, and (2) to create space for critical and reflexive subjective and inter-subjective growth to occur.

In the rest of the paper I discuss a classroom example that will help, I hope, to illustrate the previous ideas.

TEACHING-LEARNING ALGEBRA

The example that I present here comes from a 6-year longitudinal program where our research team followed a class of students from Grade 2 on. The research program revolved around the teaching and learning of algebraic pattern generalization and equations. I focus here on the latter.

As mentioned previously, within our cultural-historical perspective, algebraic knowledge (in this case knowledge about equations) is not considered a psychological or mental entity. It is a culturally codified and historically evolved way of thinking and doing that features an *analytical* manner of calculating with determinate and indeterminate quantities (Radford, 2013b). For our Grade 2 students, algebraic knowledge was a pure possibility—possibility to bring forward forms of action, understanding, and interpretation.

The culturally codified and historically evolved ways of thinking are *general* or *abstract.* As such, they cannot be sensed (perceived, touched, heard, etc.). They are pure possibility. To become objects of consciousness and thought, they have to be set into motion through activity. In common parlance, however, the term activity is often used with different and sometimes contradictory meanings. Sometimes activity refers to a mere set of actions carried out by an individual, sometimes by various individuals. Activity appears hence as a mere background or as an empirical phenomenon. This is not the dialectical materialist meaning. In dialectical materialism it is *in* and *through* cultural-historical activity that individuals produce knowledge and co-produce themselves. To emphasize this dialectical materialist meaning of activity, I shall use rather the original Hegelian term *labour*. I can hence rephrase my previous sentence by saying that, to become objects of consciousness and thought, algebraic knowledge has to be set into motion through classroom *joint labour*.

In joint labour teaching and learning are *fused* into a single process: the process of teaching-learning—one for which Vygotsky used the Russian word *obuchenie*. In this sense, teachers and students "are simultaneously teachers *and* students" (Freire, 2005, p. 76). They are simultaneously teachers *and* students, but not because both are

learning (Roth & Radford, 2011). They are, of course. However, the real reason is because teachers and students are *labouring together* to *produce* knowledge.

Through students-and-teachers' joint labour algebraic knowledge is hence going to be *produced*, that is, knowledge is going to be *brought forward*. The bringing forward of knowledge, however, is always partial, incomplete, unfinished (Radford, 2013a). It is an actualization of possibilities made up of concrete and singular or individual interpretations, which depend on the characteristics of joint labour. These characteristics depend, in turn, on the didactic quality of the mathematical questions, the richness, deepness, and variety of forms of mathematical inquiry. To ensure a didactic quality we conduct an *a priori* analysis that allows us to gauge the epistemological density (e.g., sedimented meanings) of knowledge (in this case, algebraic knowledge). The epistemological analysis also provides us with the ground to select and conceptually organize the questions that will serve as the starting point for joint labour to occur. The forms of production of algebraic knowledge are hence related to the collective and emergent forms of thinking that will arise in the classroom out of conceptually charged and epistemologically informed questions that the students tackle with the teacher.

One important point to notice is the *division of labour* that arises as a result of the manner in which teachers and students are aware of the *object* (in Leont'ev's (1978) sense) of their joint labour. The teacher and the students have a different grasp of this *object*. The object, which has a didactic intention, is not necessarily clear for the students from the outset. The students' and teachers' difference vis-à-vis the object of labour creates tensions and (dialectical) contradictions (see Williams & Ryan, in press). In our example, the object of joint labour is to foster deep and varied mathematical interpretations and understandings about equations. These include those algebraic interpretations that have been built historically and culturally. It is in the course of joint labour that those historical interpretations may be brought forward and become objects of consciousness and thought. The culturally and historically built mathematical interpretations have been the object of successive refinements, organized theoretically in complex forms of thinking—in our example, historically constituted ways of algebraic thinking cast in more and more complex semiotic systems. Because these historical forms of thinking are not natural but cultural, they are not necessarily clear for the students. The teacher, hence, has a particular role to play in joint labour. But regardless of how much the teacher knows about algebra, she cannot set algebraic knowledge in motion by herself. She *needs* the students—very much like the conductor of an orchestra, who may know Shostakovich's 10^{th} Symphony from the first note to the last, needs the orchestra: it is only out of joint labour that Shostakovich's 10^{th} can be produced or brought forward and made an object of consciousness and aesthetic experience.

Now, the 'need' to which I am referring here is not merely rational. The 'need' is thoroughly *emotional*. It requires a deep emotional connection between participants. Thus in the best musical performances, the conductor and the musicians work truly

collectively, attuning and responding to each other. Maybe one of the best examples of joint labour is the amazing tuning of Venezuelan conductor Gustavo Dudamel and the Simón Bolivar Youth Orchestra (see http://www.youtube.com/watch?v=XKXQzs6Y 5BY#aid=P9NiWZi3QJQ particularly from 27:50 to 35:00). This musical example intimates that forms of (musical) knowledge production are deeply entangled with forms of human interaction and cooperation. The same is true of classroom mathematical knowledge. The mathematics teacher and the students *need* to labour together to bring forward various mathematical interpretations and make them the object of an intellectual, reflective, and aesthetic experience. This joint labour is, simultaneously, intellectual and emotional; they cannot be separated. They are two sides of the same coin. We may conclude, then, by noticing that, although there is a division of labour that is induced by the manner in which teachers and students engage in their joint labour—division of labour that has to do with the teacher's awareness of the didactical intentions, etc.—the teacher and the students *need* each other to bring knowledge forward.

The communitarian ethic mentioned in the previous section finds its full expression in the theoretical articulation and practical elaboration of this mutual 'need' of teachers and students. We are interested in an ethic that fosters modes of collaboration of a non-utilitarian and non self-centred nature—modes of human collaboration and interaction that rather promote solidarity, critical stance, and responsibility.

Indeed, since the communal ethic we target is not something that will necessarily emerge in the classroom naturally, we have to create the conditions for it to appear. To promote forms of human collaboration aligned with our ethical perspective, since the beginning of research program, the class that we worked with was divided in small groups of 2 or 3 students since the beginning of the research program. The students were encouraged to discuss the emerging ideas, to listen to and try to understand the other students' perspectives, to compare them critically to what they produced and to engage in dialogue to improve the ideas generated in the classroom.

At the very beginning of the program the Grade 2 students were confronted with equations having the unknown on one side of the equation only. Then they tackled equations having the unknown on both sides. Here is an example of the latter. The equation was introduced under the form of a story that the teacher (T) read to the students:

> Sylvain and Chantal have some hockey cards. Chantal has 3 cards and Sylvain has 2 cards. Her mother puts some cards in three envelopes making sure to put the same number of hockey cards in each envelope. She gives 1 envelope to Chantal and 2 to Sylvain. Now, both children have the same amount of hockey cards. How many hockey cards are in an envelope?

The equation was illustrated in the blackboard (see Figure1).

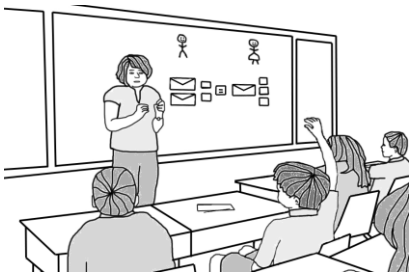

Figure 1: The Sylvain and Chantal equation

In the beginning the students were resorting to interpretations of an arithmetic nature: they were resorting to trial and error methods. Thus, Willy (W) suggests the following:

1 W: Um, I think there is… there is 1, um is, 1 hockey card in each card (*meaning envelope*), because… euh, there are 3 cards just there (*Chantal's cards*) and if there is just 1 in the cards (*meaning envelopes*), that means there are 4, and there are 2 cards just there (*meaning Sylvain's cards*) and 2, and … there are 2 in the 2 envelopes.

2 T: Uh, huh (*trying to make sense of W's strategy*). So, if I understand well Willy, you used the trial and error strategy?

3 W: Uh? Huh…

4 T: That is, you said: Ah! I will pretend that there is 1 card here, 1 card here, 1 card here (*referring to the envelopes*). Is that what you did?

The teacher asks for other ideas. Aided by Sue (S), Joe (J) suggests removing one envelope from each side of the equation:

5 J: Um, I think there is 1 [card] in each [envelope], because I would like to remove Chantal's envelope there…

6 T: OK.

7 S: And Sylvain's envelope and…

8 T Why do you remove an envelope here, and an envelope here?

9 J: Um, because if, because Chantal has 3 [cards], and Sylvain has 2 [cards], and if, and if there is a card in this [envelope], (*pointing to the remaining envelope in Sylvain's side of the equation*), it will make equal to, it's equal.

10 T: […] Ok, so you found the solution like that? You, you isolated a little bit, but you didn't isolate completely, eh? That was your solution, you removed the envelopes, eh?

11 J: Yeah…

Joe's strategy seems to draw on a discussion that the class had the previous day about removing envelopes to *simplify* the equation. In turns 5 and 7 he suggests removing one envelope from each side of the equation. Since this is a crucial idea in solving an equation algebraically (Filloy, Rojano, & Puig 2008), the teacher invites Joe to articulate the idea in an explicit way (Turn 8). Yet, removing the envelopes from each

side of the equation is made to *simplify* the equation, but not to *deduce* the value of the unknown: in Turn 9 Joe indeed *assumes* that the number of hockey cards in an envelope is 1 and concludes that both sides of the simplified equations are equal.

The design of the activity provided the class with the opportunity to produce or bring forward mathematical interpretations of a varied nature and to become aware of the limitations of trial and error methods. Naturally the students did not know that they were resorting to trial and error methods (see Willy's surprise in Turn 3). Working together with the students, the teacher brings forward terminology and conceptualizations that allow the students to make sense and better understand the methods they imagine. In labouring with the students, the teacher also provides room for the classroom to collectively become conscious of the subtleties of a historically and culturally constituted way of thinking about equations. Thus, in Turn 10, the teacher distinguishes between isolating "a little" and "completely" the unknown. In doing so, the teacher calls attention to the fact that the equation was simplified but not totally algebraically solved—although of course, she does not use those terms. Rather, she calls attention to a subtle aspect of a way of thinking where equations are successively simplified until the value of the unknown is *deduced* (as opposed to *guessed*).

The previous short excerpts illustrate the forms of knowledge production that we foster: they are not based on the private owner logic of the transmissive and the "progressive" educational programs. Knowledge is not something that an individual possesses: knowledge is potentiality that is actualized through joint labour. By being actualized, knowledge (in this case algebraic knowledge) becomes the object of consciousness and thought for the students. In labouring together, the teacher and the students have accomplished important things. In Turn 4 the teacher articulates for the students a strategy that so far remained ostensibly shown. She uses the term "pretend," which allows the students to better seize the conceptual nature of the trial and error method they imagined. As pointed out above, in Turn 10 a delicate distinction is made apparent to differentiate between isolating the unknown "a little" and "completely." Although knowledge is being bestowed with mathematical determinations and, in doing so, is progressively becoming an object of consciousness for the students, the teacher and the students still have a long way to go.

Naturally, the teacher is not interested in the particular equation under discussion. Actually, she is not interested in *any* particular equation. The interest is rather put on *ways of thinking* about solving linear equations. But the understanding of such ways of thinking (the historically and culturally constituted way of thinking that we term algebraic included) can only appear through the solving of particular equations. The teacher and the students talk hence about particular equations (like the equation in the dialogue), but what is becoming object of consciousness is not how to solve *that* particular equation but the manner in which to think about equations like *that*. More precisely, what is becoming an object of consciousness for the students is a system of ideas: something general (i.e., algebraic knowledge). We have termed this lengthy

social and sign-mediated process of becoming conscious of systems of ideas, in this case algebraic ideas, *objectification* (Radford, 2008). Now, the fact that the teacher is not interested in particular equations *per se*, does not mean that the discussed equations do not bear any relevance. The depth and scope of the awareness that is produced in the course of a process of objectification depends on the quality of the didactic choice of equations, their conceptual organization, etc. It would have been, for instance, of limited interest not to tackle equations with the unknown on both sides of the equation, given the object (i.e., the didactic purpose) of the joint labour. But the depth and scope of the awareness that is produced in the course of a process of objectification depends *also* on the characteristic and complexity of the classroom discussions, which in turn depends on forms of classroom interaction and human collaboration.

To foster sophisticated forms of classroom interaction and collaboration, we encourage the students to produce ideas in small groups and to discuss them with other groups in a deep manner. To provide a concrete example, I turn now to an example that comes from the following year—when the students were in Grade 3 (the students were 8–9-years old). In the beginning, the teacher plays a salient role in suggesting what is to be discussed and in the organization of the forms of interaction. The teacher's organizing role decreases as the collective gains cohesion and a common understanding of their joint labour. Thus, in the Grade 3 example, organization of interaction was divided in four steps. In the first step, the students worked in small groups to produce a text that included: a story of their invention, the translation of the story into an algebraic equation, and the solution of the equation (see Figure 2, pictures 1 and 2) (for more details, see Radford, 2012). Each group had a "corresponding" group with which an exchange in consecutive phases will occur.. In the second step, one text goes to the corresponding group, and vice-versa. Each group proceeds to read and evaluate the other group's production (see Figure 2, pic. 3). We ask the students to assess the corresponding group text on the basis of several elements, such as:

> (1) Is the text clear? (2) Do they find the answer to be right? (3) Do they find the solution convincing? (4) Do they find the solution beautiful?

Once they have finished critically studying the other group's text, the two groups get together (see Figure 2, pic. 4). The groups take turns presenting their results, emphasizing what they liked about the text and what they think should be improved and how. The teams also react to the critique and the teacher may also be part of the discussion. After having discussed the groups' texts, as a last step, they work together in trying to come up with a text that would be an improvement of what was initially submitted. They are also encouraged to share the final text with other groups.

The question, of course, is not merely how to come up with a better mathematical solution. Although this is important, equally important is the fact that in going through this process, the students have an opportunity to understand others and in understanding others to better understand themselves.

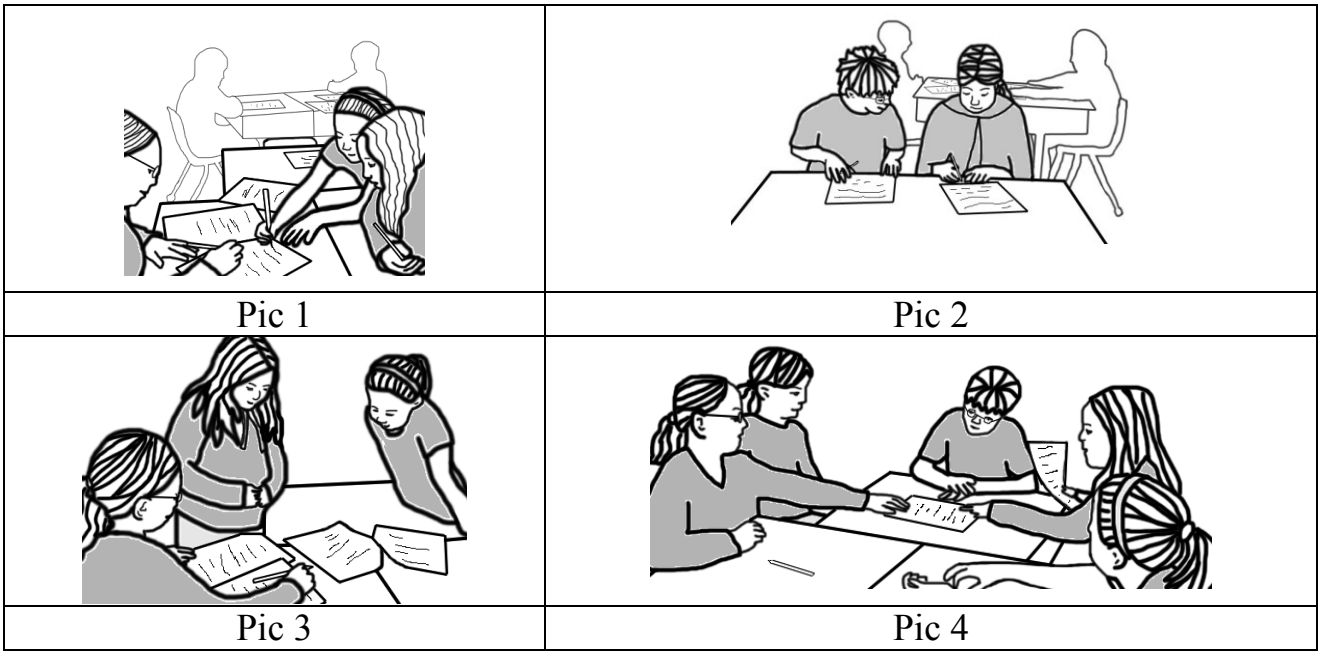

Figure 2: In Pics 1 and 2 Teams 1 and 4 work independently towards the production of a mathematics text. In Pic 3, Team 1 critically examines Team 4's text. In Pic 4, the members of Teams 1 and 4 meet to discuss their texts.

Team 1 (made up of Christina (Ch), Elisa (E), and Sara (S)) produced the following story: "Martine has a collection of 10 stamps. For her birthday she receives an envelope with stamps. Cassidy has 6 stamps in her collection. And [she receives] 2 envelopes with stamps for Christmas. How many stamps are there in each envelope?"

Team 2 (made up of Carl (C) and Sandra (Sd)) produced the following story: "For Christmas Calin received three boxes with *Webkinz* and Samantha received one box. He [Calin] has already 4 *Webkinz*. And Samantha has already 28 *Webkinz*. Now both have the same amount of *Webkinz*."

The translation and solution of the equation appear in Figure 3.

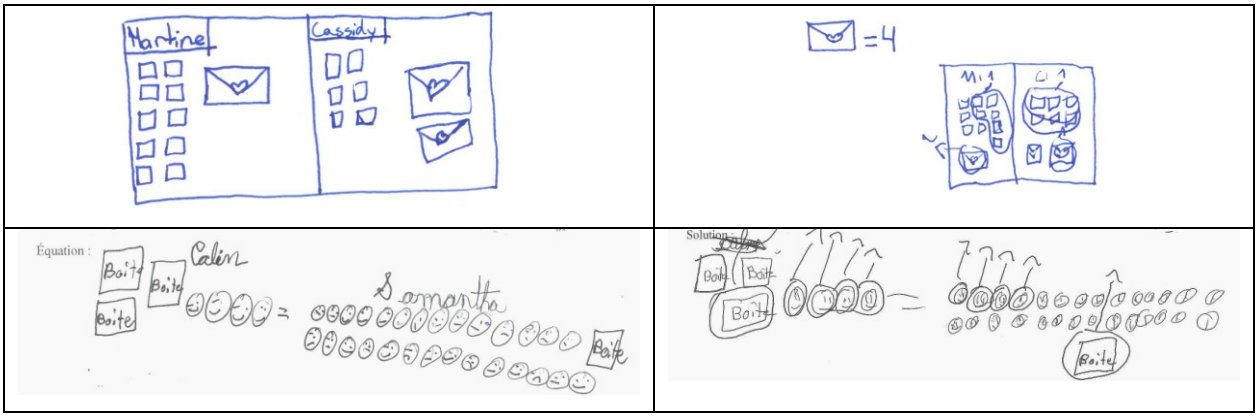

Figure 3: Top row, Team 1's equation and solution.
Bottom row, Team 4's equation and solution.

During the discussion, three elements were discussed. (1) Team 4 pointed out that Team 1's equation does not have an equal sign and that they did not specify the nature

of the answer (they wrote that an envelope is equal to 4, but that such an answer is ambiguous). (2) The teams discussed the validity of removing 6 items at once from the equation (as Team 1 did), as opposed to removing items one at the time (as Team 4 did). (3) Team 1 complained that Team 1 did not ask a question.

The teacher (let's call her T', as she was different from the Grade 2 teacher) was attending to other groups when Team 1 (T1) and Team 4 (T4) started the conversation:

1 C: (*Addressing theme 1*) Um, what we liked about your story is that it was clear, it was nice, there were no mistakes, we could read it well. That's about your story.

2 Sd: (*Talking about the solution and addressing theme 2*) Here, here what we liked is that you put "envelope = 4."

Then, they pointed out what they did not like:

3 C: What we did not like… You did not put the equal sign in the equation.

4 Sd: And you have to put it.

5 C: (*Addressing theme 3*) You did not [remove the equation terms] one at the time.

T1 agreed with T4's remarks. When it was T4's turn, T4 argued that T1's story did not include a question in the story and that without a question one cannot know what one is looking for. When the teacher (T') arrived, she found the students in a vivid and unsettled discussion. They summarize their discussion for her.

6 T': So (*talking to Team 1*), is there a question missing?

7 Ch: There is no question! (*Answering the teacher's question*) Yes!

8 E: Yes, the question is missing!

9 T': Ah! but why do you think that…

10 C: (*Interrupting*) Yeah, but…

11 T': (*Talking to Carl*) We'll ask the question here (*meaning T1*)… That's OK, you will be able to defend yourself. (*Talking to T1*:) Why do you think that it is important to ask a question?

12 Ch: Because if you don't, what are you going to do?

13 Sd: You don't need to ask a question!

The discussion continued without agreement for a while before the teacher decided to ask:

14 T': For someone who is reading the story … do you think that it is important to ask the question?

15 C: I would say no…

16 T': I do think that in a story like this, it is important to have a question if …

In the end, the question remained unsettled. And the goal was not to settle it. The goal was to create the conditions of possibility for the students and the teacher to engage in classroom debates underpinned by non-utilitarian forms of human interaction and collaboration out of which the teacher and the students speak out and position themselves in the public space.

SYNTHESIS AND CONCLUDING REMARKS

My goal in this paper was to inquire into current conceptions of mathematics teachers and students. My inquiry was based on the idea that conceptions of what individuals are within a definite historical period are related to the manner in which human existence is culturally produced. I scrutinized hence the conceptions of teachers and students through the lenses of what I termed the cultural logics of production, which included *forms of production* and *forms of human collaboration*. The examination of the traditional educational model (which Freire called the "banking concept" of education) through this methodological approach reveals that teachers and students are produced by an alienating educational structure that turns them into private owners. When we apply the same inquiry to the Piagetian "progressive" educational reform we cannot fail to notice that the educational structure is basically kept intact. The illusions of change appear only in terms of re-distribution of power and agency, but in the end students are subjected to the same utilitarian production logic—namely the private ownership of production logic—with the same alienating results. It turns out that, upon closer scrutiny, the progressive educational reform is not the antithesis of traditional teaching but its dual model. In the second part of the paper, I discussed a historical-cultural conception of teachers and students that is based in a non-utilitarian logic of production of knowledge and subjectivities. The forms of production of knowledge in the classroom rest on a dialectical materialist conception of knowledge as historically synthesized labour. Mathematical knowledge is not something possessible. It is not yours or mine. Mathematical knowledge appears as pure potentiality—virtual possibilities for mathematical understandings, meanings, and course of action.

To be materialized, knowledge has to be set into motion through teachers' and students' labour. It is through classroom labour that knowledge moves from an indeterminate form of possibilities to a determinate singularized form filled with concrete determinations (in our examples, in Grade 2, the concrete determinations revolved around guessing the answer through trial-and-error methods and issues about the algebraic simplification of equations; in Grade 3 the concrete determinations revolved around symbol-use, the translation of story-problems into algebraic non-formal symbolism, etc.).

The classroom labour that mediates knowledge and makes it an object of thought and consciousness may be alienating or fulfilling. All will depend on the classroom forms of knowledge production and forms of human collaboration. The dialectical materialist approach to the production of knowledge and subjectivities outlined in this paper puts

forward a communitarian ethic that is based on a non-essentialist conception of the individual that allows us to understand teachers and students as engaged intellectually, emotionally and ethically in joint labour. Teachers *and* students are in the same boat, *producing knowledge and learning together*. In their joint labour, they sweat, suffer, and find gratification and fulfillment with each other.

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