Chapter 17
Theories and Their Networking: A Heideggerian Commentary

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Abstract The chapter briefly discusses the construct of theory and the contribution of networking theories to mathematics education research. It starts by a reflection on the meaning of theories in general and in mathematics education in particular. Dwelling upon Heidegger's etymological analysis of theory, it stresses the ineluctably tension between the phenomena a theory tries to account for and the manner in which the account is carried out. The comment concludes by suggesting that networking mathematics education theories offers a unique possibility to grasp a thematized and systematic array of sides of educational problems.

Keywords Networking theories • Heidegger • Semiosphere • Methodologies

17.1 Theory

The concept of theory is an elusive one that often escapes the realm of definitions, regardless of how hard we try to pin it there. Buried under numerous layers of meaning, theory seems to appear differently depending on the discipline that evokes it. Some of us grew up thinking of theory as a kind of lens through which we perceive, interpret, and interact with our surroundings. This is the meaning of theory that we inherited from the ancient Greeks, who, as we well know, cast knowledge in a metaphor of vision. Martin Heidegger reminds us, indeed, that the word *theory* derives from the Greek verb *thēorein*, a verb that comes in turn from two root words: *thea*

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and *horaō*. *Thea*, from where the word theater derives, "is the outward look, the aspect, in which something shows itself, the outward appearance in which it offers itself" (Heidegger 1977, p. 163). *Eidos* is the name Plato uses to refer to which shows itself in the phenomenological realm, that is, a *presence that makes itself present* (e.g., an idea, a thing), more specifically the "aspect in which what presences shows what it is" (p. 163). To know, *eidenai*, is to have seen this aspect. *Horaō* means "to look at something attentively, to look it over, to view it closely" (p. 163). For the ancient Greeks, then, theory consisted in looking "attentively on the outward appearance wherein what presences becomes visible and, through such sight—seeing—to linger with it" (p. 163).

The modern term observation, which comes from the Latin word *contemplatio*, refers to the Greek lingering vision metaphor and moves the term theory into new territory. Although it stresses the visual metaphor through which theory is conceived, as in *vita contemplativa*, it adds a new array of efforts that have to be made in order to render visible the thing to be seen. With da Vinci and Galileo the meaning of theory changes: the border line separating *bios theōrētikos* (a theoretical form of life) and *bios praktikos* (a practical and productive form of life) somehow vanishes and theory appears as an endeavor where one strives to manipulate something, to work over it, to pursue it, "to entrap it in order to secure it" (Heidegger, p. 167). And it is *objectness*, that is, this feature of entrapping something as an object to be secured, that, according to Heidegger, characterizes the modern concept of science—a concept that "would have been as strange to medieval man [*sic*] as it would have been dismaying to Greek thought" (p. 168).

Thus, when Euclid proves the Pythagorean Theorem, he resorts to the original *bios theōrētikos*: Euclid's proof consists indeed in attentively looking at the outward appearance of the right triangle and the squares built on the sides; when areas are compared, he is looking at the relations over and over, closely, lingering, waiting so to speak for the relational presences to become visible through sight. When Galileo is busy measuring time using a large pail filled with water descending along a channel carved on an inclined plane, he resorts to a conception of theory or theoretical approach where the original senses of *bios theōrētikos* and the *bios praktikos* have merged. Galileo's deeds illustrate very well how manipulation and planning become important in the modern concept of theory. From the Renaissance on, mathematics moves too from its central place within a *bios theōrētikos* to a synthesis that starts featuring the dimensions of a *bios praktikos* where reckoning comes to the fore: mathematics becomes not merely the science of reckoning:

> in the sense of performing operations with numbers for the purpose of establishing quantitative results …. Mathematics [becomes] the reckoning that, everywhere by means of equations, has set up as the goal of its expectation the harmonizing of all relations of order, and that therefore "reckons" in advance with one fundamental equation for all merely possible ordering. (Heidegger 1977, p. 170)

There are several aspects that come to the fore in the concept of theory. A theory is always a theory of *something*—an object-area. A theory is always about the mattering and happening of specific kinds of entities that Heidegger calls *presences*. It is in this sense that theories work as filters that discriminate between presences and

their importance. But when we do so, we highlight presences and link them in ways that make them appear as congruent wholes. These links that we create between presences comprise *meaning*.

To bestow meaning on what otherwise would remain an overwhelming flux of sensorial data, we codify our experience of the world in more or less explicit ways: we create patterns of understanding and action. Although culturally codified shared experience comprises a vast territory, some parcels of it are highlighted and expressed through language; they acquire the status of *principles*. Acting hence as filters, these principles (P) allow us to refer to *presences*—problems, questions, tasks, situations. As a result, questions, problems, and tasks (Q, in short) are already imbued with a theoretical layer. It is this theoretical layer that allows us to recognize for instance two tasks as *similar* or even a task *as such*. But this does not mean that the theoretical principles of a theory predate genetically the problems or the tasks. There is a fundamental dialectical relationship between them. P and Q are formed simultaneously; they co-emerge.

This picture, however, is incomplete in an essential way. For the systematic actions that we undertake to cope with a task—i.e., the methodology, M—is consubstantial with the principles P and questions Q that we use to recognize or formulate a task as such. This is why a theory—or a theoretical approach—can, analytically speaking, be thought of as a triple (P, M, Q) (Radford 2008) only if we do not forget that there is a profound entanglement between these three "components" of a theory and that none of them can be reduced to the others or serve as the constitutive basis for the others. Because of their mutual genetic constitution, we should talk about these components as being in *trialectical* existence.

Now, to talk about a theory as a trialectical entity means to conceive of it as something dynamic, an entity in movement with layered descriptions of reality that emphasizes at certain times P, Q, or M, or two or all three of them. What is characteristic of a theory is that, in its movement, it produces results. Results may refer to new interpretations of presences (i.e., the *objects* of the theory), the identification of new presences or relationships between presences, etc. The results of a theory may require some adjustments and the transformation of its components, P, Q, and/or M.

The dynamic dimension of a theory, however, cannot be limited to the manner in which it is affected by its own results. Theories develop not only through the internal trialectical relationship of its own components. Theories are produced within cultural formations and live and interact with other theories.

17.2 The Semiosphere

Following semiotician Yuri Lotman's (1990) ideas, I have suggested (Radford 2008) that we can think of theories in general, and theories in mathematics education in particular, as evolving in a *semiosphere,* that is, a multi-cultural, heterogeneous, and dynamically changing space of conflicting views and

meaning-making processes generated by theories and their different research cultures. It is in a semiosphere that theories live, move, and evolve. It is in a semiosphere that theories come into a relationship.

What characterizes what has been termed the networking of theories is the explicit goal of bringing theories together. That is, to put them in explicit relationship so that theories get connected or networked within a same research project.

There are different possible forms of connectivity. In their seminal paper, Prediger et al. (2008) identify some of them, including "comparing" and "contrasting," "coordinating" and "combining," "integrating locally" and "synthesizing."

As suggested previously (Radford 2008), the possible forms of connectivity are constrained and afforded by the nature of the theories, but also by the research goal of the connectivity research project. In general terms, a network N of theories T_1, T_2, T_3, … can be seen as a set of connections c_1, c_2, c_3, …, where c_k involves at least two theories T_i, T_j (in what follows, to simplify, I will assume that only two theories are networked).

Using the semiosphere's spatial metaphor, theories T_i and T_j can be visualized as being "closer" or "further" depending on their own (P_i, M_i, Q_i) and (P_j, M_j, Q_j) structures. The connection c_k of T_i and T_j requires the identification of research questions Q_{ij} (tasks, problems, etc.) that guide the enterprise as well as the building of a new methodology M_{ij} to answer the research questions under consideration.

One of the key research questions that have been investigated within the networking theories research community is the manner in which the analysis of classroom events differs when conducted through different theories or theoretical lenses. At the level of methodologies a typical example (used in this book) has been the analysis of a common videotaped lesson or segment of it under different theories. Another example of methodology consists in the creation of educational tools within a theory that are then used in the classroom and analyzed through the lenses of that and other theories (Radford 2014). This endeavor has led the corresponding research teams to learn from each other, to improve and refine their own theories, to understand them better, and to become more sensitive to other ways of theorizing.

As the chapters of this book show, the networking task is not easy, but it is rewarding. The task is not easy, among other reasons, because theories may use the same theoretical names with different meanings. They may resort to different theoretical principles and conceptualize differently the basic phenomena under scrutiny; they may also resort to different methodologies or to have a different set of concerns leading to different research questions. A networking task hence requires an open mind from the outset. It requires the capability of opening oneself to others and moving across theoretical approaches in a cautious and reflective way. By being confronted by, or immersed into, new theories, new predispositions towards new emerging shared interpretative situational contexts become available. The structuring background of shared reality shifts and new forms of action and understanding become possible. Researchers become endowed with new possibilities to look at their home theories and to see the familiar through new stances that make the familiar look unfamiliar and hence open to scrutiny, critique, and change.

In a networking task there is always a tension that results from putting together different theories. The tension is not, however, something to be seen in negative terms. It is this tension that pushes the networking task further and moves the theories to each other. The tension does not need to end up in a harmonious synthesized point in the semiosphere. In his thoughtful critique Kenneth Ruthven (Chap. 16 in this book) notes that the synthesis of theories is the kind of connectivity that does not seem to appear in the examples shown in the book. Certainly, a synthesis is the most difficult kind of connection to achieve. But there may be an *unresolved synthesis*, that is, a synthesis where theories do not disappear to create a new entity, yet the theories are radically shaken and transformed. The synthesis appears not in a new single entity, but in the imprint that the other theories leave in the transformed theory.

17.3 The Question of Learning

In this section, to illustrate the tensions that ineluctably arise in the networking of theories, I would like to comment on learning.

Learning mathematics is indeed one of the central concerns of most theories or theoretical approaches featured in this book—other important concerns revolve around knowledge and how it appears or is practiced in a manner conforming with the institutional dimension in which it operates, as in the *Anthropological Theory of the Didactic* (ATD). Learning can be conceptualized in different ways and operationalized in distinctive manners. Even the questions that are asked about it vary from one theoretical approach to another. Following Heidegger's ideas, let me suggest that learning mathematics can be considered as a *presence* whose presencing is differently framed by educational theories in accordance with the lenses they provide and the manipulative (i.e., methodological) endeavors that they make to reveal its presence in the classroom—to make it come to stand and lie in unconcealment (Wrathall 2011), that is to become object of thought and consciousness. Let me also suggest that, at its most general level, learning is a process of tuning with life and that its being is interwoven in threads of objective, subjective, conceptual, aesthetical, ethical, and political matters. What theories provide us with are not really truths, but *moments* of learning-as-being. For instance, when researchers resort to the *Abstraction in Context* theoretical approach (AiC), they posit the problem of learning as distinctive kinds of students' deeds and focus on actions that can be identified as "building-with," "recognizing," and "constructing." Learning appears—or is expected to appear—through these lenses that unavoidably transform it into an object of specific form. The expected object, regardless of the theory, is always partial, as it has undergone a process of filtration or a process of entrapping that seeks to secure its recognition in the advent of its presencing. Hence, we recognize some aspects of learning, but not the whole of it.

Referring to the sciences in general, Heidegger notes their impotence in grasping their topical presences in their totality and suggests that "this impotence of the

sciences is not grounded in the fact that their entrapping securing never comes to an end" (Heidegger 1977, p. 176). And he goes on to dispel the idea that the problem would be merely methodological. The problem, in fact, is *ontological*. He continues:

> [the impotence of the sciences] is grounded rather in the fact that in principle the objectness in which at any given time nature, man [*sic*] history, language, exhibit themselves always itself remains only *one* kind of presencing, in which indeed that which presences can appear, but never absolutely must appear. (Heidegger 1977, p. 176; emphasis in the original)

Regardless of their theoretical sophistication, concepts remain, and will remain, precarious vis-à-vis the presences they strive to reveal.

But again, the transcendence of the presences they strive to reveal does not stem from the insufficiency of our methodologies or concepts, but from the presences' ontological constitution. The reason is not to be found in the idea that presences such as learning are Kantian "things in themselves" or immutable Platonic beings. Their transcendence has rather to do with their own fluid nature: they are moving pointers refracting the complexities of life—not objects to be grasped, like apples with our hands, but mobile pointers that invite us to historical journeys through which to explore our place and possibilities as humans in the historical, cultural world of practice.

Within this line of thought, if learning is a process of tuning to life, learning changes with life, nature, and the individuals that come to inhabit and transform nature and the cultural world.

It is this unique possibility of offering us a thematized and systematic array of sides of learning and other crucial problems that I find of vital importance to mathematics education in the networking theories research field.

References

Heidegger, M. (1977). *The question concerning technology and other essays*. New York: Harper Torchbooks.

Lotman, Y. (1990). *Universe of the mind. A semiotic theory of culture*. London: I. B. Taurus.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: First steps towards a conceptual framework. *ZDM – The International Journal on Mathematics Education, 40*(2), 165–178.

Radford, L. (2008). Connecting theories in mathematics education: Challenges and possibilities. *ZDM – The International Journal on Mathematics Education, 40*(2), 317–327.

Radford, L. (2014). On the role of representations and artefacts in knowing and learning. *Educational Studies in Mathematics, 85*(3), 405–422. doi:10.1007/s10649-013-9527-x.

Wrathall, M. (2011). *Heidegger and unconcealment. Truth, language, and history*. New York: Cambridge University Press.