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Intercorporeality and ethical commitment: an activity perspective on classroom interaction

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Abstract In this article, we present a sociocultural alternative to contemporary constructivist conceptions of classroom interaction. Drawing on the work of Vygotsky and Leont'ev, we introduce an approach that offers a new perspective through which to understand the *specifically human* forms of knowing that emerge when people engage in joint activity. To this end, we present two concepts: *space of joint action* and *togethering*. The *space of joint action* allows us to capture the collective and sensuous or *inter*corporeal dimensions of thought and feeling in interaction. We resort to the concept of *togethering* to capture the ethical commitment participants make to engage in and produce activity. These concepts are illustrated through a discussion of concrete episodes from an elementary mathematics classroom.

Keywords Activity theory · Ethical commitment · Interaction · Intercorporeality · Objectification · Leont'ev · Vygotsky

1 Introduction

In a grade 2 class, the students and the teacher explore the sequence shown in Fig. 1.

In the first part of the exploration, working in small groups, the students are required to continue the sequence up to term 6. When the group of Cindy (left in Fig. 2, first picture), Carl (middle in Fig. 2, first picture), and Erica (right in Fig. 2, first picture) start tackling the question of drawing term 6, Carl says:

1. Carl: We do 6 plus 6 equals 12, plus 1.
2. Erica: Yes… No…
3. Cindy: Yes!

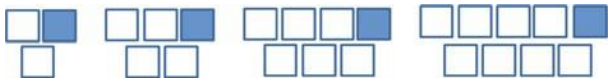


Fig. 1 The first four terms of a sequence investigated in a grade 2 class

4. Carl: Yes, that is what we did in the other ones. Look! … Look (*referring to term 4*), 4 plus 4 equals 8. In there [term 4], there are 8 (*and pointing to the successive squares of Term 4 in Erica's sheet he continued*) 1, 2, 3, 4, 5, 6, 7, 8, plus 1, which is equal to 9 (See Fig. 2).

This short example brings to the fore many aspects that have been emphasized in current research on learning, such as interaction (Schwarz, Dreyfus, & Hershkowitz, 2009; Steinbring, Bartolini Bussi, & Sierpinska, 1998), the function played therein by gestures (Roth, 2001), and, more generally speaking, the role of embodiment and multimodality (Edwards, Radford, & Arzarello, 2009). Now, the reasons for paying attention to interaction and embodiment are varied; so are the interpretations of the role that these aspects come to play in accounts of how learning occurs and, more pragmatically, how it could be enhanced. Thus, in contemporary constructivist approaches inspired by Kant, which assume that the individual mind constructs itself, the students' cognition and the social are kept apart: they remain at most "reflexively related," two poles that can be explored from a psychological or social perspective. As Cobb and Yackel (1996) explain, "from one perspective [the social], we describe the joint or collective processes constituted by actively cognizing individuals and, from the other [the individual], we describe the interpretations and construals of individuals as they participate in those collective processes" (p. 178). Within this perspective, knowledge results from the students' own actions. Knowledge is always something personal. As a result, interaction is reduced to a space of mere subjective experience. Sociocultural theorists question the dualism between the mind and the social that this alluded posture entails. In sociocultural approaches of Vygotskian descent (e.g., Bartolini Bussi & Mariotti, 2008; Radford, 2008a; Roth & Lee, 2007), the social and the individual are conceived of as organically and irreducibly imbricated. Without being causally produced, any higher-order psychological function is considered to have its root in the societal-historical means and processes of the activities in which individuals take part (Leont'ev, 1978). Any thing and any way in which the individual can think and communicate is already enabled and mediated by cultural possibilities (e.g., practices, signs).

Fig. 2 *To the left*, Carl starts counting the squares on the bottom row; then he counts the white squares on the top. *To the right*, the moment in which, after finishing counting from right to left the top white squares, he returns to the dark square and says "plus 1"

As may be expected, this sociocultural view of the social and the individual results in a different conception of interaction. Far from being a space of subjective experience, interaction constitutes for sociocultural theorists the social fabric of consciousness and thought. This is why the individual's cognition and the social realm are *coterminous*. For socioculturalists, the specifically human form of the psyche develops in interaction with others, interactions that change their form in the course of history, and, in so doing, change the very form and content of cognition. Cognition is historical and cultural (Radford, 2008b).

Naturally, these general theoretical tenets need to be made operational to enable us, mathematics educators, to investigate classroom interaction from a sociocultural perspective. This is the purpose of this article. To do so, we draw on the category of *activity* (Leont'ev, 1978) and sketch a concept of interaction that is embedded in the more general idea of classroom activity. Using concrete examples from our classroom research, we introduce two central ideas: the *space of joint action*, which allows us to capture the collective dimensions of thought and consciousness (Section 3), and *togethering*, which we use to capture the ethical commitment participants make to engage in and produce joint activity (Section 4). We begin by articulating the category of activity and its role in orienting actions and interactions.

2 The category of *activity* and interaction in classrooms

Interaction is frequently thought of in terms of a "negotiation" of meanings. At first sight, the metaphor of "negotiation" seems interesting in that it emphasizes the active role of the parties involved. However, it also carries with it the vestiges of individualistic thinking: I can only negotiate something if I *possess* it. It is only through my possessing it that I can sacrifice a bit of it and expect you to sacrifice a bit of your own possessions in turn. In this case, the other (the person with whom I am negotiating) appears as a competitor or antagonist in a traffic of personal goods. The term "negotiation" derives from a Latin word for "to do business," which, during the sixteenth century, acquired the sense of "bargaining" in the context of commercial practices. It was precisely to this idea of bargaining that Piaget resorted in his conceptualization of the cognitive role of interaction. In his *Sociological Studies*, Piaget (1967) conceived of interaction in terms of schemas of reciprocal exchange of ideas based on service, value, personal effort, sacrifice, satisfaction, and self-interest. Thus,

> [i]n the most general form, the exchange schema can be represented as follows: each action of [the individual] x on x' constitutes a "service," i.e. a value r(x) sacrificed by x (time, labor, objects or ideas, etc.) that results in a (positive or negative) satisfaction s(x') of x'. Conversely, x' sacrifices the values r(x') when acting on x, who experiences the satisfaction s(x). (p. 51)

Similarly, the famous equilibration mechanism was studied in terms of equalities and inequalities between r(x) and r(x').

As we can see, interaction is portrayed as obeying an individualistic logic of best self-interest. I interact with other individuals, giving and receiving services, expecting to end up with increased meanings and a more robust personal way of thinking. In the end, interaction is no more than a space for the exchange of meaning. For sociocultural theories, this line of inquiry is problematic: it not only remains entrapped in the individualist psychological and educational paradigms of modernity, but also, at the ethical level, reduces the other to a

means of my own ends. As von Glasersfeld (1995, p. 127) put it, from a constructivist perspective, "others have to be considered because they are irreplaceable in the construction of a more solid experiential reality." Were I not in need of a more solid experiential reality, the others would not come to be part of my subjective world of interests. I would be able to do without them. This line of inquiry is problematic in that it fails to take notice of the fact that

> interaction is the founding process of the self, the constitutive process through which I become a continuously transforming and transformed entity … not as a substance, not in the substantive sense of *être* (to be, essere); rather in the sense of the never ending and always changing *étant* (be-ing), a being-with-others. (Radford, 2010)

In what follows, we lay down a conception of interaction that avoids the individualist, economical substrate of the "negotiation" metaphor that conceptualizes the classroom as a kind of business center. To do so, we draw on the category of *activity* of Leont'ev (1978). Indeed, the category of activity captures interaction in ways that are more appropriate for educational purposes. It is used to theorize action and interaction in a way that overcomes the problems of dualistic approaches while offering room for understanding the growth of subjectivity and agency. The starting point is a reconceptualization of the relation between the individual and the social. Human activity does not constitute a relationship that opposes the individual and society: As Leont'ev (1978) contends, "This must be stressed because psychology is now being flooded with positivist conceptions that are in every way imposing the idea of opposition of the human individual to society" (p. 51). Leont'ev criticizes positivist conceptions for reducing society to a mere "external environment to which [the individual] is forced to accommodate, in order not to appear 'nonadapted,' and to survive in exactly the same way as an animal is forced to adapt to an external, natural environment" (p. 51). This positivist perspective misses precisely the main point: "the fact that in society a man finds not simply external conditions to which he must accommodate his activity, but that these same social conditions carry in themselves motives and goals of his activity" (p. 51).

In Leont'ev's perspective, a distinctive trait of activity is that it is stimulated by the pursuit of a *collective* endeavor. Activity conceived of as joint pursuit runs against the individualistic views of the social and the individual and is articulated around what Leont'ev called the *object* of activity—that is to say, the *intentional object* to which all the individuals' efforts are directed. According to Leont'ev, this object appears twice: in material and ideal (reflected) form.[1] Returning to the opening classroom example, the *object* of activity consists in reflecting on the sequence of terms in a cultural mathematical way. The object of activity can only be attained through *actions* directed to specific *goals* (drawing terms 5 and 6). In our example, the actions include counting, gesturing, and perceiving. Activity and actions stand in a mutually constitutive relation: Actions presuppose, and draw their sense from, the activity that they concretely realize; but activity exists only because of the concrete actions. Whereas counting and adding in general may be used to realize different activities, Carl's counting of 4 and 4 and adding of these numbers take on a specific sense in an activity where the students will end up articulating an efficient

[1] This is so because the material/intentional object (designated in Russian *by predmet* in Leont'ev's works) is distinct from *objekt*, which refers to material objects. Leont'ev grounds his work in Marx, whose German makes the same distinction between the nouns *Gegenstand* and *Objekt*, respectively. *Predmet* and *Gegenstand* have both material and ideal dimensions.

model in the form of a generalization of the type $2x+1$ (even if the generalization is not expressed in an alphanumeric way).

Yet clearly, the individuals who participate in activity are different—both cognitively and emotionally. As a result, the goal and the object of activity are *refracted* differently in the individual consciousness of each participant (the teacher, the students). This is why interaction in activity is investigated as part of a joint endeavor directed toward the reaching of a common object (in our example, an algebraic form of thinking) that is variously refracted by the specificities of the individuals and the manners in which they become positioned in sociocultural webs of knowledge and action.

Activity as the joint pursuit of a material and ideal object leads to an altogether different conception of interaction, subjective phenomena, and the world. As activity unfolds and develops, so do the participating individuals. It is precisely this developmental horizon that was of interest to Leont'ev. His main objective was not to offer a *theory* of social practice or activity, as it is often mistakenly understood. Rather, his objective was to pursue the *organic connection that exists, in activity, between the material world, the individual's mind, and the self*. As he argued, activity is not merely the substrate of cognition, the layer from where the psyche would unproblematically originate, but rather an overarching unit that constitutes an evolving and dynamic space of joint action "containing in itself those internal, impelling contradictions, dichotomies, and transformations" that create the conditions for consciousness and the self to emerge (Leont'ev 1978, p. 6). In this view, "instead of being a collection of 'mental processes,' the human mind emerges as biased, striving for meaning and value, suffering and rejoicing, failing and hoping, alive, real" (Kaptelinin, 2005, p. 5). As a result, the world is inherently shot through with significations, and it is through these significations that words and signs acquire personal "meanings."

At the educational level, one of the clearest advantages of Leont'ev's category of activity is that it allows us to reconsider classroom interaction through new lenses. In particular it allows us to reconceptualize interaction as a key component of classroom activity and brings in new ideas concerning the roles of the teacher and the students. It is in the accomplishment of the material/ideal object in and by means of joint practical activity that the students become acquainted with inherently collective significations ("meanings"), and concretize them in the form of personal sense. These significations ("meanings") are collective, because the signs used in and for communication (words, intonation, gestures, body position and orientation) are the results of, and are marked by, cultural-historical processes, which also mark personal sense. And like "meaning," personal sense corresponds to something in the world. In our example, personal senses refer to a generalized cultural form of counting ("*2x+1*"). The difference is that they capture the ongoing process of joint activity from the viewpoint of the individual's consciousness; and, in so doing, they appear not as "things" or "commodities" to be traded off, but as dynamic *relationships* between the individual and her world.

3 The space of joint action

The *space of joint action* is more than a spatial notion where interaction would occur. It is a space of relations and embodied reciprocated tunings occurring in the concrete space of interaction. It is based on the *inseparability* of "consciousness-for-others" and "consciousness-for-myself" (Vygotsky, 1986, p. 256). This inseparability results from the possibilities for action, reflection, and emotion that language and other sensuous multimodal forms

of communication, like touching, eye contact, and gesturing embed. Ontogenetically speaking, the inseparability of the individual's consciousness and that of others have been largely studied in children and newborns. Thus, it is well known that a few hours after birth, newborns show a propensity to interact with others through corporeal signs —e.g., the imitation of facial gestures (Meltzoff & Moore, 1977). A recent study shows that twin fetuses plan and execute movements directed towards each other. This intra-pair contact in the physical space of the uterus is the emergence of an elementary form of proto-consciousness that evolves as forms of interaction become more and more complex —from head-to-head and head-to-arm contact at the 11th week of gestation, to more sophisticated and constant features of corporeal contacts around the 22nd week (Castiello et al., 2010), up to the "discovery" of inner speech and interior life, when 4-year-old children realize that they can *talk* to themselves (Flavell, Green, Flavell, & Grossman, 1997; Trevarthen & Reddy, 2007).

The concept of *space of joint action* that we introduce here draws on this idea of the inseparability of consciousness and stresses the fact that interaction is based on an evolving, tuning, and reciprocating of the participants' perspectives, making thinking a *collective* phenomenon. The fabric of the *space of joint action* is made up of bodily resonance and intercorporeal coordination accomplished at different levels: speech, posture, gestures, artifact- and sign-mediated actions, joint perception, etc.[2] We exemplify the proposed category in the following example from a fourth-grade classroom—students aged 9–10 years—engaged in mathematical modeling, which plays a central role in the Ontario mathematics curriculum. Learning how to model contextual and familiar situations in an efficient mathematical way was in fact the object of the activity.

The teacher organizes the class in small groups of three students and invites them to work on a problem based on the following story:

> For his birthday, Marc receives a piggy bank with one dollar. He saves two dollars each week. At the end the first week he has three dollars; at the end of the second week he has five dollars and so on.

Using bingo chips of two colors (blue and red) and numbered plastic goblets, the students are asked to model the saving process until week 5. Then, they are required to answer questions so as to find the amount of money saved at the end of weeks 10, 15, and 25.

The students begin modeling the saving process in the manner of a "real situation": they start placing the bingo chips in the goblets (three bingo chips in the goblet that corresponded to the piggy bank of week 1, etc.). Although interesting, the model proves to be of limited use to answer the questions about the amount of money saved in some distant week (like week 25). Indeed, the bingo chips pile up inside the glass, making it hard to discern any structure, let alone a mathematical one. The students' attention is directed to the sequential additive actions (adding two bingo chips) that remain unsynthesized in a more abstract multiplicative structure. The teacher is in the process of talking to another group at the other end of the class. One of us (LR) approaches the group formed by Albert (Fig. 3, to the right on pics 1–4), Krysta (in the middle), and Manuel (to the left). He suggests putting the bingo chips in front of the goblets. The students accept the suggestion and start piling them up without distinguishing between colors. The researcher proposes using a blue bingo chip to signify the initial dollar in the piggy bank. Following this

[2] The recent need to come up with non-individualistic conceptions of interaction has led several researchers to conceptualize the space of interaction in different ways. See, e.g., the *theory of joint action* (Ligozat & Schubauer-Leoni 2010) and the *cognitive space of action, production and communication* (Arzarello, 2006).

Fig. 3 Pics 1–4. The student modeling the saving process up to Week 5

suggestion, the three students create a model of the saving process through active participation. After putting a blue bingo chip in front of each plastic goblet, they continue:

Episode 1:

1. Albert: (*Pointing to a space in front of the goblet that models week 3).* Now, we need 6 [red bingo chips] there, mmm...
2. Krysta: 1, 2 (*With the help of Albert, Krysta starts putting in the red bingo chips, while Manuel gives them more bingo chips; see pic 1 in Fig. 3*), 3, 4, 5, 6 red here. (*Pointing to modeling space of week 4*) And 8 red here. 1, 2 (*pic 2 in Fig. 3*), 3, 4, 5, …
3. Manuel: Do you need some more? (*He opens his right hand and offers some bingo chips; see pic 3 in Fig. 3*).
4. Krysta: (*Continuing counting*) 6, 7.
5. Albert: And just...(*Krysta takes some bingo chips from Manuel's hand*).
6. Krysta: (*Continuing counting*) 8. (*Pointing to week 5.*) And 10 reds here (*pic 3*).

There is insufficient space to model the process, so the students move the artifacts to an adjacent desk. In pic 4 (Fig. 3), Manuel adds the required red bingo chips.

The preceding six turns and the accompanying pictures show how the students interacted to come up with a saving model using artifacts. These interactions are not made up of three different and juxtaposed individual perspectives. Rather, the participation in this collective activity creates a *space of joint action* in the confines of which the students—speaking *to* and *for* the benefit of *others*—think, and act *together*. In joint action, each child not only comes to cope with the task at hand, but also gives some part of him/herself to the other participants (Leont'ev, 1978). Thus, in turn 1, Albert articulates the number of bingo chips that are required for week 3. Krysta and Manuel agree; and in turn 2, Krysta and Albert add the bingo chips in coordinated, joint actions (pic 1). When done, Krysta says "And 8 red here," and points to a place in front of the goblet that signifies week 4. Again, Krysta and Albert coordinate actions to add the bingo chips (pic 2). In the meantime, Manuel changes his body position and brings more bingo chips, while asking if more of these are needed (pic 3). In turn 6, after taking some chips from Manuel's hand and adding them to week 4, Krysta points to a place where the bingo chips corresponding to week 5 should be placed. Albert also points to the same place with a red chip, then moves in a circle to signify the red bingo chips' place (pic 3). In pic 4, Manuel brings more chips and puts them in front of the goblet that signifies week 5. We see here how the three students create a space of joint action through a complex array of corporeal dimensions around the artifacts: this array includes a sophisticated coordination of very different sign forms (i.e., gestures, words, actions, and perceptions). In this space where intentions co-vary as activity unfolds, thinking appears as something *collective*, as something *inter*-corporeal and *inter*-objective.

The resulting model is much more efficient than the "piling models" produced by the other groups. Yet, the mathematical relationship between the variables (i.e., the number *n* of the week,

and the amount saved *a(n)*) might, at this point, still be difficult to discern for the students. The researcher returns to the group and suggests grouping the red bingo chips in two.

Episode 2:

7. L.R.: It would be easier if you made small groups of two (*he groups the red bingo chips of week 3 in groups of two*) given that he [Marc] adds two each week, yeah?
8. Manuel: Okay.
9. Albert: (*Acquiescing*) Mmhu.
10. Krysta: So, we add 2 each week (*she and Albert reorganize in groups of two the red bingo chips of week 4, while Manuel intensely observes the actions; see pic 5 in Fig. 4. When they finished, they started working on week 5)...*
11. Albert: (*Silently organizes in groups of two the red bingo chips of week 5.*)
12. Krysta: (*Accompanying Albert's actions she says*) 2, 4, 6, 8 (*pic 6 in Fig. 4*), 10.
13. Manuel: (*Counting with two fingers the groups of two in week 4*) There are 1, 2, 3, 4 (*pic 7 in Fig. 4*). (*Counting with two fingers the groups of two in week 5*) 1, 2, 3 (*pic 8 in Fig. 4*), 4, 5.

The previous episodes show in a clear way the students' undergoing process of objectification. That is, the social process through which, in this case, the students become aware of a cultural mathematical way of modeling sequences. Indeed, the students have moved from a model in which the bingo chips appear in one pile to one in which the bingo chips are grouped in twos. The new organization allows Manuel to make a quick and efficient count of the chips (turn 13). Naturally, the students may not be aware yet of all the intricacies and advantages of the new form of modeling. Thus, the mathematical relationship between the number of the week and the number of groups of two is not yet articulated in an explicit manner. Becoming aware of this is part of the process of objectification. Yet, the students find meaningful the new form of modeling, for, as Krysta argues, "we add 2 each week." The multiplicative relationship between variables appears in an insinuated form at the *horizon of the students' conceptual potentialities*—that is to say, a phenomenon that without being clearly articulated emerges into the students' horizon of attention. It is a phenomenon that, as Merleau-Ponty (1960) notes "was previously present in the culture in the form of a haunting impression or anticipation, and the process of becoming aware that poses it as an explicit signification does nothing but achieve its long incubation in an operant sense" (p. 67, our translation). The multiplicative relationship between variables that insinuates the new spatial organization of the chips is a new possibility for consciousness.

The students then spend some time writing down the answers on the activity sheet. Talking about week 10, Krysta begins episode 3:

Episode 3:

14. Krysta: So, we should do... That (*pointing to the bingo chips in front of week 5; see pic 9 in Fig. 5*) times two. So 11…
15. Albert: 11 plus 11… 22.

Fig. 4 Pics. 5–8. The students refining the model

16. Krysta: 22.
17. Albert: (*He laughs.*)
18. Krysta: At the end of…, okay at the end…
19. Albert: Well, wait... No. It would be 11 plus 10 because…
20. Krysta: (*Pointing to week 5*) 5.
21. Albert: (*Pointing to the blue bingo chip*) We always start with the ... [blue chip] (*pic 10 in Fig. 5*).

To answer the question about week 10, the students resort to a "doubling strategy." The strategy can be applied to other cases, as the students noted. Hence, to find the amount of saved money at the end of, say week 25, you can start from week 5; by doubling, you get the amount of saved money in week 10. You double again and add the amount of week 5, without forgetting to remove the extra blue chips that were added in the process. The strategy works just fine but is cumbersome to determine the amount of savings in "distant" weeks such as week 78 or 103. In general, the cumbersome nature of a strategy tends to be insufficient to persuade students to search for more powerful ones. Children's cognition is not organized around, and driven by, needs that can only by satisfied through mathematical optimizing strategies—we are reminded here of the dieters who use a variety of idiosyncratic rather than mathematical powerful strategies to "calculate" the appropriate fractions of ingredients (Lave, Murtaugh, & de la Rocha, 1984). In most practical activity, people do not naturally default to mathematical optimizing strategies. The use of such strategies is the result of lengthy cultural evolutionary processes. Doubling strategies were developed by ancient civilizations (e.g., in Egypt of the Pharaohs). New strategies developed at the end of the Middle Ages and early Renaissance subsequently replaced the older ones—when new commercial activities required more powerful forms of computation. To think in terms of mathematical optimizing strategies, hence, is not a natural part of children's cognitive behavior but the result of the children's cultural development. This is why the students' encounter with a sophisticated form of mathematical thinking, such as the one targeted here (i.e., $2 \times 5 \div 1$ or more generally $2 \times n \div 1$), requires more than a "good" mathematical context. It is precisely through joint action with the teacher that existing, cultural-historical strategies come to be enacted in activity and objectified by the child.

4 Togethering

In the preceding section, we discuss the manner in which the students reached a doubling strategy in the context of a modeling process. In our analysis, we make an effort to show that through a complex sensuous evolving coordination and tuning of speech, gestures, gaze, and actions, interaction unfolds in what we term a space of joint action—a true space of intersubjectivity where the reaching of others is accomplished and where thinking appears as a collective phenomenon. In terms of learning, however, such a coordination, as

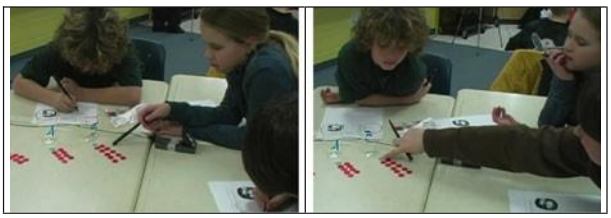


Fig. 5 Pics 9–10. The students suggesting the formula "11 +10."

complex as it may be, may not necessarily result in the attainment of the goal of the activity. The involvement of the teacher may be required. But how can the teacher become involved in the students' learning that we have described in the previous section? To describe the teacher's involvement from our sociocultural perspective, we shall introduce the concept of *togethering*. As developed below, *togethering* is an analytical category that accounts for the *ethical* manner in which individuals engage, respond, and tune to each other, despite their cognitive, emotional, and other differences. As an analytic category, *togethering* does not refer to just any form of getting together to do something, but is used specifically to capture joint practical activity that has the purpose of realizing a collectively motivated object. It theorizes those events that systematically transcend the boundaries of the *here* and *now*.

The starting point is the following: in contrast to the approaches that favor the idea of the student as the self-determining autonomous constructor of his/her own knowledge—approaches that end up creating a dividing line between the realm of the student and the realm of the teacher—activity perspectives take a different turn. In activity perspectives there is no such dividing line (Roth & Radford, 2010). As a result, it is not surprising to find the role of the teacher theorized in different terms. The students and the teacher work *together* towards the attainment of the same material/ideal object of the activity. But there tend to be differences in the way the object comes to be reflected in the consciousness of the teacher and the student (Bartolini Bussi, 1998). That is, since the division of labor that organizes classroom activity positions the teacher and the student differently, the object of activity does not appear exactly in the same manner to each of these participants. In joint activity, therefore, the object of activity is refracted differently in and by the participants' consciousness. Were not the object of activity to appear differently refracted, there would not be anything to learn, for learning is always an attempt at grasping and overcoming differences (even if these differences may never disappear). In our example, whereas for the students the goal of the activity appears to be the mobilization of doubling strategies, for the teacher the goal appears to be an algebraic strategy in which a synthetic multiplicative structure becomes salient. [3]

The different trajectories of the psychic reflections in joint activity allow us to overcome the two forms of reductionism concerning the teaching–learning situations that can be found in the literature. On the one hand, in constructivist approaches, learners are emphasized at the expense of the teacher; as a result, it becomes impossible to understand the temporal continuity of culture and its transformation in and by new generations. On the other hand, sociological and behaviorist approaches tend to reduce learning to transfer, transmission, and external shaping. The reductionism is present even in those approaches that use concepts such as scaffolding, as teaching is conceived as providing help, holding up a scaffold, and learning is still done on the student's part.

In activity approaches, the dichotomy is overcome because *joint* activity inherently reproduces culturally possible forms of participation, which requires both *active* student participation and asymmetry (Roth & Radford, 2010). The latter is inevitable and infuses the activity with those "impelling contradictions, dichotomies, and transformations" that Leont'ev placed at the basis of the evolving dynamics of activity. A mere knowledge "transmission," however, cannot overcome the contradictions. The transmission view, Vygotsky (1997) argues, entails a tremendously misleading idea of the student—one in which he or she is reduced to a kind of sponge. "The old pedagogics," he writes, "treated

[3] These ideas can also be expressed as a difference in activity. But while individualist student-centered approaches locate this difference in the realm of *beings*, activity perspectives locate it in the *division of labor* that underpins joint activity and the manner in which the object of activity becomes refracted.

the student like a sponge which absorbs new knowledge" (p. 48). The issue is that, from activity perspectives, teaching and learning are *not* two separate processes (Bartolini Bussi, 1998). Vygotsky used the Russian word *obuchenie*, which has often been inadequately translated as "teaching," whereas it refers in fact to a simultaneous "double-sided process, one side of which does indeed refer to learning (a change in the psychological processes and knowledge of the child), but the other of which refers to the organization of the environment by the adult" (Cole, 2009, p. 292). In activity approaches, the two sides are irreducible, so that investigating either teaching or learning would be incomplete: it would give a one-sided picture of the whole phenomenon.

There are several ways in which to elaborate the theoretical idea of *obuchenie* or teaching/learning (e.g., as internalization, discourse participation, enculturation). Resorting to the theory of knowledge objectification (Radford, 2008a, 2009b), we follow here a different line of theorizing classroom activity. In this theory, the idea of teaching and learning is articulated around the shared work of the students and the teacher in a space of joint action, in the course of which the students attend to and become familiar with historically and culturally constituted forms of thinking—in our case, an algebraic form of thinking about patterns. Because we are concerned with *obuchenie* activity, the material/ideal object will be related to knowledge, concretized in the material object that is reflected psychologically (ideally) in consciousness. Now, how can the students attend to or perceive something as an algebraic formula in the piggy-bank problem? In the previous section, the students perceived the bingo chips as numbers, not as tokens of a type, that is, the general algebraic structure *2n+1*. The researcher's suggestions opened the possibility for the students to develop a more structured way of perceiving the saving process. However, the objectification of the formula was not achieved. For this to occur, a deeper exposure of the dialectic contradictions in the activity is required—one exposing in more detail the participants' different perspectives. We now turn to the involvement of the teacher.

In episode 4, the students are already done with the question about week 10 when the teacher appears at their desks to see what they have done:

Episode 4:

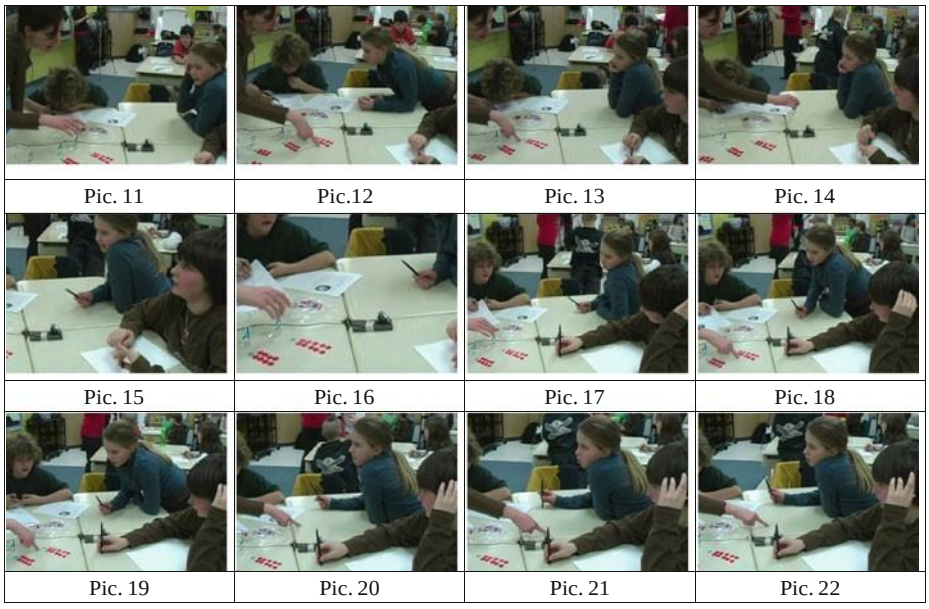
22. Mrs. Giroux: What I find interesting here is that [in your model] you have bingo chips of two colors. What does it mean?
23. Krysta: Because blue was what he already had.
24. Manuel: Yeah, because it [the story problem] says that the piggy bank had 1 dollar.
25. Albert: The piggy bank had 1 dollar, so those (*he points in sequence to all the blue bingo chips from weeks 1 to 5*) are all the 1 dollar that he had already (*now he points to the red bingo chips from weeks 1 to 5*) added to 2, 4, 6, 8, 10.
26. Mrs. Giroux: Okay, okay. What would happen if it was week 10?
27. Albert: Well, we added all this again (*pointing to week 5*), because we know that 5 þ 5 ¼ 10, so…
28. Krysta: (*Interrupting*) plus… we added… We added all this (*she points to the red bingo chips in week 5*) not the blue (*she points to the blue bingo chip*)...
29. Mrs. Giroux: (*Trying to make noticeable to the students the algebraic structure*) What do you remark about week 5 (*she shows the glass corresponding to week 5*) and the number of bingo chips? (*Making the same actions*) the fourth week and the number of bingo chips?
30. Albert: It's always twice…
31. Mrs. Giroux: (*Repeating*) it's always twice.

32. Krysta: It's the double of what you… No! (*she watches the artifacts intensely for a while and says*) I am confused!
33. Albert: Yeah! It's twice, look! (*Counting the red chips*) 1+1, 2; 2+2, 4; 3+3, 6; 4+4
34. Krysta: (*Interrupting*) 8.
35. Albert: (*At the same time*) 8.
36. Krysta: 5+5, 10.
37. Albert: (*Pointing to the bingo chips in week 5*) 5+5, 10.
38. Krysta: Cool. Its twice the week...
39. Mrs. Giruox: So, if the reds are twice [the number of the week], what happens to the bingo chip… (*she points to the blue bingo chip in week 5*)?
40. Krysta: Plus 1.

The teacher's actions display her understanding that the students are now ready to produce the expected formula. Therefore she invites them to write an equation that would indicate the calculations to be carried out in response to the question about week 10. However, the students exhibit difficulties in writing the formula. Manuel is busy writing on his activity sheet and asks Krysta for some help. Talking to Albert, Mrs. Giroux says:

Episode 5:

41. Mrs Giroux: (*Grabbing the glass of week 5; see pic 11 in Fig. 6*) What did you do here? 5… (*Pointing now to the red bingo chips; see pic 12 in Fig. 6*) times…?
42. Albert: … 2.
43. Mrs. Giroux: (*Pointing to the blue bingo chip; pic 13 in Fig. 6*) Plus?
44. Albert: 1
45. Mrs. Giroux: (*Taking the glass of week 5, she moves it to her left to a place where one would expect to find week 10 if the sequence would materially be extended*) What would you do for week 10, if week 10 was here? (*pic 14 in Fig. 6*). What would be the equation… that you would use?
46. Albert: to get 21?
47. Mrs. Giroux: To determine how much money there will be in the piggy bank.
48. Albert: (*Looking at Mrs. Giroux; pic 15 in Fig. 6*) Ummmh… 10 divided by 2, minus…no, plus 1?
49. Mrs. Giroux: (*Grabbing the glass of week 5 again*) What did you do here? (*pic 16 in Fig. 6*)
50. Albert: (*Taking a deep breath and hitting the desk with the back of the pen, while Mrs. Giroux holds the glass of week; see 5 pic. 17 in Fig. 6*) Ok.
51. Mrs. Giroux: (*Still holding the glass, she utters softly*) 5…
52. Albert: (*In synchronization with Mrs. Giroux' gesture that points to the side of the red bingo chips; pic 18 in Fig. 6*) Times 2…
53. Krysta: (*Who has been following the discussion for a while*) Times 2 equal…
54. Mrs. Giroux: (*Pointing now to the blue bingo chip*; *pic 19 in Fig. 6*) Plus 1.
55. Albert: (*Almost at the same time*) Plus 1.
56. Mrs. Giroux: (*Pointing now to an empty space where week 10 would hypothetically be*; *pic 20 in Fig. 6*) 10?
57. Albert: (*Mrs. Giroux points silently to the place where the red bingo chips would be; pic 21 in Fig. 6*) Times 2.
58. Krysta: (*At the same time*) Times 2.
59. Mrs Giroux: (*Silently pointing now to where the blue bingo chip would be; pic 22 in Fig. 6.*)



Pic. 11	Pic.12	Pic. 13	Pic. 14
Pic. 15	Pic. 16	Pic. 17	Pic. 18
Pic. 19	Pic. 20	Pic. 21	Pic. 22

Fig. 6 Pics. 11–22. Mrs. Giroux and Albert working together

60. Krysta: Plus 1.
61. Albert: (*Looking at the teacher*) Minus 1? Times 2 minus 1? Plus 1?

This episode can be divided into two parts: the first part runs from turn 41 to turn 48; the second part runs from turn 49 to 61. In the first part, the teacher draws on the multiplicative relationship between the variables "the number of the week" and "the number of red bingo chips" that the students noticed in the previous episode. The idea is to develop the activity in a way that enables and encourages the students to generalize the multiplicative relationship up to week 10 (and eventually to any particular week). In turns 41–44, the teacher works with Albert on week 5: she grabs the fifth glass (pic 11), articulates the number of the week ("5"), puts the glass back on the table, points to the red bingo chips and asks: "times?" Albert's answer emerges quickly: "2." She points to the blue bingo chip and asks "Plus?" Again, Albert's answer comes quickly: "1." In turn 45 she invites Albert to imagine the model of week 10. She is tense. There is much at stake. If the interaction fails, the activity fails. It would not just be Albert's failure. It would be her failure as well. What would fail is the *obuchenie* process.

In turn 45, Albert follows with his eyes the teacher's hand movement as she places the glass to simulate week 10. To keep him with her, she keeps talking: "What would be the equation... that you would use?" Hesitantly, Albert responds to the call with another question: "To get 21?" We distinguish two *interwoven* levels here: the one of understanding and the one of emotions (see also Seeger's discussion of emotions in his contribution to this special issue). At the emotional level, by responding to the call, Albert is making concretely available to the teacher his willingness to understand. At the cognitive level, the question reveals that the understanding is not clear yet. In turn 47, the teacher elaborates the idea of the equation, gesturing in circles over the bingo chips in front of week 5, while watching Albert's eyes. In the middle of turn 48, the eye contact is lost and Albert, with his eyes directed to a spot to the left of the teacher, as if trying to remember something, suggests a formula.

The second part starts with the teacher grabbing again the fifth glass and inviting Albert to start anew. We are at the heart here of the (dialectical) contradiction—which, consistent with the theory presented, becomes the driving force of the activity. She asks: "What did you do here?" (turn, 49, pic 16). Albert exhibits acceptance of the teacher's invitation with his entire body: He takes a deep breath and hits the desk with the back of his pen (pic 17). The way the teacher asks the question is encouraging: it conveys the idea that Albert knows, but has not yet sufficiently attended to what is marked in the configuration and what is intended to be remarked. To remark the remarkable, that is, the mathematical structure, and to grasp it, to make it an object of consciousness is, of course, what objectification is about. But because the teacher cannot inject such a structure into the student's consciousness, *both the student and the teacher have to work together*. To be in the student's consciousness, its material aspects have to be apparent in the joint space. It is implicit that the teacher knows the structure. At this instant in their interaction, both orient to the activity in ways that take into account her knowing and his learning. It is her knowing the structure that refracts the object of activity in a way that is different from Albert's refraction of the object. But knowing it is not enough. The teacher and the student have to engage in a process of objectification. Objectification is the event of the thing in consciousness, that is, the simultaneous realization of the object at its two levels according to Leont'ev (1978). It will happen when the sought-after object leaves the realm of latent attention and crosses the threshold of explicit attention in Albert's consciousness—the moment at which the sought-after object will find a similar form of refraction in Albert's and the teacher's consciousness.

In turn 51, the teacher engages in joint action with a soft and inviting word: "Five," that she utters while holding the fifth glass. Without talking, she moves the hand to point now to the red bingo chips (pic 18). Albert's voice fills the space left behind by the teacher's silence. He says "Times 2." The teacher moves the pointing gesture to the blue bingo chip (pic 19) and caringly says, almost at the same time as Albert, "plus 1." She now moves her hand to an empty space where the model of week 10 would be (pic 20) and softly says "10?" Without speaking she points to the imagined position of the red bingo chips (pic 21), while Albert looks at the hand and says "Times 2" (turn 50). She moves again in silence and makes the pointing gesture toward the imagined position of the blue chip (pic 22) and Albert hesitantly says "Minus 1? Times 2 minus 1? Plus 1?".

At this point of the activity, the objectification has almost succeeded. Albert still has to better secure the various elements of the formula. That does not take long. A few minutes later, the teacher organizes a general discussion. She invites several students to present their ideas. At a certain point, she asks Albert to explain the calculations to determine the amount of money at the end of week 2.

Episode 5, continued:

62. Albert: It's 2, the second week, it's times 2 because you add ... Two euh, dollars
63. Mrs. Giroux: Okay...
64. Albert: And one, plus one, like one.
65. Mrs. Giroux: Ok.... Do it for [Week] 4. Same idea. 4.
66. Albert: 4 times two...
67. Mrs. Giroux: 4 times 2 because it's the double…
68. Albert: Plus one. 4 times 2+1 equals… 9.

The lesson ends at this point. On the following day, the students in this class work on an isomorphic problem. This time the piggy bank had $6 when Marianne received it and she

saved $3 per week, so that at the end of the first week she had $9, at the end of the second week she had $12, and so on. Talking to his group mates about how to calculate the savings at the end of week 10, Albert said: "She adds 3 dollars each week. So I will do it like this,'kay, 3 times 10 is 30 [plus 6] it's 36. Okay, it's 36."

In this episode, Albert's progressive awareness of the mathematical structure is embedded in a space of joint action where various perspectives meet and where there is a keen sense of *togetherness* expressed in the "most subtle nuances and twists of style, in intonation, in the speech gesture, in the body (mimic) gesture, in the expression of the eyes, the face, the hands, the entire external appearance, in the very way the body is carried" (Bakhtin, 1984, pp. 294–295). Expressing oneself leads to new forms of consciousness, because "for the speaking subject, to express means to become conscious; the subject expresses not only for others, it expresses in order to know itself what it intends" (Merleau-Ponty, 1960, p. 146, our translation). The space of joint action is a requisite for the encounter of various perspectives. But it does not in itself guarantee a successful outcome of the classroom learning activity. The continuous ethical commitment to a common cause is still required. And it is based on an emotional, cognitive, and ethical engagement variously attended through and in language, body, signs, and artifacts, a commitment out of which objectification and subjectification are realized. We call such a commitment *togethering*.

5 From personal constructions to togethering in collective practical activity

In individualistic student-centered approaches, students' learning has been put under the banner of the autonomy of thought that comes from Kant and the Enlightenment—a concept that Piaget (1973) endorsed in his own views of epistemology, cognitive development, and education. The autonomy of thought is the result of theorizing language and words as denoting "meanings" [*significations*], a relation that is always posed by an individual consciousness (Merleau-Ponty, 1960). In these approaches, therefore, the only road to knowledge is that of the student's *personal* constructions. The teacher then appears as someone who cannot interfere with the students' own constructions. Seen through these lenses, Mrs. Giroux's actions may be considered inappropriate in that they would be putting at risk the students' autonomy and the whole educational project. For sociocultural perspectives, autonomy is conceptualized differently—as heteronomic committed action tuned towards others and the individual's community (see Radford, 2009a). In this context, autonomy is not considered a necessary condition for knowledge attainment. At most, autonomy is its result. This, of course, is the central idea behind Vygotsky's concept of zone of proximal development.

Collective understandings, marked by their historical and cultural contingency, are not "constructed" by the students on their own or as a result of interactions with peers who, as negotiator agents, trade epistemic goods. Thus, the students in episodes 2 and 3 stay with their doubling strategies rather than spontaneously developing, as Piaget's child subjects appear to do, culturally advanced forms of mathematics. Joint practical activity, when it involves the teacher and cultural–historically marked artifacts (Bartolini Bussi & Mariotti, 2008; Vianna & Stetsenko, 2006), realizes historical collective motives that come to be reflected in the consciousness of the students *as a result of joint practical activity*. Thus, the ideal object of the activity, which reflects *historical collective* understandings, emerges for the students in and as a result of interactions with the teacher. But the sense of these interactions arises from and presupposes the activity, which is realized in and through the

concrete (mathematical, discursive, signifying) actions that we exhibit in Sections 3 and 4. It is not a matter of deterministic knowledge transfer, as the teacher's participation only guarantees that existing cultural significations enter and thereby somewhat structure the joint activity. But these significations only exist latently until the point when they become explicit in the students' awareness. Students' contributions, inherently offered in the form of material signs and carriers of cultural significations, also enter into and thereby structure the activity. As a result, learning occurs as these collective, cultural significations come to be concretized in the individual consciousness as part of the psychic reflection of the activity as a whole.

In Section 4, we propose the concept of *togethering* to theorize the intricate aspects of the coordination of perspectives: Mrs. Giroux is tuning into the idea of doubling and trying to intimate other ways of attending that Albert expresses. This requires an attunement of perspectives (as opposed to direct teaching); such an attunement is precisely the outcome of the joint attention to the emergence of a common object for a common understanding of the activity that it stimulates. This joint attention to the emergence of a common object is one of the reasons why *obuchenie* cannot be reduced to teaching or learning. *Obuchenie* requires togethering, the committed engagement in joint activity with a common ideal/material object, as there is only hope for instruction when the teacher speaks in a way that is anticipated to be comprehensible to the student, and when the student speaks in a way that is anticipated to be comprehensible to the teacher (Roth & Radford, 2010). But neither teacher nor student can be certain that what they say makes sense to the other. Each participant, in speaking to the issue at hand, thereby exposes not only a way of thinking but also him/herself. Joint activity is an opportunity for producing and ascertaining that the object is reflected similarly in the consciousness of all participants. *Togethering* captures the ethical commitment all parties make to the object, which becomes a common object of activity because of *togethering*.

In the teaching–learning process as we conceive it here, neither participant can anticipate the precise nature of others' actions; the *obuchenie* activity has emergent qualities as it unfolds in unforeseeable ways. For the students, the result of joint activity is the emergence of the object, whereas for the teacher it is a continuous unforeseeable repositioning of the object so that it becomes an object of consciousness for the students. And, of course, this repositioning can only happen through an understanding of the students and their current needs. The two forms in which activity refracts its object depend on each other: the joint activity produces the object for both the students' and teacher's understanding. Neither is guaranteed and both therefore lead a precarious existence until they come together.

6 Overcoming the individual–collective divide

Constructivist approaches—from Kant to the present day—cannot explain the dynamic nature of cultural and individual development; nor can they explain the irreducible relationship between individual and collective consciousness. These approaches remain mired in an opposition of individual and collective, forever in search of "taken-as-shared" understandings and increasing fit of personal constructions with experiences in the world. Kantian inspired approaches were initially subject to considerable critique by the German philosopher Hegel (1977), who realized that the *development* of consciousness, both at the individual and collective levels, required a process that simultaneously realizes *objectification* and *subjectification*. Marx (1973) pointed out that objectification and subjectification

could occur only in joint practical activity, which, because of the concurrent psychic reflection, comes to shape the consciousness of all participating subjects.

The concept of activity, as Leont'ev understood it, integrates and develops those foundational conceptions of practical activity, which has both material and ideal dimensions that cannot be reduced to each other. This position overcomes both problems of the current Kantian educational approaches: the gap between individual and collective and the gap between (ideal) knowledge and the material world it reflects. Here, we have developed a way of accounting for classroom processes, which are understood as the result of joint practical activity. Naturally, there are many types of human activities. We have focused here on one of them: practical school teaching–learning classroom activity. What makes this activity distinctive is the manner in which the object of activity is refracted among the participants. In other settings, for instance in research laboratories, the goal of the activity may be refracted differently. In classroom teaching–learning activities the teacher has a grasp of the object of activity (here, an efficient algebraic way of modeling) that impinges classroom interaction with a certain teleology that is not necessarily present in researcher laboratories, where things are still being "found out." This specific distribution of knowledge and division of labor makes teaching–learning activities distinctive.

In the course of our analyses, we show how practical learning activity provides *a space for joint action* and why it requires *togethering*. We insist on the fact that practical learning activity both presupposes and produces historical collective and individual consciousness. At this point in our discussion, we hasten to remind the reader that by consciousness we are not referring to a metaphysical entity. Consciousness is an unpopular concept in both psychology and mathematics education. Its unpopularity may be attributed to the fact that consciousness is often thought of as something lying in the depths of the soul and hence incapable of experimental investigation. It is not to this metaphysical concept of consciousness that we are referring. We refer rather to consciousness as something concrete: it is a subjective reflection of the world that *expresses* the concrete affective relationship between the individual and her sociocultural, historically situated setting. And since learning is more than knowing, since learning is also the process of becoming (Radford, 2008a), any account of learning, we claim, must comprise the realm of consciousness, for consciousness includes the students' thinking and their emotional orientations. Consciousness, Vygotsky writes, is "a sphere that includes our inclinations and needs, our interests and impulses, and our affect and emotion... A true and complex understanding of another's thought becomes possible only when we discover its real, affective-volitional basis" (Vygotsky, 1987, p. 282). Consciousness can thus be grasped through its overt manifestations—speech, gestures, and all sensuous actions of the sort we scrutinized here.

In providing accounts of teaching/learning processes, the sociocultural theory we propose goes beyond transmissive formats of direct teaching and individualistic student-centered perspectives on learning. From the activity perspective articulated here, teaching and learning are but two sides of the same teaching/learning activity (*obuchenie*), which allows individual consciousness to emerge as a concrete realization of historical collective consciousness. Interactions in the mathematics classroom do not constitute negotiations of significations ("meanings"), the bargaining of knowledge stuff that epistemic subjects exchange. Rather, significations always already exist in culturally and historically contingent activities from, and even before, the instant a baby arrives in the world. They exist in crystallized forms in artifacts, legal structures, moral and esthetic norms, or forms of governmental and political organization (Ilyenkov, 1977). By employing and interacting with communicative signs that have their specific place in specific *obuchenie* activities, students concretize collective significations in their personal sense.

Togethering is a theoretical category in our theory of knowledge objectification that aims to account for the teacher-students embodied-, sign-, and artifact-mediated interaction that includes both co-knowing and co-being. *Togethering* is not the result of some social contract or norms evolved from classroom communities. *Togethering* is both entailed by and the outcome of a joint form of ethical engagement, a collectively motivated activity based on trust and responsibility. For the teacher's engagement to make any sense, the students have to exhibit commitment to a common object (even if it has yet to reveal itself), and their commitment presupposes that of the teacher. To realize the object of the joint activity, each party has to enact its part in the *irreducibly collective responsibility*: Without teacher or student commitment, the object of *obuchenie* activity cannot be realized.

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