RHYTHM AND THE GRASPING OF THE GENERAL

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In this paper we deal with the genesis of students' algebraic generalization of patterns. Our aim is to better understand the way students attend to the perceptually given (e.g. the three first elements of a geometric or numeric sequence) and start moving beyond it in their attempt to grasp a possible general mathematical structure. We provide a multi-semiotic microanalysis of the work done by one Grade 9 student and her small-group mates and show how rhythm accounts for a subtle semiotic device which helps the students project –at the aural, kinesthetic and visual levels– a regularity *which proved to be crucial in conveying a sensuous meaning of mathematical generality.*

INTRODUCTION AND THEORETICAL FRAMEWORK

To account for the progressive manner in which the perceptually given is transcended in generalizing tasks, Kieran et al. (1996), Love (1986), Mason (1996), and Mason et al. (1985) talk about "seeing" or "noticing" the general in/through the particular. Following this line of enquiry and drawing from Husserl's phenomenology and Vygotsky's psychology, in what follows, we investigate the students' production of algebraic generalizations as a process of *objectification*.

Our theoretical construct of *objectification* refers to an active, creative, imaginative and interpretative social process of gradually becoming aware of something (Radford, 2003). Within this context, the objectification of a general mathematical structure in a generalization task amounts to noticing or becoming aware of general mathematical properties that are not directly visible *as such* in the realm of the concrete and the particular. In the overcoming of the particular, the visual stimuli (numbers, shapes, etc.) are continuously being transformed by an *interpretative* and *intentional* contextual process anchored in our own personal biography and cultural history. It would be misleading, however, to think that the continuous modification of the perception of the objects in front of us is accomplished through the organ of vision alone. Vision does not merely transform brute perception into conceptual objects. Human perception, as well as all higher psychological functions, are indeed characterized by a sophisticated collaboration between our historically evolved senses (e.g. vision, touch and audition) and also between our senses and the complex cultural artifacts and semiotic systems that we use. Thus, language, Mikhailov suggested, "constantly participates in converting the perception and understanding of the external object into self-awareness and self-consciousness." (Mikhailov, 1980, p. 236).

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As a result of the distinctive historically and culturally mediated nature of human cognition, in the objectification of mathematical knowledge recourse is made to body (e.g. kinesthetic actions, gestures), signs (e.g. mathematical symbols, graphs, written and spoken words), and artifacts of different sorts (rulers, calculators and so on). All these signs and artifacts used to objectify knowledge we call *semiotic means of objectification* (Radford, 2003).

To understand the students' grasping of mathematical generality, some of our previous works dealt with the phenomenological import of language and gestures and their various mechanisms to ground generalization. We put into evidence two important linguistic functions to which students resort in order to take notice of a mathematical structure: a *deictic function* (based on an intensive use of deictic terms such as "this", "that") and a *generative action function* (based on adverbs of repeated action like "always"; see Radford 2000, 2002). In subsequent articles we dealt with the role of gestures (Radford et al., 2003, 2004) and studied the generalizing function of what we termed 'objectifying iconic gestures', i.e. hand motions that depict a *new* referent by stressing some of its essential features (Sabena et al. 2005). In terms of the sketched theoretical framework, the research question that we want to tackle in this paper can be rephrased as follows: How do students coordinate the different semiotic means of objectification in generalizing tasks? By deepening our previous analyses, we want to better understand the collaboration between eye, word and gesture, and also explore an underlying element that proves important in ensuring the coordination between them: *rhythm*. As we shall see, entangled in words and gestures, rhythm is a crucial semiotic device through which the students make apparent the perception of an order that goes beyond the particular figures. Before going into more details, let us first summarize some aspects of our methodology.

METHODOLOGY: A MULTI-SEMIOTIC DATA ANALYSIS

Data Collection: Our data, which comes from a 5-year longitudinal research program, was collected during classroom lessons that are part of the regular school mathematics program in a French-Language school in Ontario. In these lessons, designed by the teacher and our research team, the students spend substantial periods of time working together in small groups of 3 or 4. At some points, the teacher (who interacts continuously with the different groups during the small group-work phase) conducts general discussions allowing the students to expose, compare and contest their different solutions. To collect data we use three or four video cameras, each filming one small group of students.

Data Analysis: To investigate the students' processes of knowledge objectification we conduct a *multi-semiotic data analysis*. Once the videotapes are fully transcribed, we identify salient episodes of the activities. Focusing on the selected episodes, we refine the video analysis with the support of both the transcripts and the students' written material. In particular, we carry out a low motion and a frame-by-frame fine-grained video microanalysis to study the role of gestures and words. Such a microanalysis is completed with a voice analysis using dedicated software (further details are provided below).

We will focus here on a classical pattern problem that Grade 9 students had to investigate in a math lesson (see Figure A). In the first part of the problem, the students were required to continue the sequence, drawing Figure 4 and Figure 5 and then had to find out the number of circles for Figure 10 and Figure 100. In the second part, the students were asked to write a message explaining how to calculate the number of circles in any figure (*figure quelconque*, in French) and, in the third part, to write an algebraic formula.

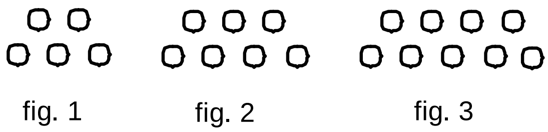


Figure A: The first terms of the pattern as given to the students.

In this paper, we provide a microanalysis of the work done on the second part of the pattern problem by one of the students: Mimi. Two other students were in her small-group: Jay and Rita. In the first part of the pattern problem, the students perceived the figures as divided into two rows and formulated a *factual generalization* (Radford, 2003), i.e. a generalization of actions in the form of an operational schema that can be applied to any concrete figure, regardless of its position in the sequence. For instance, talking about Figure 100, Jay said: "[Figure] 100 would have 101 [referring to the circles in the bottom row] and 102 [referring to the top row]". (See details in Sabena et al. 2005). This factual generalization led the students to answer that there were 23 and 203 circles in Figure 10 and 100, respectively.

RESULTS AND DISCUSSION

The coordination of word and gesture in the overcoming of the particular

The second part of the pattern problem starts with Mimi reading the question:

1. Mimi: (*reading aloud*) We have to explain clearly … how to find out the number of circles in any figure of the sequence (*she reflects for a while and says*) Add… Add three to the number of the figure! (*pointing to the results "23" and "203" already written on the paper*).
2. Jay: No! 101, 100 and (*pointing to the answer*) you got that, 203.

Although the students were satisfied with the way they answered the questions about Figure 10 and Figure 100, Mimi was intrigued by the fact that digit '3' appeared at the end of the previous answers (line 1). She hence tried to formulate a new generalizing schema that would include the digit '3' and the number of the figure. As Jay quickly noticed, the schema is faulty (line 2). Jay's utterance was followed by a long pause (5.2 seconds) during which the students silently looked at the figures. Jay became interested in Mimi's idea but, like Mimi, still did not see the link in a clear way.

Table 1 (Picture 1): Jay (in the middle) and Mimi (on the right) pointing at Figure1.

Trying to come up with something, while putting his pen on Figure 1 and echoing Mimi's utterance, Jay pensively said: "Add 3". At the same time, Mimi moved her finger to Figure 1 (close to Jay's pencil) and said: "I mean like … I mean like …" (see Picture 1 in Table 1). While Jay left the pencil on Figure 1, Mimi retrieved her hand. Then she intervened again and said:

3a. Mimi: You know what I mean? Like… for Figure 1 (*making a gesture; see Table 2, Picture 2*) you will add like (*making another gesture; see Table 2, Picture 3*) …

To explore the role that digit 3 may play, in line 3a Mimi makes two gestures, each one coordinated with word-expressions of differing values. The first couple gesture/word has an indexical-associative meaning: it *indicates* the first circle on the top of the first row and *associates* it to Figure 1 (see Table 2, left column). The second couple achieves a meaningful link between digit 3 and three "remarkable" circles in the figure. The resulting geometric-numeric link is linguistically specified in additive terms ("you will *add*") (see Table 2, right column).

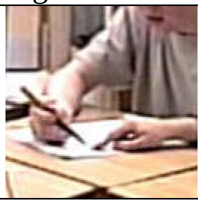
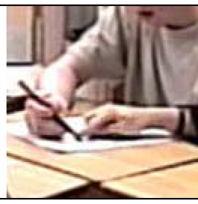
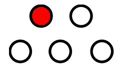
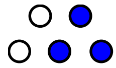
for Figure 1	you will add
	
	

Table 2 (Pictures 2 and 3): Perceptual objectifying effects of word and gesture on Figure 1.

Although Mimi has not *mentioned* or *pointed to* the first circle on the bottom row, the circle has been *noticed*, i.e., although the first circle of the bottom has remained outside the realms of word and gesture, it has fallen into the realm of vision. Indeed, right after finishing her previous utterance, Mimi starts with a decisive "OK!" that announces the recapitulation of what has been said and the opening up towards a deeper level of objectification, a level where *all* the circles of the figures will become objects of discourse, gesture and vision. She says:

3b. Mimi: OK! It would'be like one (*indexical gesture on Figure 1*; *see Picture 4*), one (indexical gesture on Figure 1; *see Picture 5*), plus three (*grouping gesture*; *see Picture 6*); this *(making the same set of gestures but now on Figure 2)* would'be two, two, plus three; this *(making the same set of gestures but now on Figure 3)* would be three, three, plus three.

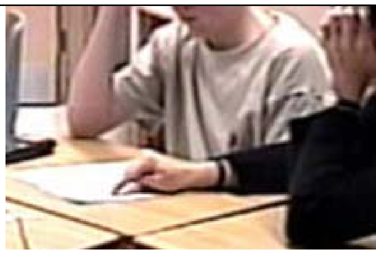
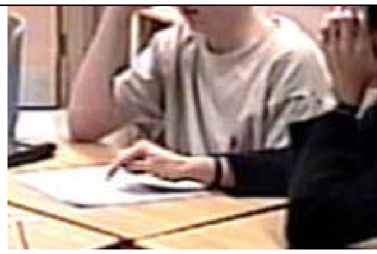
		

Table 3 (Pictures 4 to 6): In Pictures 4 and 5 Mimi makes an indexical gesture to indicate the first circle on the top row and the first circle on the bottom row of Figure 1; in Picture 6, she makes a "grouping gesture" to put together the last three circles of Figure 1.

Making two indexical gestures and one "grouping gesture" that surrounds the three last circles on Figure 1, Mimi renders a specific configuration apparent to herself and to her group-mates. This set of three gestures is *repeated* as she moves to Figure 2 and Figure 3. The gestures are accompanied by the same sentence structure (see Figure B). Through a coordination of gestures and words, Mimi thereby objectifies a general structure in a dynamic way and moves from the particular to the general.

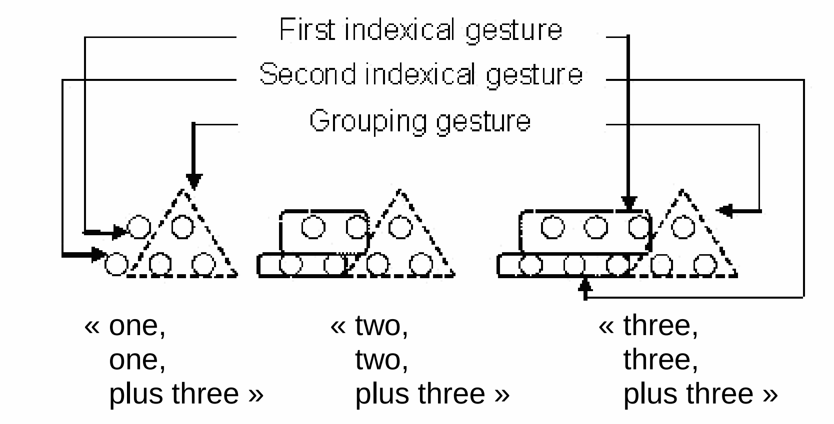


Figure B: On the left, Mimi making the (first) indexical gesture on Figure 1. On the right, the new apprehension of the figures as a result of the process of knowledge objectification.

Rhythm and the projection of the general

The genesis of algebraic generalizations entails the awareness that something stays *the same* and that something else *changes*. In order to perceive the general, the students have to make choices: they have to bring to the fore some aspects of the figures (*emphasis*) and leave some other aspects behind (*de-emphasis*). Closer attention to the previous passage suggests that the objectification of the general schema is much more than a matter of coordinating word and gesture. There is another important element: rhythm. Rhythm creates the expectation of a forthcoming event (You, 1994) and constitutes a crucial semiotic device in making apparent the perception of an order that continues beyond the first figures of the sequence.

To get a better idea of the manner in which the students emphasize and deemphasize the various features of the figures through rhythm, we conducted a *prosodic* analysis of Mimi's key utterance in line 3b ("one plus one plus three" etc.). *Prosody* refers to all those vocal features to which speakers resort in order to mark, in a distinctive way, the ideas conveyed in conversation. Typical prosodic elements include intonation, prominence (as indicated by the duration of words) and perceived pitch. Our prosodic investigation was carried out using Praat (www.praat.org) –a software devoted to voice analysis. Our prosodic analysis focused on the temporal distribution of words and word intensity. In the top part of Figure C, the waveform shows a visual distribution of words in time; the curve at the bottom shows the intensity of uttered words (measured in dB).

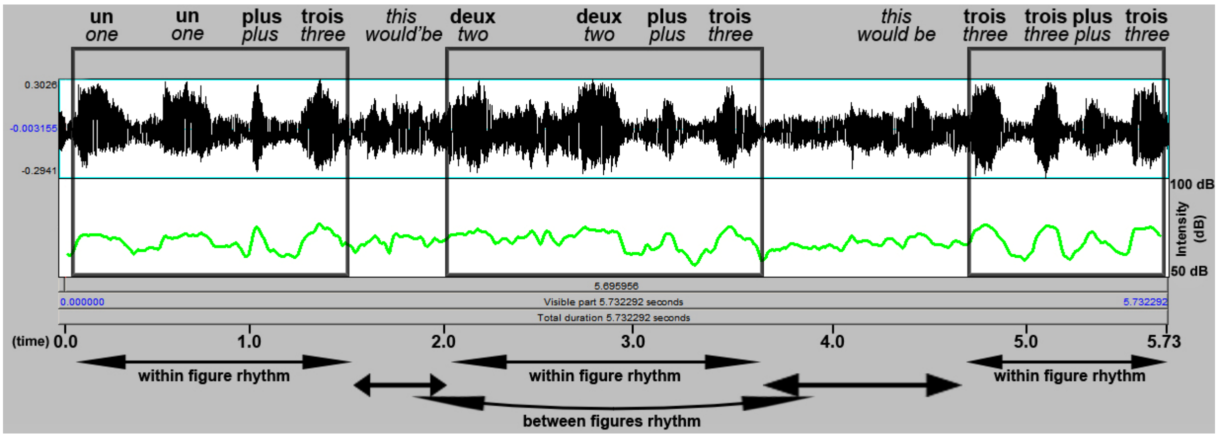


Figure C: Prosodic analysis of Mimi's utterance conducted with Praat.

The waveform allows us to neatly differentiate two kinds of rhythms: *within* and *between* figures. The first type of rhythm, generated through word intensity and pauses between words, helps the students to make apparent a structure *within* each figure. In conjunction with words and gestures (the hand performing the same kind of gesture on each figure), this rhythm organizes the way of counting. The other type of rhythm appears as a result of generated "transitions" between the counting processes carried out by Mimi when she goes from one figure to the next. To generate these transitions, at the *lexical level*, Mimi uses the same expression, namely "this would be", the semantic value of which indicates the hypothetical nature of the emerging counting schema. At the *temporal level*, this expression allows Mimi to accomplish a separation between the counted figures. At the kinesthetic level, the transition corresponds to the shifting of the hand from one figure to the next. Table 3 provides us with a precise idea of the *within* and *between* figures rhythm.

	un *one*	Un *One*	plus *plus*	trois *three*	this would'be	De ux *two*	deux *two*	plus *plus*	trois *three*	this would'be	trois *three*	Trois *three*	plus *plus*	Trios *three*
1. Intensity (dB)	76.58	77.52	80.04	81.93		78.72	78.61	77.44	80.66		81.73	81.24	77.94	80.38
2. Time (s)	0.157	0.665	1.025	1.348	0.813	2.161	2.798	3.158	3.463		4.793	5.116	5.347	5.633
3. Time (s) between consecutive words		0.508	0.36	0.323			0.637	0.36	0.305			0.323	0.231	0.286
4. Total time (s)	1.191				0.511	1.302				1.035	0.840			

Table 3: Intensity and time data of Mimi's utterance, as derived from Praat prosodic analysis. Rows 1 and 2 show the intensity (dB) and time position of words (s), both measured at the middle of the duration of the word. Row 3 gives the elapsed time between consecutive words. Row 4 gives the total time of the speech segments.

The data in row 3 indicate that $a_{33} < a_{32}$, $a_{38} < a_{37}$, $a_{313} < a_{312}$, i.e. the data show that the time elapsed between the additive preposition "plus" and the uttered number prior to it is *consistently shorter* than the elapsed time between the two uttered numbers before "plus". Thus, while the elapsed time between the second "one" and "plus" is 0.360 s (a_{33}), the elapsed time between "one" and "one" is 0.508 s (a_{32}). It is also interesting to note that, in the case of figures 1 and 2, the elapsed time between "plus" and the following word is shorter than the time between "plus" and the uttered number before it (i.e. $a_{34} < a_{33}, a_{39} < a_{38}$). The rhythmic distribution of words hence suggests that the preposition "plus" does not merely play the role of an arithmetic operation. By emphasizing and deemphasizing aspects of the figures, it plays a key prosodic role in the constitution of the counting schema.

Note that the temporal distribution of words of the two first speech segments ($0.157 \le t \le 1.348$; $2.161 \le t \le 3.463$) is quite similar to that of the third speech segment ($4.793 \le t \le 5.633$)). However, the data indicate that the duration of the latter (0.840 s) is shorter than the duration of the former (i.e. 1.191 and 1.302; see row 5). Since the students did not need to go beyond Figure 3 to objectify the counting schema, one of the reasons for this may be that an adequate objectification of the generalization was achieved during the investigation of the two first figures and the third figure hence played the role of verification. This particular status of Figure 3 is also suggested by the following facts. Firstly, $a_{410} > a_{45}$. Secondly, the intensity of the words uttered here is generally higher than the intensity displayed in talking about the first two figures (see Row 1). Thirdly, while Mimi touches the circles of the first two figures in her indexical gestures, she does not touch the circles of Figure 3. Word intensity, time duration and distant physical contact with Figure 3 seem to indicate an achieved level of awareness of the objectified mathematical structure.

CONCLUDING REMARKS

Because mathematical generality is composed of different layers of depth, the grasping of the general is a gradual process of becoming aware of something, a process that we have termed, in accordance with its etymological roots, *objectification*. An essential part of this process is the *projection* of an order into the perceptual realm. Without such a projected order, we all would be overwhelmed by the tremendous sources of stimuli in our surroundings and the richness of detail and nuances of the things in front of us (Fraisse, 1974, pp. 111-112). Three semiotic means of objectification played a distinctive role in creating such an order in Mimi's objectification of the general. These were word, gesture and rhythm. Through them, some aspects of the figures were brought to the fore; others were left in the back, giving rise to a progressive apprehension of the historically and culturally constituted mathematical general structures that were the goal of the classroom activity. Indeed, though indexical and grouping gestures, Mimi emphasized some circles in the visual realm; through words, she endowed them with theoretical content. Rhythm accounted for a subtle coordinating mechanism that produced –at the aural, kinesthetic and visual levels– a *regularity* that proved to be crucial for conveying a sensuous meaning of generality. The prosodic analysis showed how words were distributed in the temporal dimension of discourse to emphasize and deemphasize features of the figures. The ensuing aural meaning of words was synchronized with the kinesthetic and visual meanings encompassing the pointed circles and the successive position of gestures in the space. In addition to shedding some light on the genesis of the students' production of generalizations, our results speak in favor of the cognitive importance of some aspects of the students' mathematical activity –such as gesture and rhythm– that as yet are not a part of main stream studies in mathematical thinking and learning. As our analysis implies, gesture and rhythm are not only merely part of the pragmatic dimension of language and communication but of mathematical cognition as well.

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