REVIEWS

From Truth to Efficiency: Comments on Some Aspects of the Development of Mathematics Education

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Coray, D., Furinghetti, F., Gispert, H., Hodgson, B.R., & Schubring, G. (2003). One Hundred Years of L'Enseignement mathématique: Moments of Mathematics Education in the Twentieth Century. ***Monographie N° 39 de l'Enseignement mathématique.*** **ISBN 2-940264-06-6**

In October 2000, about 50 individuals attended a symposium organized by the University of Geneva and the International Commission on Mathematical Instruction to celebrate the 100th anniversary of the journal *L'Enseignement mathématique.* The symposium was based on a series of invited papers. The book under review is the result of this meeting.

Surely, there is no better way to understand an academic discipline than looking at the way in which, in the past, this discipline posed its problems and tried to solve them. Mathematics education is not the exception. What were the main questions that mathematics educators were trying to answer 100 years ago? What led them to pose these problems? How did they tackle them? What can we learn from this? The book offers valuable information concerning the aforementioned questions and provides mathematics educators with a unique perspective for understanding how the discipline has evolved in the course of the years. It contains a general introduction and three sections dealing with themes that have frequently been discussed in the journal—themes often related to areas of interest pinpointed by the International Commission on Mathematical Instruction, of which the journal became the official organ. These themes are geometry, analysis, and applications of mathematics. They are preceded by a section devoted to the birth and stakes of the journal and followed by a closing section dealing with present and future perspectives for mathematics education. Each section contains a series of papers that discuss the section's theme as it was dealt with in three major historical periods: the beginning (1900), the period leading to the emergence of 'new maths' (1950), and today. Each thematic section ends with a reaction paper and a report of the participants' discussion.

In the general introduction, D. Coray and B. Hodgson explain the general intellectual ambiance underpinning the creation of the journal and the goals of the Geneva meeting. In the first section, the articles of F. Furinghetti and of G. Schubring endeavour to show how the journal was instrumental in the efforts to achieve an international exchange of ideas and cooperation in the early twentieth-century reform movement, whose goal was the renewal of the teaching of mathematics and the training of future teachers. The section concludes with an article by G. Hanna examining the evolution of journals devoted to mathematics education from 1900 up until now, through an analysis that allows one to see the shaping and specialization of the discipline.

The papers dealing with the origins of mathematics education at the beginning of the 20th century are written by R. Bkouche (geometry), J.-P. Kahan (analysis), and P. Nabonnand (applications of mathematics). They make it apparent that the question of the teaching of mathematics was conceived of as a cultural and a social problem. Through the papers, we can see that the discussions about the teaching of mathematics were woven into the general discourse that crafted the central ideas of the modernity that arose in the late nineteenth century. Mathematics and its teaching were seen as fundamental parts of the emerging concept of *Humanity*: the concept of a civilized humanity, where civilization was equated with technological progress. There was no doubt. Mathematics could no longer be seen as a luxury for elites who managed to climb the highest peaks of education. *Mathematics was a social need.* Furthermore, the success of the civilized world depended on the inclusion of the masses. Carlo Bourlet's conference in 1910 gives us a glance at the vision of the world that the new modernity was constructing and the role that mathematics had to play in it:

> Notre rôle [celui des enseignants] est terriblement lourd, il est capital, puisqu'il s'agit de rendre possible et d'accélérer le progrès de l'Humanité toute entière. Ainsi conçu, de ce point de vue général, notre devoir nous apparaît sous un nouvel aspect. Il ne s'agit plus de l'individu, mais de la société; et, lorsque nous cherchons la solution d'un problème d'enseignement, nous devons choisir une méthode non pas suivant sa valeur éducative pour l'élève isolé, mais uniquement suivant sa puissance vulgarisatrice pour la masse. (Bourlet, as quoted by Nabonnand, p. 233)[1]

Modernity was, in fact, built on a vision of emancipation, a view of a world where all nations were walking together towards the same goal, a concert of embraced people living in civilized harmony. This vision can also be found in a paper written by Gaston Darboux a few years before World War I. Darboux says,

> [L]es nations se rapproches de plus en plus les unes des autres, elles tendent de plus en plus à former une humanité civilisée, un concert de peuples dans lequel chacun doit s'attacher à exécuter sa partie de manière à concourir à l'harmonie de l'ensemble et au bien de tous. (Darboux, as quoted by Nabonnand, p. 231)[2]

The teaching of mathematics was discussed against this background. Two trends can be discerned in the book: Along with the trend of which Bourlet was one the promoters and that we may call *socialist* in virtue of its goal, there was also a *humanist* trend. While the humanist view of mathematics emphasized the role the discipline plays in the development of logical thinking, abstraction, rigour, and other highly prized faculties that have been the clear marks of men of sophisticated spirit since the Enlightenment, the socialist trend stressed the applicability of mathematics. Its importance was seen in terms of the utilitarian ability of mathematics to master nature in the interests of mankind. Both trends, of course, claimed a central role in the mathematics to be taught. But the reasons that legitimized this claim were different. For the socialists, it was urgent to move the teaching of mathematics away from a speculative presentation of the discipline. For the humanists, mathematics should not be reduced to its practical aspects. Nabonnand quotes Bourlet:

> [I]l ne nous est plus permis maintenant de présenter à nos élèves la science mathématique sous un aspect purement spéculatif et ... il nous faut, coûte que coûte, plus encore pour rendre service à la société dans son ensemble, qu'à chacun de nos étudiants en particulier, nous efforcer de faire plier les abstractions mathématiques aux nécessités de la réalité. (Bourlet [1910], as quoted by Nabonnand, p. 239)[3]

Representatives of the humanist trend insisted that, even if applications were important, what counted in the end was the logical aspect of the theory and that, for this reason, mathematical rigour should not be compromised. Humanists feared that, in focusing on the applications, mathematics would be reduced to a series of mechanical recipes. The solution was often seen in a kind of teaching centred on the deductive organization of content. Euclid's *Elements* were the paradigm.

It is interesting to note that, although differences were important between countries, in the early discussions, the educational questions revolved around the content to be taught and answers were sought in terms of the level and orientation of the school (elementary, secondary, technological, or university education). Psychology was still an emerging field and mathematics educators did not seem to resort to it in any systematic way. This does not mean that questions concerning the way students were learning were not contemplated. Thus, one of the reports of the *Commission internationale de l'enseignement des mathématiques*, presented at Cambridge in 1912, dealt with the role of intuition and experience in the teaching of mathematics in middle schools (*écoles moyennes*) (see Schubring, p. 59). However, questions like these were often worked out in terms of a Comptean, positivist pedagogy. One of the general pieces of advice given in a French document reads,

> Le professeur ... utilisera fréquemment les représentations graphiques, non seulement pour mieux montrer aux élèves l'allure des phénomènes, mais pour faire pénétrer dans leur esprit les idées si importantes de fonction et de continuité. (see Schubring, p. 53)[4]

In geometry, educational questions centred around the role of empirical knowledge and abstract deductive thinking (see Bkouche's article). Modernity here transposed its postulate that nature is willing to unveil its secrets to us through the industrious use of the scientific method into a method that ensured the continuity between sensual and conceptual knowledge. Pedagogical discourse conveyed the firm empiricist belief that the origin of our knowledge is our senses. Reason picks up sensual knowledge and transforms it into abstract thinking. The pedagogy of modernity drew on the anti-dogmatic posture of the Enlightenment, which put the individual at the very core of knowledge. If something could be known, it could not be derived from any established authority, nor from any secondary source. It had to be known by the individual directly. Between the object of knowledge and the individual, nothing could be interposed—except his/her senses. Several years later, theorizing the role of the senses along the lines of logic–mathematical structures, Piaget would continue the Enlightenment tradition.

The period leading to the new mathematics movement (1950–1970) is surveyed in the book in the articles of G. Howson (geometry), Man-Keung Siu (analysis), and H. Gispert (application of mathematics). As in the previous period, we here find two main views of mathematics that influenced its teaching: a first one that emphasized the role of mathematics structures and a second one that promoted the application of mathematics. When these two views are compared to the ones promoted on the eve of the twentieth century, some differences can be observed. For the humanists of the early years, deduction and rigour were goals in themselves. In the structural approach, they were merely means to investigate mathematical structures. This is why Dieudonné energetically dismissed Euclid: according to him Euclid could not see that numbers and figures were, in fact, governed by sober abstract algebraic and topological structures and that mathematical objects were structural relationships. Thus, modern mathematics corrected this mistake: numbers stopped being aggregate of units and became an abundant profusion of sets of sets.

Howson weighs the impact of the abstract structural approach against important changes in the curricula, such as the introduction of affine and projective geometry and the loss of importance of deductive, Euclidean geometry. Gispert emphasizes the fact that, during this period, new branches were developed in applied mathematics, among them operational research, stochastic processes, numerical analysis, and information theory. The emergence of the new branches also resulted in an expansion of the concept of the mathematician.

The application of mathematics brought about specific pedagogical problems. It was difficult to find the right situations to be mathematized in the classroom. Furthermore, it became evident that the skills required in dealing with induction and plausible thinking could not be derived from teaching settings oriented to the study of mathematical structures. The efforts to promote a connection between teachers of mathematics and teachers of physics were not very successful (Gispert, p. 260).

As to the link between psychology and mathematics education, or as to the way in which specific learning problems were posed and tackled, it does not seem that significant progress was made. Efforts were mainly focused on organizing the curriculum in what seemed to be reasonable ways. However, as Howson (p. 127) indicates, during the two decades from 1950 to 1970, '[O]ne fact became increasingly apparent: the idea that explaining mathematics clearly and logically would automatically yield success was ill-founded.' The 'perennial controversy between computational skill and understanding, between concreteness and abstraction' (Siu, p. 188) resurfaced with great intensity.

There was a general sense that to tackle the aforementioned problems, new types of reflections were required. E. Castelnouvo, G. Papy, A. Revuz, and H. Freudenthal were among the pioneers of those who initiated the effort to seek for answers to the questions raised by the teaching of mathematics in new ways.[5] Within this context, it is not surprising that the emergence of another important journal, *Educational Studies in Mathematics*, corresponds, in fact, to this period: It was founded at the end of the 1960s by Freudenthal.

C. Laborde (geometry), L. Steen (analysis), and M. Niss (the applications of mathematics) analyse the contemporary period. The papers show in a clear way how the central questions in mathematics education have acquired great specialization. Although several of the questions debated in the previous periods continue to be discussed now (e.g., questions about the implementation of the mathematics curriculum, the place to be granted to the application of mathematics, mathematics as a formative intellectual discipline), new questions have arisen. Some of these questions are the result of new societal needs and new technologies available in schools; others are the result of the development of the psychology of mathematics. To mention but a few examples, Niss's paper gives a clear overview of how the concept of applying mathematics has been refined. Steen discusses the role of new technologies and their impact in the teaching of calculus and asks, '*How important is calculus now that graphing and symbol manipulating software is widely available?* Is calculus still as important as it was during the three hundred years between Isaac Newton and Bill Gates when it was the only tool available for most scientific models?' (p. 202). Laborde's article shows how we have become aware of specific cognitive problems underpinning the learning of mathematics, such as the development of spatial awareness, the role of symbols in making sense and becoming aware of geometric properties, and the cognitive demands imposed by geometric reasoning.

In the last section, through U. D'Ambrosio's reflection on education after World War II, we can see some of the transformations in the discourse surrounding mathematics education. The new discourse had to take into account several significant factors, such as the needs of a new kind of worker capable of dealing with the technology of the work place, the citizen as a consumer, and new conceptions of the child. In addition to this, there was also the belief in the universality of objectives and goals for mathematics education. Since mathematics, continuing the tradition of the pre-war period, was still seen as 'the dorsal spine of modern civilization' (D'Ambrosio, p. 312), efforts were made in the 1960s and 1970s to export mathematics to what was then gently called the Third World. These efforts, however, often ended in failure. Through the work of psychologists and anthropologists, it started to become evident that mathematics was much more tied to its cultural setting than expected and that mathematical knowledge is not a mere commodity of the world exchange market. Mathematics, as a product of the reflection of the world in the forms of the activity of the individuals, bears a cultural component that cannot be forgotten. Ethno-mathematics, D'Ambrosio reminds us in his paper, aims, not to displace mathematics, but to offer a complementary way of looking at the world.

In the same section of the book, J. Kilpatrick addresses, from a contemporary viewpoint, a question that, as we have seen, traversed the diverse periods of mathematics education since the beginning of the twentieth century, namely the question of *why* to teach mathematics. People learn mathematics, he suggests, for practical and intellectual reasons. Using a geological metaphor, he refers to these reasons as two tectonic plates—the practical plate and the intellectual one. A numerate citizenry is the justification for the practical reasons. But what justifies the intellectual reasons for teaching mathematics? Kilpatrick says, 'Intellectual justifications for teaching mathematics range from a society's need for an educated citizenry that understands and appreciates the role mathematics has played in building that society and developing its culture to its need for people who can extend mathematics into new realms.' (p. 319).

With this question, Kilpatrick leads us back to the question with which we started this review. We saw how, in the early twentieth century, mathematics discourse was imbued with a humanist spirit that was challenged by a socialist one. However, the socialist spirit also conveyed humanist values but for different reasons. The emancipation of mankind, as conveyed by the socialists, resided precisely in mastering nature in the interests of the whole of humanity. Regardless of whether or not we agree with the reason legitimizing the socialist claim, there was a clear motive for teaching mathematics. As we saw previously, the post-war period knew an increasing use of mathematics. The rapid development of technology made this period the one in which it was futile to ask whether it was *good* to teach mathematics. As J.-P. Kahan commented, in a plenary session held during a mathematical forum organized by the Canadian Mathematical Society on May 2003 in Montreal, '[T]hirty years ago nobody asked the question of why to teach mathematics.' Things seem to have changed. Now we have to find good arguments to convince our contemporary societies. Kilpatrick fears that the pure side of school mathematics (the side that includes pure mathematics) becomes eclipsed by the applied side (the one focusing on the applications) and asks whether the future will see 'a total eclipse or a reappearance from somewhere of justifications for developing the intellect' (p. 322).

I myself fear that if we do not find good reasons quickly, the place of 'pure' mathematics will become smaller and smaller in the school curricula. For example, deductive geometry has practically vanished from the Ontario mathematics school program. Perhaps we will have to learn to react to our contemporary societies, where questions about *truth* have been traded in for questions about *efficiency. Important* questions, indeed, seem to be no longer about whether something is true but if it works (Lyotard, 1979).

In talking about truth, I am not referring to Plato's aristocratic Athenian and reactionary ontology of unchanging forms. I am talking about truth in small capitals, as it is formed by individuals in their reflection on the world—not exactly a relativist version of truth, though (Radford, forthcoming). We might need to find ways of making mathematics a relevant part of our cultural critical reflections of the world, where mathematics can go hand in hand with other cultural manifestations, such as art and literature. Perhaps, we need to understand ourselves and, in turn, help our students to understand that mathematics is a narrative genre too and that behind a geometric proof may be an aesthètic experience as rich and enjoyable as the experience behind a poem or a painting.

It is clear that the papers included in the book under review are very stimulating and that, focusing on three important periods, they give us an exceptional overview of the way in which mathematics education has evolved since the dawn of the twentieth century. The book invites us to reflect on some of the more pressing problems that our discipline is facing today. Now, as Artigue argues, '[R]adical changes in contexts and values prevent us from simply borrowing solutions from the past' (p. 215). Our solutions have to be innovative. History shows us how certain choices were made. It does not tell us what to do next. It might be the case that, looking back wisely at these problems and their solutions, we can reinvent a kind of humanism and the values that go with it, in such a way that mathematics will again become culturally relevant.

Notes

1 Our role [i.e., the teachers' role] is extremely serious, it is fundamental, because it is a matter of making possible and accelerating the progress of the whole of Humanity. Thus conceived of, from this general viewpoint, we see our duty in a new light. It is no longer a matter of the individual, but of society; and when we are looking for a solution to a teaching problem, we have to choose a method that is not based

on its educational value for the lone student, but solely based on its popularizing potential for the masses. This and the following translations from French into English were made by Heather Empey.

2 [N]ations are coming together more and more; they are tending more and more to form a civilized humanity, a concert of peoples where everyone has to be devoted to playing his part in such a way that it works to harmonize with the ensemble and with the good of all.

3 [W]e can now no longer afford to present mathematical science to our students in its purely speculative aspects and ... we must endeavour, no matter what, more as a favour to society as a whole, than as one to our students in particular, to bend mathematical abstractions to the necessities of reality.

4 The teacher ... shall make frequent use of graphic representations, not only to let the pupils get a better understanding of the shape of the phenomena, but so as to inculcate them with the utterly important notions of function and continuity.'

5 Naturally, by then Piaget had gained some relevance in the discussions in mathematics education, but Piaget's theory was broadly used and, generally, only as a means to justify the structural approach (Howson, p. 124).

References

Lyotard, J.-F., (1979). *La condition postmoderne.* Paris: Les éditions de minuit.

Radford, L. (forthcoming). The Anthropology of Meaning. *Educational Studies in Mathematics.*