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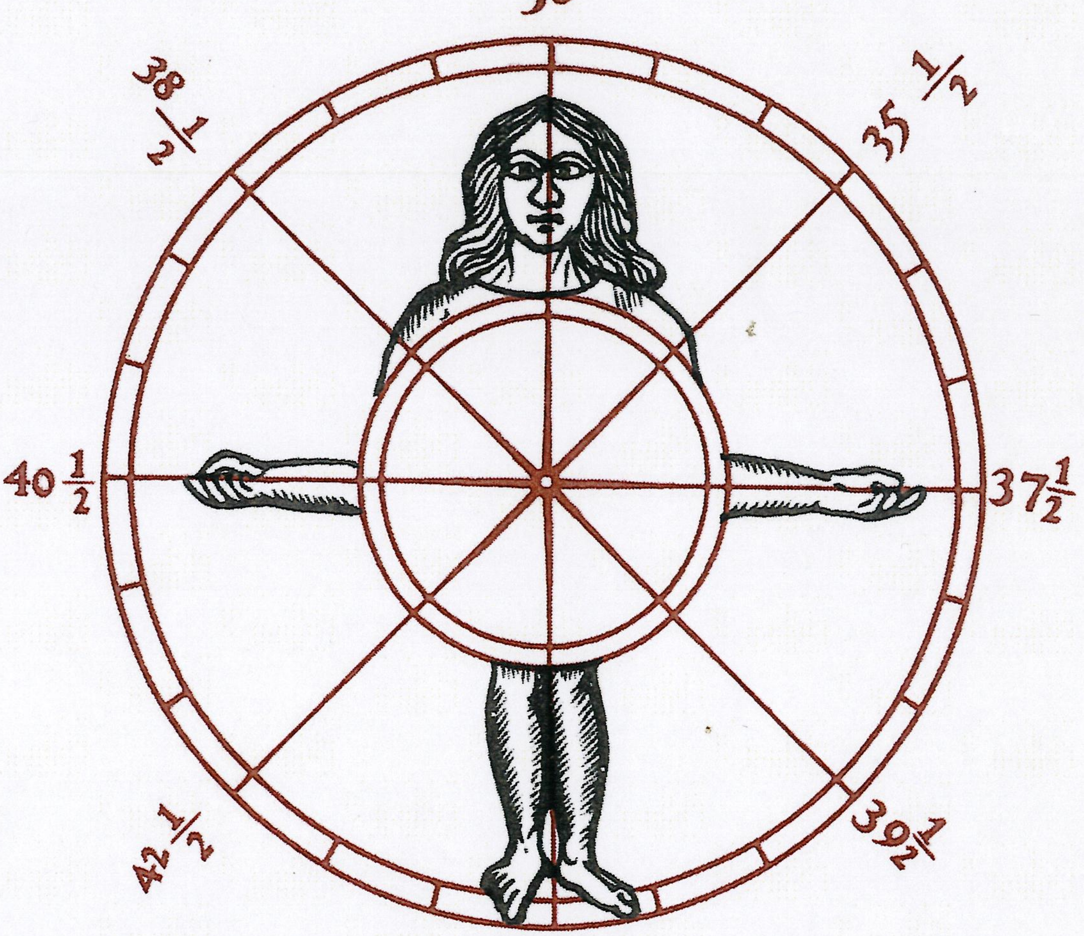

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Departamento de Matemática da Universidade do Minho

HISTORY, RESEARCH AND THE TEACHING OF MATHEMATICS

An Introduction for the Panel

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One of the most frequent ways in which the history of mathematics (HM) has been used in the classroom consists of:
(a) locating and extracting an "interesting" old mathematical episode and
(b) presenting it to the students.
Although the specific aims may vary (e.g. the aim may be that of capturing the students' attention in order to introduce them to a new curricular topic), the general goal of using HM in a teaching context is obviously to *improve* learning.

One of the problems that has been recognized concerning the use of HM is that it requires teachers to know more than only the modern mathematical content to teach. Beyond the necessary mastering of the historical mathematical content related to the episode with which we want to deal in the classroom, there are, however, two *deep* methodological problems to be considered that we want to discuss in this panel.
The first methodological problem requires us to specify what we mean by an *interesting* old mathematical episode and the way in which we are going to *locate* and *extract* it. The second methodological problem concerns the way in which we are going to present the chosen old mathematical episode to the students, something that, I claim, cannot be done by merely dropping it off in the classroom. Indeed, there should be a very delicate and complex work of "adaptation" and "handling" of old mathematical "pieces" in order that history may become a genuine and fertile tool to improve teaching.

Of course, there is not just one possible solution to the two aforementioned methodological problems. However, any possible solution must take into account (i) one's meaning of history and (ii) one's conception of the development of mathematical knowledge[1].

Often, the two aforementioned methodological problems have been avoided by assuming a simplistic and naïve view to points (i) and (ii) –leading to what we may call a Simple Teaching Model (STM). From a STM perspective, the history is confined to a sequence of events that follows a chronological order, whereas the development of the mathematical knowledge is underlined by a rather implicit standpoint according to which ancient mathematical ideas are but *imperfect* modern mathematical ideas.

Let me mention the well-known controversial "Euclidean Greek Algebra". Book 2 of Euclid's *Elements* has been seen, quite often, as a book that deals with quadratic equations. According to this view, if one does not *see* quadratic equations in Euclid's work it is merely because there were not any algebraic symbols at the time. From this interpretation, the "spirit" of Book 2 and Euclid's intentions are essentially considered as *algebraics*. However, a closer look shows that there is no such (modern) algebraic intentionality in the *Elements* (see Unguru, 1975). On the other hand, Høyrup's reconstruction of Babylonian mathematics suggests that the problems and methods that we find in Book 2 of Euclid's *Elements* are related to ancient techniques practiced by Babylonian scribes (see Høyrup, 1990). Of course, the ancient techniques were not taken over and kept intact by the Greeks. Ancient Near-East and Greek styles of mathematical thinking were very different. As Crombie said (1995), any style of thinking is determined by commitments to conceptions about nature and to conceptions about science. Hence, in order to be inserted in the realm of Greek mathematics, the pre-Greek methods had to undergo fundamental changes.

Thus, a STM avoids the aforementioned first and second methodological problems by assuming that mathematical knowledge is essentially unhistorical (which is somewhat paradoxical when we are speaking precisely about the History of Mathematics!). This allows one to link, without any problem, the concepts of modern school mathematics to their ancestors –for, supposedly,

[1] Two of the most important current non-naïve research programs used in educational mathematical circles are the *Epistemological Obstacles* and the *Reification Processes*. Both present us different accounts of the growth of knowledge. This lead them to two different *readings* of the history; a data that is *interesting* for the first may not be interesting for the second and vice-versa (see Radford, 1996).

"adding" to the latter our modern notations makes them attain the level of "perfection" of the modern concepts. From this simplistic point of view, the only problem is how to disguise ancient and old conceptualizations in modern robes. However, doing so, we evacuate all the conceptions, intentionalities and *raisons d'être* of past mathematics: we focus our attention on what we may call the "pure" mathematical knowledge. The question is: does such a thing exist ?

Some modern historiographical trends have been considering the problem of mathematical knowledge in a broader perspective and are challenging the "internalist" approach that consists in seeing mathematics as a socioculturally-free activity. Within this new perspective, a "piece" of old mathematics cannot be reduced to its mathematical content. Mathematical knowledge cannot, unlike flowers, be extracted from its own habitat and put into vases.

In this case, the link between the HM and the teaching of mathematics becomes really *problematic*. Indeed, if (modern and past) mathematical styles of thinking are rooted in their own sociocultural contexts, is it possible for us to understand them?[2] However, if past mathematical styles of thinking are understandable, *how* can we understand them and to what extent? This is the profound challenge to the first methodological problem. In this panel, we want to discuss some characteristics of *frameworks* and *methodologies* that may make it possible to understand historical mathematical achievements and the way to interpret them, bearing in mind that our work should be done for *teaching purposes*. To have a chance to succeed in such an enterprise, we must specify some philosophical and epistemological viewpoints about human cognition and the development of mathematical knowledge (something that a STM overlooks –if not, ignores completely!).

We also want to examine, in this panel, some implications related to the second methodological problem. If, once again, mathematical styles of thinking are rooted in their sociocultural factors, is it possible to compare past and modern mathematical intellectual developments? It is clear that the sociocultural factors are not the same throughout time. Thus, *what* can be

[2] In order to better understand this question, it would be worthwhile to remember, at this point of our discussion, that Oswald Spengler suggested that different cultures are incommensurable (see Restivo, 1992, pp. 3-9). Of course, we may not agree with Spengler's view, but we cannot ignore it either !

compared? On the other hand, how can we "adapt" past achievements in order to improve learning in the classroom? One may now realize why I previously said that it is no longer possible to "denaturalize" the history of mathematics and to simply extract episodes and drop them off in the classroom.

To go a step further in the use of the HM in the classroom requires us to reflect upon and to discuss very seriously the foundations of didactico-historical approaches. This panel aims to contribute to the search of solutions to these problems by gathering different scholars with different experiences and backgrounds. We hope that the discussion will allow us to elucidate some paths to overcome the difficulties which a serious use of the HM in the classroom is now facing.

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