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THE ANTHROPOLOGICAL TURN IN MATHEMATICS EDUCATION AND ITS IMPLICATION ON THE MEANING OF MATHEMATICAL ACTIVITY AND CLASSROOM PRACTICE

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Abstract. A quick glance at contemporary mathematics education makes plain that we are living in a time of important changes. The ideas conveyed by classical theories in our field, including learning as a mental adaptive construction and the conception of the teacher as a mere learning facilitator, are now questioned. There has been an important shift provoked by a profound need to regain contact with the realm of the social and the cultural. However, this shift, produced by what I term here "the anthropological turn in Mathematics Education", is not without its problems. It requires a re-conceptualization of Mathematics Education and, more specifically, of the learner, the teacher and the knowledge to be learned. In this article, I present an overview of what mathematical activity and classroom practice look like from an emerging sociocultural perspective – the theory of knowledge objectification.

Résumé. Un regard rapide sur la recherche actuelle en didactique des mathématiques montre qu'on est en train de vivre des moments de changements importants. Les idées avancées par les théories classiques dans notre domaine – par exemple, que l'apprentissage est le résultat de constructions mentales d'adaptation à l'environnement et que le professeur n'est qu'un facilitateur de l'apprentissage – sont maintenant remises en cause. Il y un tournant important provoqué par un besoin de regagner contact avec le monde social et culturel. Toutefois, ce tournant, produit par ce que j'appelle ici « le virage anthropologique dans la didactique des mathématiques », ne va pas sans poser certains problèmes. Il exige une reconceptualisation de la didactique et, plus précisément, de l'élève, du professeur et du savoir à apprendre. Dans cet article, je présente un survol de la manière dans laquelle l'activité mathématique et la pratique de salle de classe apparaissent quand celles-ci sont perçues à travers une perspective socioculturelle en émergence – la théorie de l'objectivation.

Zusammenfassung. Beim Streifblick an die heutige Art des Mathematikunterrichts erkennen wir, dass wir in einer Zeit bedeutender Änderungen leben. Die von klassischen Theorien in unserem Umfeld übermittelten Ideen, beinhaltend das Lernen als eine mental adaptive Konstruktion und die Konzeption des Lehrers als Übermittler von Lehrstoffeinheiten, werden hinterfragt. Von zwingendem Bedarf einen neuen sozial-kulturellen Kontakt zu erlangen kam es zu einer Verschiebung im Denken. Diese neue anthropologische Veränderung

in der Mathematikbildung ist nicht ohne Probleme. Sie verlangt ein neues Konzept der Mathematikbildung und speziell, eine Veränderung im Denken der Bildungsbeteiligten und der Kenntnisse die gelehrt werden. Ich präsentiere in diesem Artikel einen Überblick wie mathematische Aktivität und Klassenarbeit aus einer aufkommenden sozial kulturellen Perspektive aussehen möge – die Theorie der Kenntnis-Vergegenständlichung.

Riassunto. Uno sguardo rapido sulla ricerca attuale in didattica della matematica mostra che stiamo vivendo dei momenti di cambio importanti. Le idee avanzate dalle teorie classiche nel nostro dominio – per esempio, che l'apprendimento è il risultato di costruzioni mentali di adattamento all'ambiente e che l'insegnante non è che il facilitatore di tale apprendimento – sono ora messe in dubbio. C'è una svolta importante provocata dalla necessità di riconquistare il contatto con il mondo sociale e culturale. Tuttavia, questa svolta, prodotta da quel che io chiamo "la virata antropologica nella didattica della matematica", non si propone senza dover porre certi problemi. Esige una riconcettualizzazione della didattica e, più precisamente, dell'allievo, dell'insegnante e del sapere da apprendere. In questo articolo, presento uno sguardo alla maniera in cui l'attività matematica e la pratica d'aula appaiono quando esse sono percepite attraverso una prospettiva socioculturale in emergenza – la teoria dell'oggettivazione.

Abstrakt. Zbežný pohľad na dnešný spôsob vyučovania matematiky nás presvedčí, že žijeme v čase dôležitých zmien. Spochybňujeme myšlienky sprostredkované klasickými teóriami, nevynímajúc učenie ako mentálne adaptívnu konštrukciu a koncepciu role učiteľa ako sprostredkovateľa častí učenia. Nastal dôležitý posun vyprovokovaný vážnou potrebou získať nový sociálny a kultúrny kontakt. Akokoľvek, táto antropologická zmena vo vzdelávaní v matematike nie je bezproblémová. Vyžaduje si rekonceptualizáciu vzdelávania v matematike a zmenu v myslení žiakov aj učiteľov a vo vedomostiach, ktoré učia. V tomto článku prezentujem prehľad matematických aktivít a praktických činností v triede z pohľadu vznikajúcej sociálnokultúrnej perspektívy – teóriu objektivizácie poznania.

Key words: Constructivism, Piaget, Vygotsky, mathematics education theories, sociocultural approaches, theory of knowledge objectification

1 INTRODUCTION

Mathematics education has always shown a variety of theoretical and methodological orientations, some of them related to the local characteristics of its emergence and development. Thus, mathematics education appeared differently depending on whether or not its initial niche was a mathematics, psychology or educational department. The variety of its theoretical and methodological orientations can also be related to the regional or national context in which it was intended to operate. However, despite this variety of contexts and their ensuing orientations, with the exception of a few cases, mathematics education emerged very closely in its aims and scopes to those of dominant views of Western psychology. As a result, mathematics education imported both the concept of the mind and the concept of the individual from psychological theorizations. It is

hence not surprising that, often, learning was investigated in cognitive terms – e.g., in terms of the cognitive growth of mental structures. This form of conceiving of mathematics education is asserted with all clarity in one of the founding papers of what was to become American Constructivism – one of the current main theories in mathematics education. Thus, in a paper published by the end of the 1980s, one of the Constructivist theory's architects, Paul Cobb, talking about the role of mathematics education claimed that

> *A fundamental goal of mathematics instructions is or should be to help students build structures that are more complex, powerful, and abstract than those that they possess when instruction commences. The teacher's role is not merely to convey to students information about mathematics. One of the teacher's primary responsibilities is to facilitate profound cognitive restructuring and conceptual reorganizations.*
>
> (Cobb, P., 1988, p. 89)

However, a quick glance at what is going on in mathematics education nowadays makes plain that we are living in a time of fast changes and that framing the question of mathematics education in terms of the growth of mental structures now seems insufficient – and even misleading. Such a view entails a conceptualization of the individual and his/her mental life in ways that are not really compatible with the contemporary views of the mind as a social phenomenon. There is indeed an important shift provoked by a profound need to regain contact with the realm of the social and the cultural. In his opus magna, Phenomology of perception, the French epistemologist Maurice Merleau-Ponty argued that

> *We must ... rediscover, after the natural world, the social world, not as an object or sum of objects, but as a permanent field or dimension of existence... We must return to the social with which we are in contact by the mere fact of existing.*
>
> (Merleau-Ponty, 1962, p. 362)

The re-discovering of the social and of our ineluctable existence within it is leading us to a search for more encompassing ways of theorizing the basic problems of existence and, along with them, those of teaching and learning. It is leading us to an important shift that has become noticeable in the past few years in mathematics education and that we could summarize as the "anthropological turn". What this turn means is basically the acknowledgment of the shortcomings of rationalist and idealist Western accounts of the relationship between culture and the individual, consciousness and social practice, and teaching and learning (Brown, 2008; Presmeg & Radford, 2008; Radford, 1999, 2008a; Valero, 2009).

This turn means that we need to rethink the subject as it learns, acts, and feels in broader ways, for as Marx Wartofsky put it, even "the sensation I have, the thought I think, the desire I express, the action I perform as a human being is hopelessly infected with my personal biography, my species-history, my social and historical past and present and future" (1979, p. 115). Such a turn, however, does not go without its own theoretical and methodological problems. For one thing, in the mathematics educational realm, it is leading us to new conceptions of what mathematics activity and classroom practices are. Within this context, the traditional roles of the teacher and the student are being revisited. The anthropological turn is so significant that it seems that a substantial revision of our classical theories in mathematics education is required – or even that new paths have to be envisioned and forged. It is only against this background that the general topic of the annual 2009 conference of the International Group for the Psychology of Mathematics Education makes sense. The title of the PME topic was indeed "In search for theories in Mathematics Education". This shift marks the undertaking of an unprecedented deep process of change concerning the manner in which we are looking at the mathematics classroom, the teacher and the students.

My goal in this article is not to dwell on those aspects that have led us to see the mathematical activity, mathematics classroom and their protagonists in a different way. I am rather interested in saying something about what mathematical activity in classroom practice looks like when it is seen through the prism of sociocultural approaches that epitomize the anthropological turn. In the first part of the article I deal with the question of mathematical activity. Then I turn to new conceptions of classroom practice.

2 MATHEMATICAL ACTIVITY

Mathematical activity is, of course, one among the many kinds of human activity. Its specificity lies in its focus on a particular field of content – mathematics. To talk about mathematical activity as something specific seems to be granted by the fact that it does not appear to be the same activity as the one that can be found in other domains of knowledge – for instance literature, law or the arts. In the most general sense, mathematics, I want to contend, could be considered as a form of cultural reflection, action, and understanding that differs from others in its emphasis on, e.g., numbers, shapes, figures, time, space, and motion. Of course, such an emphasis varies from culture to culture and from one historical period to another, making the borders of mathematics not only fuzzy but also relative. Music, for instance, was considered a part of mathematics in ancient Greece and in the Middle Ages. Now it is considered a part of the arts.

However, what makes mathematical activity different from other forms of activity is not really its content or the general nature of its objects. Rather, what makes mathematical activity distinctive overall is the kind of thinking or reasoning that it conveys. Mathematical objects are general, this is true. Not even the most perfect of the particular triangles is capable of conveying the general nature of the mathematical object triangle. But the same can be asserted of the objects of law or art. Thus, not even the most perfect gesture of justice can convey the general nature of what 'justice' is. Nor can we find the general expression of beauty in any of its particular instances. The specificity of mathematical activity lies hence not in the generality of the objects it deals with, but in the forms of thinking and reflecting it conveys.

Now, the temptation here is to consider mathematical forms of thinking and reflecting as something universal, as Western rationalist epistemologies do. Such epistemologies interpret other forms of mathematical thinking as imperfect, sometimes "exotic" versions of Western mathematical thinking. Two errors are usually made in this respect. The first one is to denaturalize the mathematical forms of thinking from their own historical and cultural contexts. The second one, which is a corollary of the previous one, is to attribute to mathematicians of the past achievements that, under closer examination, do not make sense in the context of the mathematics they were practicing in their own time. The history of mathematics is full of such accounts. For instance, Euclid is often portrayed as an algebraist who did not have at his disposal the algebraic symbolism of Vieta but was nonetheless in possession of the corresponding algebraic ideas. It seems more appropriate to conceive of forms of mathematical thinking and reflecting in their own sociocultural context (Radford, 1997).

Let me discuss a short example that illustrates well, I think, the importance of understanding forms of mathematical thinking in their own habitat. It comes from the mathematics of the Loboda community of people in Normanby Island, in Papua New Guinea. Lobodan people do use mathematics, but it differs from our Western numeric-oriented mathematics in striking ways. It is not that the Lobodans are unable to count. In fact, they may count if needed. They even have a counting system with words for a few numbers. However for the most part, counting is unnecessary in Lobodan activities. In fact, Lobodan activities revolve around qualitative organizing systems and principles. As a result, their mathematical form of thinking is of a qualitative comparative nature. Thus, to describe lengths, quantities, years, time, etc. the measured object is compared to another familiar object. The length of a necklace may be compared to the length of one's arm, for instance. In a story, a man was sent back to his village, which according to our distance system was about 40 miles away. In the story, this distance is referred to as far away as "from here (Loboda) to Sanaroa Island" (Thune, 1978, p. 72). Following this same contextual comparative pattern of thought, mothers do not express the age of their children in years, but in terms

of crucial stages of aging (e.g., memeyo for infant, gwama for child, tubuhau and gomwagwehine for adolescent boys and girls, etc.). As the anthropologist Carl Eugene Thune observes, "It is not so much that one couldn't develop means for keeping track of age using the Loboda numerical terminology… as there is no interest in doing so" (1978, p. 74). When repaying a gift received previously, they are meticulous in being fair: they repay by giving back some equivalent amount (as opposed to equal). It would not make sense to say that the receiving side has to repay the same number of, say, yams, for yams are not counted. They are heaped together and considered as a collective gift. Repaying the collective gift means that a heap of yams of the same (approximate) size must be given (Thune, 1978, p. 75). It would be an ethnocentric mistake, though, to consider Loboda's concept of number as a kind of area or volume concept. Areas and volumes are ideas expressed in numerical terms, which is precisely counter to what the Lobodas do!

We see hence that the Lobodas' mathematical thinking is interested in stressing quantities in terms of practical comparisons to other situated elements rather than to absolute standards. Lobodas' mathematical thinking rests on an epistemological stance according to which their world is not conceived of as being organized by numbers. As a result their practices are not numerically oriented. It makes no sense to develop a quantitative mathematics in such a context.

In presenting this example, I wanted to stress the fact that when we say that the specificity of mathematical activity lies not only in the objects it deals with, but in the forms of thinking and reflecting it conveys, we would bear in mind that these forms of thinking and reflecting mathematically may be varied. It turns out that mathematics and its concomitant forms of thinking are not universal. Rather, as Owens put it, they are as cultural as food taboos (Owens, 2001, p. 157).

Let me now dwell upon the question of classroom practice.

3 CLASSROOM PRACTICE

The question of classroom practice is a delicate one as it depends on ontological, epistemological, and psychological assumptions concerning knowledge and the manner in which learning actually occurs. These assumptions frame conceptions about the purpose of education, the relationship between knowledge and the learners and, more generally, the role of the different actors in the classroom.

3.1 KNOWLEDGE

As we all know, most contemporary theories have adopted the view according to which the individual constructs his or her own knowledge, as he or she is moved by adaptive needs. The idea of adaptation and the mechanisms that make it

possible were introduced by Piaget in his genetic epistemology. They were borrowed from biology. Human beings are, of course, biological creatures. Thus, the mechanisms of adaptation should in principle apply to them. This is true. However, what distinguishes human beings from other biological species is precisely the fact that they are also, and are overall, cultural creatures. What this means is that mechanisms of adaptation are insufficient to account for the production of knowledge and the processes of learning. Indeed, the production of knowledge and the processes of learning occur in complex sociocultural contexts that are much more than sources of stimuli for adaptation to occur. The environment is not merely a space where the individual finds the material to produce her subjective "viable knowledge" – to borrow the term from von Glasersfeld (1995). On the contrary, the environment affects, in profound ways, the manner in which individuals come to learn or produce knowledge. The environment, Vygotsky argued in Educational Psychology – his first book, written when he was working at the Gomel Teachers' School before moving to Moscow – affects the individual both directly and indirectly. The environment affects the individual in very subtle ways, through all forms of social action and organization "that have been established in the course of historical development and have become hardened in the form of legal statutes, moral precepts, artistic tastes, and so on" (Vygotsky, 1926/1997, p. 211).

Vygotsky made those remarks in his argument against the biological views of the time where adaptation was considered a fundamental principle governing the development of organic life both in phylogenesis (i.e., the historical development of ideas and the mechanisms of their production) and ontogenesis (i.e., the development of the individual during lifespan). It is in this context that the famous recapitulation theory that advocates the recapitulation of phylogenesis by ontogenesis was formulated and that Vygotsky was objecting to it. It is interesting to note in this respect that if Piaget was interested in phylogenesis, it was largely so that he might refute history: the mechanisms of adaptation, Piaget argued, were universal, hence ahistorical and acultural. They have always been the same everywhere, regardless of the historical period we are looking at.

We see here a parting of ways. Sociocultural approaches follow here a different path from the one followed by Piagetian inspired theories. Instead of conceiving of cognition in adaptive terms, socioculturalists consider cognition as a cultural and historically constituted form of reflection and action embedded in social praxes and mediated by language, interaction, signs and artifacts. As a result, knowledge is produced by cognizing subjects who are, in their productive endeavours, subsumed in historically constituted traditions of thinking. And this, of course, leads to different conceptions of the classroom and the learner.

3.2 THE LEARNER

The Piagetian idea of knowledge as the result of subjective adaptive actions goes hand in hand with another important idea – the culturally detached, self-regulating epistemic subject of Piagetian theory. Education and psychology have drawn heavily on this concept to craft their concept of the individual and the learner. Canadian psychologist Jack Martin has, I think, best described the main characteristics of learners and individuals molded by such epistemic subjects. In Martin's words,

> *The self of self-regulated learning appears as an individual labouring in relative solitude… When social factors are considered they mostly are framed as variables that mediate or influence what is predominately a highly individualistic pattern of development... The central concern is for an individual actor capable of simultaneous action and reflection on this action, much like a stereotypic scientist in close scrutiny and judgement of experimental phenomena of interest.*
>
> (Martin, 2004, pp. 193-194)

This individual, Martin remarks, acts like a Cartesian thinker, a self

> *isolated from its surrounds even as it is shaped and affected by circumstances and events, which while always influential are not seen as in any way constitutive of the core self… one that surveys the exterior landscape for signs of personal affirmation and possibilities for expression on the one hand, and clued to strategic action on the other… its most vital resources are apparently available within its detached internality. It acts as a final arbiter over whether or not its strategies are effective or its appraisals self-sustaining…This is a self that already knows its business, one that requires only a facilitative grooming to become more fully socialized and intellectually engaged.*
>
> (Martin, 2004, p. 197)

The cognizing subject of sociocultural theories is not a Cartesian cogitator. On the contrary, it is a subject that thinks within a cultural background and that, in so doing, goes beyond the necessities of mere ahistorical adaptive urges (D'Amore, Radford, & Bagni, 2006). In other words, for sociocultural theories the "will to knowledge" (to borrow Foucault's term) and the way knowledge comes into being are neither driven nor shaped by adaptive needs or impulses to produce 'viable' hypotheses or 'optimal' results. A Loboda child sees the world through the prism of her culture and reasons mathematically in terms that are consonant with the qualitative epistemological stance that embeds the Loboda practices. I shall

have more to say on the sociocultural concepts of the learner. For now, though, I will turn to the question of how learning occurs.

3.3 HOW LEARNING ACTUALLY OCCURS

As previously mentioned, the central premise of many contemporary theories is that each student has to construct her own knowledge. Even if it is in a context of interaction and exchange with others, in the end no one can construct a piece of knowledge and then pass it on to you. The relationship between knowledge and learner is, here, a direct one. Learning and personal knowledge construction coincide. This is why the teacher is compelled to restrain herself from giving clues to the students, let along telling them the answers! To my knowledge, Brousseau (1997) has better than anyone else shown the tremendous difficulties that arise for teachers and students within this conception of learning. Teachers have to be careful not to give more to the students than what is strictly needed. Even a grimace or an involuntary gesture can ruin the learning project, for if the teacher says or does more than what she should, she trespasses the thin line that separates her from the student's autonomy, so that knowledge is no longer the personal construction of the student. Instead of genuine knowledge, what the teacher and the student produce here is the illusion of knowledge – as in the Topaz Effect. And since these difficulties are simply the logical consequences of the adopted theoretical premises, they cannot be avoided. They manifest themselves as paradoxes – heavy crosses that teachers and students have to carry on their shoulders as they interact with each other.

4 SOCIOCULTURAL THEORIES

In sociocultural theories, learning and the personal construction of knowledge do not necessarily coincide. There can be genuine learning without it necessarily coming from the student herself. Sociocultural theories have described learning in different ways. One of them puts the emphasis on internalization. Another one conceives of learning in terms of participation.

4.1 INTERNALIZATION

The idea of internalization was introduced by Vygotsky as a theoretical construct to account for the link between the individual and his or her environment. It constitutes one of the central ideas of the cultural-historical theory formulated in the early 1930s – although implicit versions of it can be found in earlier articles, such as the 1929 article "The cultural development of the child" (Vygotsky, 1929). In fact, the idea of internalization cannot be considered as an isolated

concept: it is deeply related to Vygotsky's own concept of human development and the role that signs play therein. Internalization makes operational another key theoretical construct of the cultural-historical theory, namely the genetic law of cultural development. The latter Vygotsky stated as follows: "Every [psychic] function in the child's cultural development appears twice: first, on the social level, and later, on the individual level" (Vygotsky, 1978, p. 57). Internalization as a process mediated by signs is precisely what ensures the passage from the social to the individual level: "The internalization of cultural forms of behavior involves the reconstruction of psychological activity on the basis of sign operations" (Vygotsky, 1978, p. 57).

A detailed exposition of Vygotsky's cultural-historical theory is beyond the scope of this article, as is a historical reconstruction of its main concepts. I will therefore limit my discussion here to note that, for several reasons, the theory has been subjected to criticism. Some of the criticism was expressed during Vygotsky's time while some appeared in more recent years. Some, of course, resulted from blatant misunderstandings of Vygotsky's work, the most common misreading being the interpretation of Vygotsky's theory as a theory of direct cultural transmission. Commenting on this popular and unfortunate misinterpretation, Lave and Wenger said: "learning as internalization is too easily construed as an unproblematic process of absorbing the given, as a matter of transmission and assimilation" (Lave & Wenger, 1991, p. 47). Yet, already in his early work, Vygotsky claimed a crucial role for the student in her own learning. Thus, the old pedagogy, he said "treated the student like a sponge which absorbs new knowledge." Elaborating on the idea in more detail, he claimed that "the assumption that the student is simply passive… is the greatest of sins, since it takes as its foundation the false rule that the teacher is everything and the student nothing" (Vygotsky, 1926/1997, p. 48). The teacher, Vygotsky insisted, cannot be pictured as injecting knowledge into the student's mind. Thus,

> *Just as a gardener would be acting foolishly if he were to try to affect the growth of a plant by directly tugging at its roots with his hands from underneath the plant, so is the teacher in contradiction with the essential nature of education if he bends all his efforts at directly influencing the student.*
>
> (Vygotsky, 1926/1997, p. 49)

The teacher, for Vygotsky, had an important and delicate role to play in the classroom in guiding and directing the student's environment. These ideas were developed later on and culminated in Vygotsky's concept of the zone of proximal development, introduced in his opus magna, *Thinking and Speech*.

But more interesting than these misinformed interpretations are the epistemological objections that have been made to Vygotsky's theory. Thus,

casting the relationship between the individual and her context in terms of internalization can be said to still keep traces of a form of individualistic thinking that fails to resolve the famous dichotomy between the internal and the external. As Veresov asks, "Where is the difference or even the border between external and internal then?" (Veresov, 1999, p. 225).

We need to recall that Vygotsky's theory was developed as an attempt to go beyond the reflexologist and idealist research of his time. He often complained that psychology inspired by reflexology was a psychology of behaviour without mind, and that psychology inspired by subjective idealism (introspection, for instance) was a theory of the mind without behaviour. In the footsteps of Spinoza, he was trying to overcome dualist theories (theories based on two systems, the internal and the external) and to formulate a monist theory of consciousness. But this was not without contradictions. Veresov has this to say:

> *What essentially does it mean to abandon the postulate of two system existence and to what conclusions and logical effects does it lead? This logically leads to a full rejection of the idea of the existence of the internal and the external and, consequently, to the radical refusal of the concept of internalization as a mechanism of the origin of internal structures of consciousness. Actually, the concept of internalization becomes senseless in this case.* (Veresov, 1999, p. 226)

Vygotsky's last works show his effort to overcome these difficulties (in particular his search for units of analysis). I am not going to discuss this part, as my intention was to show that Vygotsky's theory, based on the idea of internalization, is not exempt from theoretical difficulties that have implications for our conceptions of learning.

4.2 PARTICIPATION

As I mentioned before, sociocultural theories have also conceived of learning and development in terms of participation. The basic idea here is that students learn as they participate in social practices. This is the idea that was put forward by Rogoff (1990), Lave (1988) and Lave and Wenger (1991), among others. In this perspective, the shift of attention moves from the psychological to the social. I consider this perspective interesting, but all the same, a fine-grained analysis of learning in classroom processes, I want to argue, cannot omit the psychological dimension.

Some colleagues, students and I have been working out another approach that takes its inspiration from the work of Vygotsky but thematizes the question of learning in ways that are different from participation and internalization. We talk about objectification. I will try to explain as briefly as possible the main ideas of this approach in the rest of the article.

4.3 OBJECTIFICATION

The starting point is the following. At birth, we all enter a world that is not only populated by concrete objects but also by systems of thinking and modes of being. Although systems of thinking and modes of being are not visible in the way that palpable chairs, cars and other material objects are, they do exist and are entangled with the material world. Systems of thinking include forms of mathematical, scientific, aesthetic, ethical, juridical, and other kinds of reasoning – that is to say, forms of reflecting about, and acting in, the world. Modes of being intimate those manners in which we come to perceive ourselves and others. To illustrate how modes of being are culturally framed, let us consider for a moment the sense of self in ancient Greece. What was understood to be a good citizen and a good person in Athens, for instance, was very different from what is understood in, say, a liberal regime today. A good Athenian citizen was expected to take part in public life and be engaged in the multiple matters of the *polis*, i.e., the community of citizens, which provided its members with a whole and encompassing sense of belonging. A good Athenian citizen was expected to run his life in accordance with standards that would be at odds with the mores and conventions of contemporary liberal regimes, their concept of self-making persons, and their cult for autonomy and individualism.

Like systems of ideas, forms of being are certainly cultural through and through. They both have been historically and culturally constituted and shape the form and content of the activities of our everyday life. This is why it might be helpful to conceive of cultures as offering models or templates of cognition and personhood to their individuals. These templates are not straitjackets – although, as history teaches us, they may become so in oppressive systems.

4.3.1 Learning and Being

For the theory of objectification, learning consists of grasping those forms of reasoning and action as well as the individual's transformative participation in sociocultural processes of agency (Radford, 2008a). The theory resists the idea that knowing and learning consist of transmitting knowledge. Or else that knowing and learning are the personal "construction" of the student. True, knowing something presupposes not only knowledge but also a knower. However, in the theory of objectification, this knower is considered to emerge in its contact with knowledge and other individuals in accordance with the modes of knowing and being that her culture makes available to her (Radford, 2008a, 2009a). How do these considerations translate into classroom practices?

Since learning cannot be reduced to the cognitive dimension, and since it also includes the dimension of the self – for knowing and being are conceived as being imbricated with each other – we seek to create the conditions of possibility

for students to become acquainted with and acquire fluency in historically constituted cultural forms of action, reflection and reasoning as they engage in complex interactional settings. We theorize the students' reflective acquaintance with cognitive historical forms of action and reasoning as processes of objectification. To investigate these processes, our research team (which includes the teachers of the classrooms in which we conduct our research) implements mathematical activities that are structured in terms of increasing conceptual difficulty. We take into consideration the fact that mathematical thinking can occur at various levels of generality. To this, we add the epistemological premise that the conceptual difficulty of the mathematical task and the semiotic systems that mediate the mathematical thinking that is thus elicited characterize those levels of generality (Radford, 2009b). Against this background, we then scrutinize the processes of objectification, i.e., the social and dynamic processes by means of which the students navigate through levels of mathematical generality. Empirically, they are investigated by studying the multi-semiotic activity that the students display. In the theory of objectification, this semiotic activity is assumed to reveal the ways in which students gain fluency in the cultural forms of mathematical thinking.

Now, the students' semiotic activity is displayed through different semiotic systems, such as language, gestures, the use of various types of mathematical sign systems (algebraic alphanumeric language, graphs, tables, signs for numbers, etc.), and other subtler semiotic resources such as rhythm. Our task is hence to account for the manner in which, through this complex and interrelated semiotic classroom activity, the students become conversant with the cultural logic behind mathematics. One of the psychological premises of the theory is that processes of meaning-making and understanding are achieved through composite meanings. For instance, language offers abstract categorical meanings while indexical, iconic, or other kinds of gestures offer analogical forms of expression. What we see when we observe the students' mathematical activity through these lenses is that mathematical meanings are forged through a subtle combination of such apparently unrelated meanings (Radford, Bardini, & Sabena, 2007; Radford, Miranda, & Guzmán, 2008; Radford, 2009c).

To address the question of being, that is, the question of subjectivity and agency, we make recourse to the idea of *subjectification* (Radford, 2008b). In practical terms, we make sure that the implementation of our mathematical activities unfolds in complex classroom social settings that allow the students to encounter other voices and understandings (Radford & Demers, 2004; Radford, 2006). Thus, classroom interaction acquires great significance. However, in contrast to other contemporary approaches in mathematics education, interaction is not considered in the theory of objectification as a "negotiation" of meanings. The idea of negotiating something is still a vestige of individualistic thinking. You can negotiate something only if you possess something. This is why it does not

make sense to negotiate something if you do not have anything with which to negotiate. We can see how the idea of negotiation of meaning is still navigating in the troubled waters of individualism while resorting to meaning in a consumerist fashion. But that is not all. From the outset negotiation assumes an antagonistic positioning between the "I" and the "non-I" (i.e., the "Other"). To pose the problem of interaction in such terms, of course, is at odds with the premises of the theory of objectification, where the pursuit of meaning is seen instead as a *joint endeavor* not only among students, but also between the students and the teacher. And this leads us to the concept of the *zone of proximal development*.

4.3.2 Zone of proximal development

Although Vygotsky's idea of the zone of proximal development (ZPD) is perhaps his most frequently used concept, it is unfortunately the least understood of all Vygotskian ideas. Usually it is quoted as the "discrepancy between a child's actual mental age and the level he reaches in solving problems with assistance" (Vygotsky, 1986, p. 187). And often, it is understood as a simple space of knowledge transmission: the space where the teacher dispatches knowledge to the student. We have already discussed why this idea is alien to Vygotsky's own ideas. In other no less unfortunate interpretations, the ZPD appears as something intrinsic to the student. Indeed, the concept of ZPD is often presented as if the student has his or her own ZPD, regardless of the sociocultural context within which he/she develops. This simplification of Vygotsky's original idea overlooks the fact that the ZPD was Vygotsky's construct to account for the problem of the relationship between instruction and development. It overlooks the fundamental insight that distinguishes Vygotsky's approach from those of others, namely that instruction leads the course of development and that such a course depends on the kind of relationship that is created between the student and her context. This is why, rather than an absolute concept, the ZPD is a relational one (see also Schneuwly, 2008). In particular, it is forged out of the interaction between students, and between the students and their teacher (Roth & Radford, 2010). The ZPD is not a static thing that belongs to one particular student but rather a social, complex system in motion.

Mathematical activities in our classroom practices revolve around the idea of ZPD. It is in those zones that learning actually occurs. The philosophy behind the idea of ZPD frames the role of both the teacher and the students. Let me explain it by making a comparison.

As mentioned previously, in constructivist theories, genuine learning has to come from the student. Otherwise the autonomous construction of knowledge is put into peril. The teacher thus has to abstain herself from giving the answer to the student or mentioning something that can insinuate those aspects of the target knowledge that the student is meant to personally construct. As Brousseau tells

us, "Between the moment the student accepts the problem as if it were her own and the moment when she produces her answer, the teacher refrains from interfering and suggesting the knowledge that she wants to see appear." (Brousseau, 1997, p. 30). The disappearance of the teacher characterizes an 'adidactic' situation. In such a situation, the student is on her own in her task to conquer knowledge.

In the ZPD the situation is exactly the opposite. To conquer knowledge, the teacher participates in a direct and active matter. Perhaps we can picture the difference by imagining a 100-meter sprint race and a rowing race (see Figure 1). In the first one, the coach does not run along with the sprinter. In the second one, the racer is not alone. The racer is part of a group. One of the team members – the coxswain – is in charge of making tactical decisions, ensuring that the team races in the right direction, and provides encouragement and motivation – hence acting very much like the sociocultural teacher, who orients the direction of the discussions, readjusts the ideas when needed, and provides feedback on the dynamics of the class. The previous comments do not mean that there are no adidactic situations in the sociocultural classroom practice. They exist, but as warm ups: the teacher lets the students go as far as they can by themselves. The students' responsibility is to row as hard as they can. The teacher helps the students go farther still.

As can be seen from the previous remarks, the structure and the dynamics of a sociocultural classroom inspired by the theory of objectification are driven by conceptions of learning and being that are different from the conceptions adopted by other theories.

Figure 1. To the left, a 100-meter sprint race. The sprinter runs towards the finishing line on her own. To the right, a rowing team and its coxswain. The coxswain (to the right) fully participates in the race, orienting the boat trajectory, providing feedback on the team's performance, and making tactical decisions.

The question for us does not really concern having the students producing knowledge in an autonomous way. On the contrary, learning is conceived of as a communal event. Thus, instead of encouraging the students to make personal constructions, we encourage them to talk to each other, to try to understand other viewpoints, to encounter other voices, and to build joint projects (Radford, 2008b). We also encourage them to become critical towards themselves and the mathematical ideas circulating in the classroom.

Thus, in one Grade 3 class the students were invited to invent a story-problem and solve it in small groups using algebraic techniques. When they were done, each group was associated with another group (Group 1 with Group 2; Group 3 with Group 4, etc.). Then, the teacher invited the groups to exchange their solutions. The activity sheet of Group 1 was sent to Group 2, and vice-versa, etc. The students were asked to study the other group's solution and to try to make sense of it. They were also asked to identify the ideas with which they agreed and those with which they did not agree. After this, Groups 1 and 2 got together and, in turn, presented their agreements and disagreements to the other group. Learning to disagree is one of the values that we promote. Disagreeing entails the development of cultural forms of mathematical argumentation, justification, and reasoning. Let us note that the teacher is not shy of becoming involved in those discussions, if the students find themselves in a dead-end. For, in the end, what has been learned is not the property of this or that student or the teacher. Learning has been weaved in with the voices, perspectives, agreements and disagreements of all the members of the classroom community – to which, of course, the teacher entirely belongs.

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