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# Book Review: Classroom Interaction: Why is it Good, Really? Baruch Schwarz, Tommy Dreyfus and Rina Hershkowitz (Eds.) (2009) *Transformation of knowledge through classroom interaction*

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## 1 Introduction

Interaction is one of those topics that captures and reveals very well recent theoretical differences and paradigmatic shifts in mathematics education. Thus, if one were to understand the evolution of, and new trends in, mathematics education, conceptions about interaction are worth taking into account. However, it would be misleading to think that interaction has been a research item since the origins of our discipline. Mathematics Education was born at the dawn of the twentieth century out of two perspectives that pursued different goals. The first was constituted by a *humanist* view of mathematics; the second by a *social* view. The humanist view of mathematics emphasized the role this discipline plays in the development of logical thinking, abstraction, and rigor. The social trend stressed the applicability of mathematics in light of a world that was heading towards massive industrialization and technological progress (Radford 2004). The central focus of these foundational trends was curricular innovation—for example, the introduction of new concepts (such as functions) and the search for suitable ways in which to present them. Classroom interaction was not on the research agenda.

A quick look at the first issues of *Educational Studies in Mathematics* shows clearly that, in tune with the key ideas of modernity and the role that mathematics came to play therein, one of the main foci was the question of the applicability of mathematics. Another

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important focus, also a token of the modern spirit, was the design of teaching situations allowing the students to grasp the structural nature of mathematics epitomized in Bourbaki's work.[1] In the 1970s and 1980s, Piaget's work became more and more influential. Yet its epistemology, centered as it was in the relationship between the knowing subject and the object of knowledge, did not translate directly into the devising of teaching and learning practices at the heart of which social interaction could be found. For classroom interaction to gain some interest in mathematics education, a drastic shift in the manner of conceptualizing teaching and learning had to occur. It was the merit of the German social interactionism (Bauersfeld, 1980; Voigt, 1985, 1989), the North American socioconstructivism, as developed by Cobb and his collaborators (see, e.g., Cobb & Yackel, 1996), the Italian school of mathematics education (Bartolini Bussi, 1991), and the theory of didactic situations (Brousseau, 1988) to have called attention to the importance of interaction in the teaching and learning of mathematics. Naturally, the aforementioned shift was not without its own problems. It required going beyond the methodological settings of clinical interviews and written tests. But it also required a tremendous theoretical repositioning of mathematics education and its objects of study, in particular a reconceptualization of the role of the students and the teacher. Thus, as long as meaning was considered to come from within mathematics—that is to say, as something stable lying in the depths of mathematical structures and syntax—there was no room for interaction as a research subject. Drawing on symbolic interactionism Bauersfeld (1980, p. 32) contended, however, that "if problems and concepts become functions of the situation instead of being constant and stable, it then becomes necessary to consider the social constitution of meaning, i.e., the constitution of meaning through human interaction." Bauersfeld's claim was consistent with the burgeoning educational research of the time, which was preoccupied with understanding and developing collaborative or cooperative learning settings in the classroom (e.g., Johnson & Johnson, 1980; Slavin, 1980). Now, even though interaction research was not specific to mathematics education, mathematics education researchers became interested in specific questions such as the role of language in communication, the relationship between individual and collective reflections, forms of mathematical argumentation, and the study of routines of classroom interaction. Among the publications dealing with interaction in the 1990s, let me mention the book edited by Cobb and Bauersfeld (1995) and the book edited by Steinbring, Bartolini Bussi and Sierpinska (1998). It was obvious that, since its inception, interaction was conceived in different theoretical terms. Thus, in the 1998 book, Sierpinska offered an analysis of the theoretical differences underpinning the concept of interaction in three theories: constructivism, sociocultural approaches, and interactionism (Sierpinska, 1998).

In the past few years, influenced by both the increasing use of videocameras and changes in conceptions about learning and cognition, research on interaction has come to pay more attention to theoretical and methodological questions. The latter includes more refined tools to investigate verbal and non-verbal interaction (e.g., Maschietto & Bartolini Bussi, 2009; Roth, 2010; Sabena, 2008). The former includes a reconceptualization of the relationship of the teacher and the students from perspectives that depart from individualist views of learning (Ligozat & Schubauer-Leoni, 2010; Roth & Radford, 2010). The recent book *Transformation of Knowledge Through Classroom Interaction*, edited by Schwarz, Dreyfus, and Hershkowitz (2009) picks up some of the new problems raised by the aforementioned changes. In the following section, I offer an overview of the book and comment on some of the various contributions that it makes to current research on interaction. Although I believe that there is

[1] See, e.g., the teaching design in the article of Roumanet (1968) in the first issue of ESM.

nowadays a general consensus concerning the benefits of classroom interaction, I argue that interaction is often taken to be good for diverse and sometimes opposite reasons.

## 2 Transformation of knowledge through classroom interaction: an overview

The book Transformation of Knowledge Through Classroom Interaction (hereinafter TKCI) comes from a workshop held at the Hebrew University of Jerusalem in February 2008 attended by leading scholars in mathematics and science education. The book is divided into four parts. Each part is comprised of several chapters and some include a commentary paper. The first part deals with the problem of knowledge construction in classroom interaction. The second deals with the role of the teacher in classroom interaction. Part three is devoted to the role of argumentation and dialogue in transformation of knowledge. Part four focuses on questions of methodology. There is also a fifth part that offers general reflections on the chapters from the previous four parts of the book.

As expected from a book of this type, the perspectives of the authors are varied, and so are the problems that they tackle and the tools they use to investigate them. Naturally, because of the complexity of the classroom and its phenomena, there are no clear topical frontiers between the five parts of the book. It is hence not surprising that a same problem (such as the role of the teacher or the role of artifacts) appears in more than one part.

The book makes various contributions to current research. I will focus on four: (1) the development of methodologies and theoretical tools to investigate interaction; (2) the role of the teacher; (3) the purpose of classroom interaction and dialogue, and (4) the relationship between thinking and speech.

### 2.1 Methodological and theoretical innovations

At different levels, the chapters of the TKCI book introduce innovative methodological tools and theoretical constructs to investigate classroom interaction. This is the case of, for example, Schwarz, Dreyfus, and Hershkowitz's chapter where a detailed presentation of the nested epistemic actions model for studying abstraction in context is discussed. The model is based on some constructs (Recognizing, Building-With, and Constructing plus Consolidating) that are illustrated through concrete and well-refined classroom episodes. The analyses presented suggest that the genesis of abstraction stems from a "need" that can only be fulfilled through the appearance of a new construct. The origins of this need, the authors argue, have to be explored further and may be related to dialogical, dialectical, and motivational processes. This chapter is a good representation of the theoretical and methodological refinements surrounding current interaction research. So is the chapter written by Tiberghien and Malkoun, where the authors introduce a sophisticated methodological approach based on the idea that "a classroom is an 'entity' viewed as a community of practice where two joint actions take place: *teaching and learning within a communicative process*" (p. 43; emphasis as in the original). Tiberghien and Malkoun's approach is underpinned by the ecological assumption that knowledge "lives" within groups of people. They offer a distinction between types of knowledge. "Taught knowledge" is the knowledge specific to a classroom; "knowledge to be taught" is knowledge conveyed by the curriculum; "scientific knowledge" is the knowledge that lives in the scientific community. Against this theoretical background and a division of scales that possess their own "granularity", the authors present an analysis of knowledge development from the students' perspective and from the perspective of "taught knowledge".

The refinement of methodological tools and theoretical constructs can also be observed in the other chapters of the book. Unfortunately, I cannot enter here into details and do justice to each of the contributions. Let me hence mention, by way of example, the chapter by Saxe, Gearhart, Shaughnessy, Earnest, Cremer, Sitabkhan, Platas, and Young, where a methodological framework for studying the "travel of ideas" in classroom communities is presented. The authors argue that "investigating the emergence, travel and transformation of ideas is essential to understanding learning and teaching in inquiry-oriented classroom communities" (p. 203). Their approach is rooted in the socioconstructivist assumption that by working both with others and individually, "students develop understanding as they produce, coordinate and adapt representations … to serve mathematical functions … in communicative and problem-solving activities" (p. 204). They present a six-phase lesson model that includes independent (i.e., individual) work, small-group discussions, and teacher-orchestrated discussion of solutions.

Readers interested in methodological questions will certainly appreciate Rina Hershkowitz's contribution, where the distinction between theoretical frameworks and methodologies is addressed. Readers will also welcome the authors' efforts to show the articulation between their theoretical principles and their methodologies. For example, in the Saxe et al. chapter, the traveling of ideas is considered according to the well-known socioconstructivist understanding of cognition as an adaptive endeavor that in the pedagogical design translates into the creation of a space for both individual and group work. In Tiberghien and Malkoun's chapter, the methodology follows a well-established tradition according to which collective and individual processes can be disentangled in order to be studied. Based on this assumption, their methodology leads in the end to the question of the articulation of collective and individual perspectives—a legendary difficulty with which socioconstructivists have also been confronted. A question that arises in this context is the following: How is a meaningful integrative synthesis of analytic elements possible when the classroom is, as they contend, an *entity*? The practical articulation of various processes poses a delicate question indeed. As they acknowledge, "Studying the articulation between the collective and individual processes that happen in the classroom is still an open question" (p. 44). In his chapter in the last part of the book, Michael Roth mentions some of the difficulties attached to this problematic issue (what he calls the passage "from units of analysis to the analysis of units"). Of course, the question is not merely methodological: It is not really about how to investigate the individual or the social in classroom interaction but rather about how realistic the assumptions underlying such a separation are.

Let me move now to the role of the teacher in classroom interaction.

## 2.2 Group work and the role of the teacher

Contemporary pedagogies in mathematics education encourage more and more group work and communication. Indeed, these two elements are often taken as tokens of the reformed approaches that emerged in the 1980s against the traditional settings of direct teaching. It was precisely against direct teaching that the North American constructivism and the French theory of didactic situations were shaped (see, e.g., Brousseau, 1986; Cobb, 1988). Yet, group work is not necessarily enough to ensure that learning will happen. In the opening chapter of Part 2 of the TKCI book, entitled "Expert support for group work in elementary science: The role of consensus," Christine Howe notes

> no matter what its significance, group work among children will never be sufficient to deliver the science curriculum. Children working with each other are not going to construct Newton's laws or Darwin's theory of evolution, nor, given the difficulties that adults are known to experience (Dunbar & Fuselgang 2005; Kuhn, Amsel, &

> O'Loughlin 1988), are they going to master the full intricacies of hypothesis testing. Expert guidance will be required in addition, primarily from teachers but perhaps also from computers. (p. 93)

Howe's remark is at the heart of one of the most delicate conceptual problems of interaction. It questions the idea of the possibility of children attaining cultural knowledge through their own deeds. To grasp the scope of Howe's remark it may be worth recalling that in neo-Piagetian theories, knowledge is the result of the child's interaction with the environment. Knowledge results from the child's pragmatically adapting his/her own deeds to the surroundings or, as Saxe et al. put it, knowledge is constituted as the individuals "produce, coordinate and adapt representations" (p. 204).

Now, how can we be sure that interaction (conceived of as driven by adaptation) will end up in the curricular target mathematical knowledge (the "knowledge to be taught" in Tiberghien and Malkoun's terms)? In other words, how can we ensure that the idiosyncratic knowledge resulting from the students' interactive adaptation will converge with the scientific knowledge? Piaget himself was confronted with this question by René Thom in the famous Piaget–Chomsky debate (Piattelli-Palmarini, 1982). Piaget's unconvincing response rested on the allusion that all knowledge starts from practical actions carried out on a biological basis common to the human species, a response that goes back to his co-authored book with logician Evert Beth (Beth & Piaget, 1966):

> From the beginning, mathematics thus seems to be regulated by internal laws and to escape the arbitrariness of individual wills … it starts not *ex nihilo* but from a system of schemes of action, the roots of which must undoubtedly be sought in the nervous and biological organization of the subject. (p. 238)

As we can see, to the dismay of Glasersfeld, the answer supposes that there is a kind of "natural" evolution that will lead us to the same concepts and a same conceptual world—what Glasersfeld calls Piaget's "metaphysical realism" (von Glasersfeld 1988, p. 27). In his book, *The Future of Education,* Piaget (1973) said:

> if mathematics teachers would only take the trouble to learn about the "natural" psychogenetic development of the logico-mathematical operations, they would see that there exists a much greater similarity than one would expect between the principal operations spontaneously employed by the child and the notions they attempt to instil into him abstractly. (p. 18)

This "naturalistic" assumption, needless to say, is what justifies Piaget to talk about the allegedly *a*cultural and *a*historical universal mechanisms of knowledge production (coordination, assimilation, and equilibration) and what he called the "universal or epistemic subject" (Beth & Piaget, 1966, p. 238).

Of course, the idea of a natural convergence toward science has been questioned not only by educators but also by cross-cultural psychologists and anthropologists (see, e.g., Lizcano, 1993; Lloyd, 2006; Thune, 1978). Social, cultural, and historical factors have come to be considered as playing an ampler role than the limited one they usually play in neo-Piagetian and individual-centered approaches, where, as Jack Martin (2004) notes:

> When social factors are considered they mostly are framed as variables that mediate or influence what is predominately a highly individualistic pattern of development. There certainly is little here that might speak to the possible socio-cultural, political, and moral constitution of personhood. The central concern is for

> an individual actor capable of simultaneous action and reflection on this action, much like a stereotypic scientist in close scrutiny and judgment of experimental phenomena of interest. (pp. 193–194)

In neo-Piagetian approaches to interaction, the environment is reduced to "a facilitative grooming," an instrumental space that helps the individual "to become more fully socialized and intellectually engaged" (Martin, 2004, p. 197). What is missing in this account, Martin argues, is the critical inception of culture. For "human cultures are much more than the sum of immediate interactions. Cultures are not merely influential; they are constitutive" (p. 194).

Now, if we cannot rely on children's group work as a road that will ineluctably lead to complex cultural forms of knowledge, such as Darwin's theory of evolution, as Howe argues, the question is: How to frame the role of the teacher? This point is one of the most controversial in conceptions of interaction. It is controversial in that the role that we ascribe to the teacher depends on the idea that we convey of the student. In turn, the idea that we convey of the student is related to the ontological question of the *nature of the self*.

For Piaget and the educational theories that he inspired the self is considered as an autonomous agent. This is why, Piaget argued, education should not be about being told, but about re-constructing or re-discovering (Piaget, 1973). Within this context, the usual answer to the question of the role of the teacher is often given in terms of a guide of students' ideas. Yet, this way of theorizing the students and their relationship to knowledge puts the teacher in a difficult juncture that has been clearly pointed out by Brousseau (1997) in addressing the role of the teacher in the theory of didactic situations (TDS). Brousseau starts from a conception of cognition that is thematized along the lines of Piaget's genetic epistemology. And although Brousseau conceives of adaptation as driven not by urges of a biological type but by optimizing needs (in a mathematical sense), he dives far enough into Piaget's constructivist waters to inherit the problems that come along with its idea of the autonomous student. Thus, talking about the teacher, Brousseau says:

> everything that she [the teacher] undertakes in order to make the student produce the behaviours that she expects tends to deprive this student of the necessary conditions for the understanding and the learning of the target notion; if the teacher says what it is that she wants, she can no longer obtain it. (p. 41)

For knowledge is something that has to be constructed by the student. This way of theorizing the student–teacher interaction is further explained by Kidron, Lenfant, Bikner-Ahsbahs, Artigue and Dreyfus (2008) as follows:

> according to the TDS, significant mathematical learning cannot be achieved if the students' work is too much dependent on the teacher. This basic assumption is embedded in the TDS through the notions of *adidactical* [or a-didactical—LR] *situation* and *milieu*. In a-didactical situations (an ideal type) students are expected to be able to test, reject, progressively adapt and refine their models and solutions thanks to the potential offered by the milieu of the situation in terms of action and feedback, without relying on the teacher's guidance, and without trying to guess the teacher's expectations. (p. 248)

Becoming too dependent on the teacher appears to run against the idea of the students' autonomy. The a-didactic situation is indeed the space *par excelle*nce where the student can and should show her autonomy *vis-à-vis* the object of knowledge. This does not mean that in the TDS the teacher is evacuated from the arena of knowledge production. The teacher is

active within the limits allowed by the student's autonomy and comes certainly to the fore in the phase of knowledge institutionalization—that is, the phase where the teacher makes the student aware of the conceptual constructions they have accomplished.

Within this general context informed by Piaget's epistemology, it is hence not surprising that, with the exception of the TDS (see e.g., Brousseau, 1986; Hersant & Perrin-Glorian, 2005; Margolinas, 2005), little theorizing has been done on the role of the teacher. As Schwarz et al. remark in their contribution to the TKCI book, in a passage where they refer to research on abstraction but that is true of research in general,

> Literature on research in mathematical abstraction has traditionally avoided treating the role of the teacher in guiding processes of abstraction … The tradition is understandable since researchers in this field have generally belonged to a constructivist approach according to which high-level processes involve autonomy. (p. 38)

Before we move forward, it might be worth asking why many contemporary educational approaches hold so strongly to the idea of autonomy. By asking this question I do not mean to dismiss autonomy as an important element in the education of children. The challenge is rather to realize that foundational ideas in education do not come out of the blue and that perhaps there are also other ways in which to conceive of the teacher–student relationship and of the role of the teacher more generally. In this line of thought, let me recall that autonomy has come to be positioned as a cornerstone of self-perceptions and perception of others through a lengthy process that involves cultural, political, and economical spheres. Autonomy was the chief feature that characterized the individual in the philosophy of the Enlightenment—this historical–cultural movement where for the first time in Western history the individual assumed himself/herself as a reflective and autonomous being. This movement was one of rupture with medieval and ensuing cultural formations where individuals and what they knew (truth) depended on the webs of power established by a hierarchical monarchical society. The Enlightenment portrayed a new idea of the individual—it conceived of it as a rational agent whose actions and projects of life could only be authentic if they came from within. Autonomy came to be understood as the capacity for doing something without the recourse to others. The idea of autonomy was a central thread in the shape of the modern self and its relationships to others. It has had an important impact in education, particularly in the idea of learning as an autonomous endeavor and in conceiving of the ensuing teacher–student interaction. In other theoretical paradigms, such as in Vygotskian research, autonomy is not the pre-condition of learning. Autonomy is the outcome. In this paradigm autonomy is understood in a broader sense, not as an "individual property", but something that includes a responsible engagement in sociocultural projects and collective decisions. Such a concept allows one to envision a different role for the teacher and new forms of student–teacher interaction.

In some chapters of the book, the authors turn to Vygotsky's work. Yet, the ghost of the intrusive and authoritarian teacher transmitting knowledge haunts many of the chapters. Thus, in summing up her theoretical position, Howe says:

> In general then, the research points to a theoretical position that is a rapprochement of Piagetian and Vygotskian thinking, but goes beyond this thinking in several respects. Social products are emphasized, but children are viewed as responding to these products actively rather than passively assimilating them. (p. 102)

Yet it was Vygotsky himself who argued against a transmissive view of teaching and a passive view of learning (Radford, 2010). In his first book, *Educational Psychology,* he

argued that "Strictly speaking … [i]t is impossible to exert a direct influence on, to produce changes in, another individual" (Vygotsky, 1997a, p. 47). He complained that "the old pedagogics … treated the student like a sponge which absorbs new knowledge" (p. 48). Vygotsky came back several years later to question of the relationship between the teacher and the student, when he devised the concept of zone of proximal development. Under individualistic interpretations, the zone of proximal development has been seen as a space of potentialities *possessed* by the individual. What is wrong with this interpretation is not only that it loses sight of the social, cultural, relational and contextual nature of interaction and development, but also that it ignores the fact that the zone of proximal development is a space of production of subjectivities: It is not only an interactional zone allowing the students to become familiar with historically and culturally constituted forms of thinking and being but also a zone of novelty, conflict and resistance.

Unfortunately, the view of children passively assimilating knowledge has become a stamp of Vygotsky's work. It may be a token of the manner in which mathematics and sciences education have evolved along with the enduring individualist forms of theorizing that they are attempting to overcome.

### 2.3 Dialogue, communication, and thinking

In chapter 12 of the book TKCI (a chapter entitled "A dialogue on dialogue and its place within education"), Rupert Wegerif engages in a discussion with Paolo Boero, Jerry Andriessen, and Ellice Forman, and goes on to claim that "dialogue should be treated as an *end-in-itself*" (p. 184), and that teaching should be "teaching for dialogue" (p. 185). He argues that

> to be more dialogic is not necessarily to be more productive in constructing knowledge but it is to be more open to other voices, more able to question and to listen and so more able to allow new unanticipated meanings to emerge. (p. 185)

Such a position, Andriessen replies, is problematic insofar as it cannot ensure that oppositions would eventually be bridged. In this context "dialogue sounds like people in trenches throwing ideas to each other" (p. 185). Wegerif's position rests on a contemporary (post-modern) view of dialogue not as *consensus* but as *difference*. Yet, to make differences meaningful and not merely the dialogical impasse of the entrenched people voicing meaningless opinions to each other, Boero suggests three mechanisms that may promote "productive dialogue" (p. 188): (1) evolution of personal interpretation of the situation; (2) from a situation to the opposite one, to a wider perspective; and (3) from single case to generalization. These mechanisms of classroom dialogue do not evolve as they are moved by natural causes but rather cultural ones, the teacher carefully becoming involved with the students' contributions. Although Wegerif's proposal is attractive in some respects, it still has to address many questions, such as the relationship between dialogue and disciplinary knowledge. It is indeed Forman who reminds us that it might be more accurate to talk about teaching for *disciplinary dialogue* instead of teaching for dialogue *tout court*. In a different vein, since not all verbal exchanges qualify as dialogue, Andriessen asks the question of what it takes for two participants to enter into dialogue. Equally important, he asks: "what does such dialogical thinking look like, how do we define it relative to different activities and practices?" (p. 191). Andriessen's question is of course of interest to mathematics educators, as it suggests that thinking is not only dialogical (and hence related to language) but it may acquire different forms as both the context and the practice of dialogue change. Anna Sfard's enlightening commentary paper on Part 3 of the book

answers Andriessen's question in part. She suggests that thinking can be defined as "the individualized form of the activity of communicating" (p. 174). This definition asserts that social relations precede cognitive processes and leads us back to the relationship between thinking and speech. Of course, speech is not the only form of human communication. Communication is more general and includes forms of non-verbal communication largely used by animals and humans alike. In her book, Sfard (2008, pp. 86–87) defines communication as a performed patterned activity in which an action A belonging to a certain well defined communicational repertoire of an individual is followed by action B of another individual (belonging to a communicational repertoire of reactions). In this sense, if I apply this definition to two computers playing chess, I can imagine them drawing on communicational repertoires of actions and reactions. The computers would be communicating. And they would even be thinking. Their corresponding programs are individualized forms of the communication activity of chess players. Yet, curiously, the computers do not know that they are playing chess! This is why IBM supercomputer Deep Blue still does not know that it defeated world champion Garry Kasparov in 1997. What Deep Blue lacks is a *reflective* dimension that Vygotsky considered to be at the heart of thinking and consciousness. In contrast to Deep Blue, Kasparov was conscious of the defeat and in the famous movie *Game Over* (Yayanti, 2003) offered some explanations for it. What this example suggests is that speech, and communication more generally, is crucial in the development of thinking. However, there is still something underneath thinking that makes it irreducible to speech and communication; this, Vygotsky argued, is consciousness.

Let me revisit, in the next section, Vygotsky's distinction between thinking and speech and how he conceived of their relationship to consciousness.

### 2.4 Thinking and speech

The main focus of Vygotsky's lifetime work was the understanding of consciousness—since his early writings, such as his study on Hamlet (Vygotsky, 1971), to the 1925 seminal article "Consciousness as a problem in the psychology of behaviour" (Vygotsky, 1979), up to his opus magna *Thinking and Speech* written towards the end of his life (Vygotsky, 1987). It is, hence, not by chance that he closed the last chapter of this book with a discussion about the relationship between thinking, speech, and consciousness. The chapter begins with a discussion on the relationships between word and thought. Vygotsky starts by arguing that the relationships between word and thought are neither inherent nor primal and goes on to say that "On the contrary, these relationships emerge and are formed only with the historical development of human consciousness" (p. 243).

For Vygotsky (1987), thinking and speech are not reducible to each other. Their relationship is not one of coincidence but one of embodiment: "thought is connected with the word and embodied in it" (p. 244). He suggested that the movement from thought to word could be analyzed in terms of a differentiation of planes "through which thought passes as it becomes embodied in the word" (p. 149). He started by focusing on two planes: external and inner speeches. In the development of the external aspects of speech, the child, Vygotsky observed, uses single words, then moves to an articulation of two or more words, then to simple and later on more complex phrases. As Vygotsky said, in mastering the external aspects of speech, the child moves from the part to the whole. Now, the *meaning* of the child's speech undergoes a process in the opposite direction: "The semantic aspect of speech develops from the whole to the part or from the sentence to the word" (p. 250). Thus, "the child's thought emerges first in a fused, unpartitioned

whole. It is for precisely this reason that it must be expressed in speech as a single word" (p. 251). As the child's speech develops, "he can move in his thought from an unpartitioned whole to parts" (p. 251).

But the semantic plane of speech is only the first of the internal planes of speech. Beyond it, Vygotsky (1987) noted, lies the plane of inner speech. Inner and external speech are of course related. For Vygotsky, external speech "is a process of transforming thought into word." Inner speech goes somehow in the opposite direction: it "involves the evaporation of speech in thought" (p. 257). But Vygotsky argued that it was a mistake to consider inner speech as external speech minus vocalization. Their structures are deeply different. And to elucidate those differences he turned to the study of egocentric speech, which he claimed to be the "key" to the study of inner speech: "Egocentric speech is still vocal and audible. Though internal in function and structure, egocentric speech is external in manifestation" (p. 258). At this point, he argued that egocentric speech is not something that disappears with socialization, as Piaget suggested. Egocentric speech is not a form of self-centered speech but an overt communicative and interactional phenomenon that is to be transformed into inner speech—it is "one aspect of the general transition from inter-mental functions to intra-mental functions, one aspect of the transition from the child's social, collective activity to his individual mental functions" (p. 259).

Within this context, Vygotsky (1987) went on to suggest that inner speech *is* thought that is connected with the word and that "inner speech is thinking in pure meanings" (p. 280). And when all seemed to point to the equating of inner speech to thinking, he argued that a further plane had yet to be distinguished—the plane of thought itself. And between this plane and speech rests always an "uncrossable Rubicon" (p. 280). It is at this point that Vygotsky offered the famous cloud metaphor: "Thought can be compared to a hovering cloud which gushes a shower of words" (p. 281). In other terms, the connection between speech and thinking is always indirect. Inner speech is at most part of the interface between the individual's thought and external communication. For behind thought lies something that is not even made up of words or thoughts: behind thought lies consciousness.

> We must now take the final step in the analysis of the internal planes of verbal thinking. Thought is not the last of these planes. It is not born of other thoughts. Thought has its origins in the motivating sphere of consciousness, a sphere that includes our inclinations and needs, our interests and impulses, and our affect and emotion. The affective and volitional tendency stands behind thought. Only here do we find the answer to the final "why" in the analysis of thinking … A true and complex understanding of another's thought becomes possible only when we discover its real, affective–volitional basis. (p. 282)

To sum up, what Vygotsky was arguing is that speech (or communication for that matter) as an inter-personal phenomenon constitutes the social origins of egocentric and inner speeches. But speech or communication (in any of their various forms) cannot be equated to thinking itself. The question of thinking is rather a question of consciousness, something that he understood not in metaphysical terms but in concrete ones, as the subject's *reflection* of reality. And it is here that the word along with its meaning come to play a fundamental role. For "the word's distinguishing feature is a generalized reflection of reality" (p. 285). The question that Vygotsky's collaborators attempted to elucidate later on was the manner in which the forms of generalized reflection of reality that the word and communication mediate are carried out. It became clear that the question could not be answered by investigating speech itself but what Vygotsky referred to as the *functional* role of speech in the act of thinking, something that could be studied only

through the investigation of the individuals' *activity* (*Tätigkeit*) that speech mediates (Kozulin, 1986). In the hands of Leont'ev, the problem became one of the relationships of consciousness and activity. The inter-personal plane was theorized in an ampler manner, as social consciousness, which appeared as the condition for the existence of individual consciousness. Within this context, consciousness appeared not as a thing, "not a plane, nor even a volume, filled with images and processes" (Leont'ev, 1978, p. 95), but as a *relation*—the relationship of the individuals to their reality as revealed to them in the subjective form of their real life.

Drawing on these ideas I have suggested that thinking can be considered as a sign-mediated *reflection* of the world in the form of the individuals' activities (Radford, 2008). The idea of reflection in this definition corresponds to that which Vygotsky and the historical–cultural school of thought elaborated. It relates to the various ways in which we orient ourselves, interpret and act in our environment. It includes sensing, perceiving and all sensorial forms up to complex forms of psychic reflection such as practical and theoretical thought. The concept of reflection conveys the idea of the mind as a culturally evolved form of sensing and acting (Leont'ev, 1981) entangled with the emergence of consciousness, something that happened for the first time "in some specific stage of animal development," when "a qualitative change in the development of brain processes took place … [and] marked the development of a new quality that could not be mechanically reduced to more simple phenomena" (Vygotsky, 1997b, p. 113).

The idea of thinking as reflection involves consciousness at various levels. And it is because computers cannot reflect that they cannot think. This is also why they fail to know that they may be playing chess and that Deep Blue does not know what it accomplished.

Thinking as reflection stresses the fact that thinking is unavoidably mediated not only by the meaning of words and artifacts and the historical intelligence embedded in them but also by the goal and the societal motive of the activity in the context of which individuals think, feel, and act.

Now, what is the importance of thinking as reflection within the general problem of classroom interaction? The importance of the question rests on the fact that thinking is not a kind of individual cogitation. Because of its reflective, activity-meditated nature, thinking is truly social and develops in interaction only. Yet, we still have to specify the meaning of this mediation and how it relates to learning. There are several ways in which to do this. In the following section I will consider two.

## 3 Why is interaction good, really?

Interaction can be seen as providing a facilitative space of exchange. In this case, interaction appears as a *tool*—a *pedagogical instrument* that helps the teacher to create the conditions for learning to occur. It is in this sense that several contributors to the book TKCI consider it (see, e.g., the contribution of Saxe et al. or that of Sohmer, Michaels, O'Connor, & Resnick).

But interaction could also be seen as something to be promoted in order to go beyond knowledge properly. It is in this sense that it is considered by other contributors to the book TKCI. We saw, for instance, Wegerif promoting the idea of teaching for dialogue—dialogue as an "end-in-itself" (p. 184). Some recent approaches to interaction have indeed been motivated by a need to move away from perspectives on education that confine teaching to the search and implementation of *efficient* actions towards knowledge acquisition. This is an important ongoing shift in which mathematics education is no longer considered as a

kind of 'methodology'—a sort of appendix of mathematics whose goal is to ensure the diffusion and acquisition of disciplinary knowledge. In his contribution to Part 3 of the book, Baker seems to espouse this position and argues that

> the point of argumentation-based cooperative learning is not necessarily to change the students' beliefs or other attitudes, but rather to get them to broaden and deepen their views, to make them more reasoned and reasonable, to enable students to know of and understand others' views to reflect upon them and (sometimes but not always) respect them as a worthy of debate. (p. 155)

In this context, mathematics education becomes something rather more general, concerned by the mathematical content but also by more general issues of the students' education. Yet, I would argue that, although interesting, neither Wegerif's nor Baker's position go far enough. The point is neither to use dialogue as an end, a discursive device that satisfies itself with the encounter of irreducible differences, nor to enlarge our own perspective by being confronted with that of the others. Let me explain.

In the first case, I am afraid that differences may remain unquestioned, where we might lose sight of the fact that identity and differences are cultural constructs produced in social practices. The point is not to find comfort in the encounter of differences but rather to unveil the social, political, and cultural mechanisms that produce and reproduce them.

In the second case, I am afraid that conceiving of interaction and cooperative learning as a form of reflection on others' views may lead us to see differences in the unproblematic spirit of dialogue, consensus, and communication of naïve multiculturalism; it may lead us to reduce the other to an exotic instrument of our own intellectual growth. Education in general, and mathematics education as part of the general educational of students, should include questions of citizenship, agency, power, and political matters (Brown, 2010; Valero & Stentoft, 2010). This idea has been clearly expressed by Alrø and Skovsmose (2002) in their classroom approach: "We are not simply considering the most efficient way for students to come to grasp certain mathematical facts. Nor are we only considering the learning of mathematics, where the content of the learning is interpreted strictly in mathematical terms" (p. 8). Drawing on Freire's work, for them, learning must be an *empowering act*: as they contend, "we do not want to consider only mathematical concepts and techniques in isolation, we also want to include the social contexts in which they might be operating" (p. 8).

Naturally, this polemical and controversial shift that positions mathematics education *in* and *beyond* the mathematical is not limited to our field. It is rooted in a now long-standing debate concerning the goal of education and a critique of the technocratic paradigm that has dominated perspectives on education for more than a century. Thus, Canadian educator Ted T. Aoki argued that in the technocratic paradigm (the one in which interaction appears as a facilitative tool for knowledge construction), a competent teacher is reduced to

> one who has skills and techniques oriented toward efficient control. Such a know-how-to-do view of [curriculum] implementation is embedded in scientific and technological thought/action framework that reduces human competence to instrumental reason and instrumental action. Here, the teacher is seen as a rule-oriented, rule-governed being cast within a manipulative ethos, an ethos in which even his [or her] future is conceived in terms of rules. (Aoki, in Pinar & Irwin, 2005, p. 113)

It is within this debate that the question of mathematics teaching and learning has become broader. The problem has become broader as to encompass a critical positioning of

the students in the public space of the classroom, society, and scientific and mathematical practices. "Pedagogy and curriculum," argues the Brazilian educator Tadeus da Silva (2000), "should be able to offer opportunities for the students to develop capacities of critique and of questioning of the dominant systems and forms of identity and difference representation" (pp. 91–92).

But it would be a mistake to reduce the benefits of interaction to a space of communal reflection on conceptual matters (the mathematical content) and political ones (e.g., who is in power in the classroom or the sociocultural mechanisms of identity and difference production). There is still an ontological sense of interaction that we need to consider. Entangled with the conceptual and the political, interaction is the founding process of the self, the constitutive process through which I become a continuously transforming and transformed entity in the historical and cultural chain of Being. It is in the course of interaction that I come into being, but not as a substance, not in the substantive sense of *être* (to be, *essere*); rather in the sense of the never ending and always changing *étant* (*be-ing*), a *being-with-others* (Radford, 2008). Interaction is more than anything else a *call* (Lévinas, 2006), the call of the Other; and this voice, this call, is my only possibility to enter the world of beings, to distinguish and recognize myself as another, through an unsettling process of readjustments taking place in a space similar to the amorphous unpartitioned conceptual space that Vygotsky described in the emergence of thinking in the child. But here, the space is the space of Being, populated by contrasting contemporary and historical voices and presences. My becoming is not only a question of entering the cultural world of Reason. My becoming is rather a sensing that senses the world through my own body and those of others. The call of the other is a fleshy call. It is not an instrumental call: It is ethical. It means the continuous attentive turning towards the other—my *answerability*, as Bakhtin would call it. The ethical constitutive dimension of interaction is a movement of openness that entails my involvement in activity (*Tätigkeit*)—that is to say, our *communal* life, our only possibility of *be-longing*—and the attempt at achieving the unachievable empathetic understanding that involves both reason and feeling.

## 4 Summing up…

In this article, I have discussed some of the contributions that the book Transformation of Knowledge Through Classroom Interaction makes to current debates on interaction. My intention has not been to go from chapter to chapter and then to attempt to synthesize some common ideas. I opted for a more modest option—to offer a general panorama according to specific areas. I chose to focus on the development of methodologies and theoretical tools to investigate interaction; the role of the teacher, the purpose of interaction and dialogue, and, finally, the relationship between thinking and speech.

The structure of the book organizes in a clear manner the various topics addressed by the contributors. The commentaries on the various parts are very helpful in summarizing, contrasting, and challenging the ideas presented. Taken together, the chapters offer a state-of-the-art discussion of classroom interaction research. The variety of theoretical approaches is an indication, I think, of the way in which mathematics education has come to maturation. The approaches reflect the different intellectual traditions that constitute the richness of our research field. Readers interested in methodological and theoretical questions will find in the book deep insights and suggestions.

## References


Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. New York: Kluwer.

Bartolini Bussi, M. (1991). Social interaction and mathematical knowledge. In F. Furinghetti (Ed.), *Proceedings of the Fifteenth Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1–16). Assisi: PME.

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics, 11*, 23–41.

Beth, E. W., & Piaget, J. (1966). *Mathematical epistemology and psychology*. Netherlands: Reidel.

Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques [Foundations and methods of didactics of mathematics]. *Recherches en Didactique des Mathématiques, 7*(2), 33–115.

Brousseau, G. (1988). Le contrat didactique: Le milieu [The didactic contract: The milieu]. *Recherches en Didactique des Mathématiques, 9*(3), 309–336.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Brown, T. (2010). Truth and the renewal of knowledge: The case of mathematics education. *Educational Studies in Mathematics*. Springer Online First. DOI 10.1007/s10649-010-9259-0.

Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. *Educational Psychologist, 23*(2), 87–103.

Cobb, P., & Bauersfeld, H. (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale: Erlbaum.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*(3/4), 175–190.

da Silva, T. (2000). *Identidade e diferença. A perspective dos estudos culturais [Identity and difference. The perspective of cultural studies]*. Petrópolis: Editora Vozes.

Hersant, M., & Perrin-Glorian, M.-J. (2005). Characterization of an ordinary teaching practice with the help of the theory of didactic situations. *Educational Studies in Mathematics, 59*, 113–151.

Johnson, D. W., & Johnson, R. T. (1980). *Promoting constructive student–student relationships through cooperative learning*. Minneapolis: Minnesota University (ERIC Document Reproduction Service No. ED 249 216).

Kidron, I., Lenfant, A., Bikner-Ahsbahs, A., Artigue, M., & Dreyfus, T. (2008). Toward networking three theoretical approaches: the case of social interactions. *ZDM—The International Journal on Mathematics Education, 40*, 247–264.

Kozulin, A. (1986). The concept of activity in Soviet psychology: Vygotsky, his disciples and critics. *American Psychologist, 41*(3), 264–274.

Leont'ev, A. N. (1978). *Activity, consciousness, and personality*. Englewood Cliffs: Prentice-Hall.

Leont'ev, A. N. (1981). *Problems of the development of the mind*. Moscow: Progress.

Lévinas, E. (2006). *Totalité et infini. Essai sur l'exteriorité [Totality and infinity. Essai on exterioriry]*. Paris: Le livre de poche.

Ligozat, F., & Schubauer-Leoni, M.-L. (2010). The joint action theory in didactics: Why do we need it in the case of teaching and learning mathematics? In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the 6th Conference of European Research in Mathematics Education (CERME 6)* (pp. 1615–1624). Lyon: Institut National de la Recherche Pédagogique.

Lizcano, E. (1993). *Imaginario colectivo y creación matemática [Collective imaginary and mathematical creation]*. Barcelona: Editorial Gedisa.

Lloyd, G. E. R. (2006). The use and abuse of classification. In D. R. Olson & M. Cole (Eds.), *Technology, literacy, and the evolution of society* (pp. 141–164). Mahwah: Erlbaum.

Margolinas, C. (2005). La dévolution et le travail du professeur [The devolution and the work of the teacher]. In M.-H. Salin, P. Clanché, & B. Sarrazy (Eds.), *Sur la théorie des situations [On the theory of situations]* (pp. 329–333). Grenoble: La pensée sauvage.

Martin, J. (2004). The educational inadequacy of conceptions of self in educational psychology. *Interchange: A Quarterly Review of Education, 35*, 185–208.

Maschietto, M., & Bartolini Bussi, M. G. (2009). Working with artifacts: gestures, drawings and speech in the construction of the mathematical meaning of the visual pyramid. *Educational Studies in Mathematics, 70*(2), 143–157.

Piaget, J. (1973). *To understand is to invent. The future of education*. New York: Grossman.

Piattelli-Palmarini, M. (Ed.). (1982). *Théories du langage, théories de l'apprentissage: le débat entre Jean Piaget et Noam Chomsky [Theories of language, theories of learning: the debate between Jean Piaget and Noam Chomsky]*. Paris: Seuil.

Pinar, W. F., & Irwin, R. L. (Eds.). (2005). *Curriculum in a new key: The collected works of Ted T. Aoki*. Mahwah: Lawrence.

Radford, L. (2004). From truth to efficiency: Comments on some aspects of the development of mathematics education. *Canadian Journal of Science, Mathematics and Technology Education / Revue canadienne de l'enseignement des sciences, des mathématiques et des technologies, 4*(4), 551–556.

Radford, L. (2008). The ethics of being and knowing: Towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in mathematics education: Epistemology, history, classroom, and culture* (pp. 215–234). Rotterdam: Sense.

Radford, L. (2010). The anthropological turn in mathematics education and its implication on the meaning of mathematical activity and classroom practice. *Acta Didactica Universitatis Comenianae—Mathematics, 10*, 103–120.

Roth, W.-M. (2010). Incarnation: Radicalizing the embodiment of mathematics. *For the Learning of Mathematics, 30*(2), 8–17.

Roth, W.-M. & Radford, L. (2010). Re/thinking the Zone of Proximal Development (Symmetrically). *Mind, Culture, and Activity*, 17(4).

Roumanet, A. (1968). Une expérience d'enseignement de mathématique avec des enfants de 11 à 13 ans [A mathematics teaching experience with 11–13-years old children]. *Educational Studies in Mathematics, 1*(1–2), 222–236.

Sabena, C. (2008). On the semiotics of gestures. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in mathematics education: Epistemology, history, classroom, and culture* (pp. 19–38). Rotterdam: Sense.

Schwarz, B., Dreyfus, T., & Hershkowitz, R. (Eds.). (2009). *Transformation of knowledge through classroom interaction*. New York: Routledge.

Sfard, A. (2008). *Thinking as communicating*. Cambridge: Cambridge University Press.

Sierpinska, A. (1998). Three epistemologies, three views of classroom communication: Constructivism, sociocultural approaches, interactionism. In H. Steinbring, M. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 30–62). Reston: The National Council of Teachers of Mathematics.

Slavin, R. E. (1980). Cooperative learning. *Review of Educational Research, 50*, 315–342.

Steinbring, H., Bartolini Bussi, M., & Sierpinska, A. (1998). *Language and communication in the mathematics classroom*. Reston: National Council of Teachers of Mathematics.

Thune, C. E. (1978). Numbers and counting in Loboda: An example of a non-numerical oriented culture. *Papua New Guinea Journal of Education, 14*, 69–80.

Valero, P., & Stentoft, D. (2010). The 'post' move of critical mathematics education. In A. O. Ravn & P. Valero (Eds.), *Critical mathematics education: Past, present, future* (pp. 183–195). Rotterdam: Sense.

Voigt, J. (1985). Patterns and routines in classroom interaction. *Recherches en Didactique des Mathématiques, 6*(1), 69–118.

Voigt, J. (1989). The social constitution of the mathematics province—a microethnographical study in classroom interaction. *The Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 11* (1&2), 27–34.

von Glasersfeld, E. (1988). Introduction à un constructivisme radical [Introduction to a radical constructivism]. In P. Watzlawick (Ed.), *L'invention de la réalité [The invention of reality]* (pp. 19–43). Paris: Éditions du Seuil.

Vygotsky, L. S. (1971). *The psychology of art*. Cambridge: The M.I.T. (First published in 1925).

Vygotsky, L. (1979). Consciousness as a problem in the psychology of behavior. *Soviet Psychology, 17*(4), 3–35.

Vygotsky, L. S. (1987). Thinking and Speech. In R. W. Rieber & A. S. Carton (Eds.), *Collected works of L. S. Vygotsky, Vol. 1* (pp. 39–285). New York: Plenum.

Vygotsky, L. S. (1997a). *Educational psychology.* Boca Raton: St. Lucie.

Vygotsky, L. S. (1997b). Mind, consciousness, the unconsciouss. In R. W. Rieber & J. Wollock (Eds.), *Collected works of L. S. Vygotsky* (Vol. 3, pp. 109–119). New York: Plenum.

Yayanti, V. (2003). *Game over: Kasparov and the machine*. Montreal: Alliance Atlantis.