

## **On Signs and Representations A Cultural Account**

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l'écriture, la lettre, l'inscription sensible ont toujours été considérées par la tradition occidentale comme le corps et la matière extérieurs à l'esprit, au souffle, au verbe et au logos.

J. Derrida, *De la grammatologie*, 1967, p. 52.

### **1. The ghost of the metaphysics of pre-sence**

One of the oldest discussions on the relationships between language and ideas is found in Plato's dialogue *Cratylus*.

In this dialogue, Plato deals with several conceptions of language. One of them is defended by Hermogenes, the poor brother of the rich Callias, who claims that names and language are merely *conventional* —like, he says, the names of slaves, that may be given and changed at pleasure. Another conception of language is held by Cratylus, who maintains that there is a perfect match between the things and their names. The name or the sign of a thing, according to Cratylus, discloses or uncovers the true nature of the thing. In fact, Cratylus goes further, for, in a certain passage of the dialogue, he affirms that all truth and knowledge derive from language and names. Then, Socrates, with his usual subtle spirit of controversy, replies that if knowledge comes from names, then the names must have preceded the things. "But", he adds conclusively, "how could he [who gives names to things] have learned or discovered things from names if the primitive names were not yet given?" (*Cratylus*, 347b).

The crucial point in Socrates' argument against Cratylus' alleged epistemological link between names and things is, of course, the prior existence of "the first things" over their names, and is embedded in a larger problem, namely, the problem of the relation between the signifier and the signified, a problem that goes back to earlier times. It is very well known that names, in many pre-Hellenic cultures, frequently had a magical power that allowed those who possessed the knowledge of the names to invoke the signified. The

descendants of Abraham were seen by their neighbours as a somewhat extravagant people for having a God with an inutterable name.

However, the problem of the relation between the signified and the signifier is not a specific problem of Antiquity and its sources. It is present in some contemporary communities with tribal modes of social organization<sup>1</sup>, and —with all its variations and disguises— has unrelentlessly haunted the whole tradition of Western thought, where the signifier has been considered as the the external body of a transcendental essence, as the external matter to the spirit, to the breath and to logos, as said in the quotation that this paper bares in epigraph. For instance, the 16th and 17th centuries quest for a Universal Language —a European quest, in which Leibniz was actively involved, that unfolded in the flow of the discoverings of Chinese writing, "Mexican paintings" and Egyptian hieroglyphs— was underpinned by the central idea of a language capable to go beyond the vicissitudes of the phemeral and noisy speech and to accomplish a direct and strict adequation between thought and the silent signs (V.-David 1965, p. 35). This idea was taken up by Frege, two centuries later. Thus, in an article published in 1882, he stresses the importance of a language capable of *writing* concepts without going through the non logical nature of speech. He found, in the formulas of Arithmetic, an illustrious example of such a "language" —a "language" that he called "*Begriffsschrift*" (from Begriff, concept, and Schrift, writing), and that would be translated as "ideography".<sup>2</sup>

This point is not foreign to some recent trends in linguistics and psychology. It is very hard not to recognize a modern version of Leibniz's and Frege's ghosts walking around Chomsky's generative grammars and those supposedly "deep structures" responsible for the "surface structures" of language. Indeed, generative grammars, although recognizing local differences from one language to another, postulate a *universal* kernel capable of being described in terms of formal rules from where the language under consideration would be generated. In their own way, some psychologists have been using models of the mind, that have generated an important amount of work in Mathematics Education, based on an apparently harmless distinction between "internal" and "external" representations whose parallelism with the "deep" and "superficial" linguistic structures would hardly be a mere coincidence.

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<sup>1</sup> For the Murngin of northern Australia, the act of naming is often seen as a form of creation (see Shore 1996, chapter 9). Other interesting examples in Lévi-Strauss (1955, 1962).

<sup>2</sup>"Le langage par les formules de l'arithmétique est une idéographie (*Begriffsschrift*) puisqu'il exprime immédiatement la chose sans passer par les sons." (Frege 1971, p. 68)

Let it be as it may, the relationships between the signifier and the signified is still a delicate problem. Of course, our cultural rejection of Socrates' idea as expressed in *Cratylus*, amounts to new explanations about that whose absence the sign is supposed to replace. Thus, in a book, edited by C. Janvier (1987), *Problems of Representation in the Teaching and Learning of Mathematics*, which undoubtedly still remains one of the most important works on the subject, several concepts are displayed dealing with what the representation is representing. Mason (1987, p. 75) makes use of "vestiges of complex inner experiences", DiSessa (1987, p. 84) refers to "phenomenological primitives", Glasersfeld (1987, p. 222) has recourse to "segments of experience" and Kaput (1987, p. 163) uses "phenomenological object".

Kaput's chapter "Towards a Theory of Symbol Use in Mathematics" (1987) is particularly interesting for our cultural discussion about signs, for such a chapter devotes a section to what its author calls "shared symbols". Shared symbols, according to Kaput, carry on a dual existence: a public and a cognitive. "This duality", he says, "reflects the dual, but intimately related, uses of mathematical symbols—one for communication and the other as an assist to cognitive processing." (1987, p. 172).

Unfortunately, the question of the "dual, but intimately related" life of symbols is no longer taken up again, leaving us without knowing what kind of intimacy Kaput had in mind.

Kaput's emphasis on the cognitive role of signs in detriment to the communicative one can hardly be imputed to an innocent bias. The reason, I would like to submit, is a widely shared belief according to which the "communicative" or any other "social" aspect of signs in cognition have been taken as peripheral in our classical constructivist individual-centered accounts of knowledge formation.

However, if knowledge formation is seen from a broader social context, the use of signs becomes a central cultural element in cognition. An example of such an approach is presented in the next sections.

## **2. Signs and Activity**

The main problem in developing a cognitive theory of signs within a non-cultural approach is, I believe, the link of the individual's cognitive activity to the individual's own ecological system. Probably, the only available path to attempt such a link is the one that Kaput tried, that is, the *communicative aspect* of the individuals' activity. However, since communication is not the main space of knowledge in individual-centered accounts of

cognition formation, its epistemological role is greatly compromised: communication appears as a secondary space where individuals doublecheck (or "corroborate", to use Glasersfeld's expression) if things are going okay<sup>3</sup>. As a consequence, signs cannot play an important social role. Of course, it does not mean that the very core of individual-centered accounts of knowledge formation are contradictory. On the contrary, as theoretical constructs, they display an admirable logical coherence. What we are trying to say is that these epistemological accounts favor other aspects (e.g. structural operational mechanisms) rather than semiotical categories (e.g. language)<sup>4</sup>.

In contrast to the individual-centered cognitive accounts, sociocultural approaches, from the outset, place communication and interaction as two important dimensions of knowledge formation. The theoretical elaboration of these two dimensions may lead to different accounts of the links between an individual's cognition and culture. Hence, interactionism conceives individuals as kinds of actors that engage mutually in interactive discursive processes from where knowledge negotiation is made possible (e.g. Bauersfeld 1995a, 1995b and Voigt 1985, 1989, 1995; Sierpinska 1996) while *Activity theory* presents a different approach. In the latter, the category of activity accounts for the linking between subjective and cultural cognition<sup>5</sup>. "In activity", says Payne, "subject and object interpenetrate."<sup>6</sup>

Activity is not a set of behaviors that individuals display in order to adapt to their environment, rather it appears as a complex system with its own local structure, actions, motives, and operations that have their distinctive dynamics and forms (A. N. Leontiev 1981, p. 47). Activity, a central concept in Marx's economical and philosophical writings, is a social and historical enterprise that, through labor, puts into contact individuals and their ecological setting<sup>7</sup>.

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<sup>3</sup>See Glasersfeld 1995, p. 120.

<sup>4</sup>For instance, in his comments on Vygotsky's critiques of his work, included as an appendix in Vygotsky (1962), Piaget clearly states that, after a time of work, he finally came to realize that the root of the logical operations (those operations responsible for the development of human intelligence) lies in something deeper than "the linguistic connections".

<sup>5</sup>See, e.g. Wertsch's introduction to his edited book *The concept of activity in Sovietic psychology* (Wertsch 1981), Zinchenko and Gordon (1981), A. N. Leontiev (1981, 1984), John-Steiner and Mahn (1996), Stetsenko and Arieviditch (1997).

<sup>6</sup>Quoted by Wertsch 1981, p. 12.

<sup>7</sup>"[A]ctivity is initially social in nature, that is, it is developed only under conditions of cooperation and social interaction among people." (A. N. Leontiev 1981, p. 55).

Activity is the general arena where thought is socially constructed. As Davydov (1990, p. 232) says, "Productive activity that concerns practical objects —labor— is the basis of all human cognition." Activity is underpinned by the idea that the very essence of the individual resides in her social nature. The individual does not think only in unity and contact with nature or in immediate contact with it, but through the arsenal of conceptions that her culture makes available to her. "Man", says Ilyenkov, commenting on some ideas of Marx, "only thought when he was in unity with society, with the social and historical collective that produced his material and spiritual life." (Ilyenkov 1977, p. 252)

One of the most important points concerning activity for our discussion is its mediated character. Indeed, individuals are always using instruments to do a large number of a variety of tasks. Those instruments are but *tools* that mediate actions between the individual and its ecological system in labor settings. Moreover, the tool is not merely an object allowing one to accomplish a certain task. It bears the social relations underlying the task for which it was created: the stylus with which the scribes made the signs in Mesopotamia in the third and second millennium BC, bears in itself not only the social division of work but the distinctness of the sphere of production in which it was embedded (the production of diplomatic letters, commercial transactions, legal letters, mathematical calculations, etc.). Any tool is an atom in the intersection of the web of the social relationships of its culture.

The idea of activity and their correlated actions involve not only manual but intellectual activity and actions as well. "Thinking" says Ilyenkov (1977, p. 35), "is not the *product* of an action but the *action itself*" (emphasis in the original). Like manual actions, mental actions may be considered, Vygotsky suggested, as being carried out through psychological tools (e.g. Vygotsky 1981a, 1981b). Vygotsky gave the following list of examples of psychological tools: "language, various systems for counting; mnemonic techniques; algebraic symbol systems; works of art; writing; schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs; etc." (1981a, p. 137).

Hence, Vygotsky's list of psychological tools was comprised of signs belonging to different semiotic systems and material instruments.

To better understand the Vygotskian metaphor of thought as a *mediated* activity, we must recall that one of the theoretical starting points of the theory of Activity is the "outward oriented" particularity of the human psyche. The psyche is primarily oriented towards those psychological tasks that allow the individual to interact with his or her environment and to

self-regulate his or her behavior. And social- and self-regulations are accomplished, Vygotsky said, through the use of diverse systems of signs (the classical example being that of speech). Most importantly, we must bare in mind that, as Wertsch says, "it is not the case that sign systems simply mediate some activity that would exist without them" (Wertsch 1981, p. 26). On the contrary, activity is only made possible through such sign systems.

There is another point that is worth mentioning in our discussion of signs and activity in cognition: the use of signs is not cognitively neutral. Indeed, by using signs, the individual's mental condition, in turn, found itself modified:

Man differs from animals in that he can make and use tools. [These tools] not only radically change his conditions of existence, they even react on him in that they effect a change in him and his psychic condition." (Luria, quoted by Cole 1990, p. 91).

Vygotsky also stressed this altering function of signs and tools in the psychology of individuals:

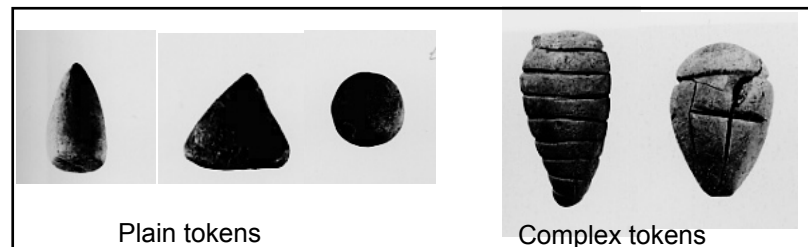
By being included in the process of behavior, the psychological tool alters the entire flow and structure of mental functions. It does this by determining the structure of a new instrumental act just as a technical tool alters the process of a natural adaptation by determining the form of labor operations. (Vygotsky 1981a, p. 137).

The altering function of signs in mind was clearly perceived by the Greeks, when writing became a social phenomenon. The increasing use of manuscripts and books by the Sophists to teach and lecture, in the 4th century BC, and the consequently abandoning of the oral tradition, was seen as an action tending to weaken or destroy memory ( $\mu\nu\nu\eta\mu\eta$ )<sup>8</sup> —that wonderful human faculty on which had exclusively relied the transmission of oral tradition and knowledge before writing. Although, of course, memory may decrease, other mental functions become more stimulated, making it possible to tackle new problems or to solve old ones through new conceptual strategies. The emergence of new social structures and the growing commercial activity of the Mesopotamian cities of the second half of the 4th millennium, led to a *diversification* of tokens. This diversification (which actually led to the first Mesopotamian technological system of accounting that preceded the Greek and Roman abacuses) could only be envisaged within the previous mental frame that had led to the elaboration of the tokens of the previous millennia. What was not previsible was the form that such diversification could have later taken. History shows us that the path followed was the perforation of some previous tokens and the invention of new shapes and drawings to differentiate the kind of goods (see fig. 1). The turn of events was, of course, constrained

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<sup>8</sup>Pfeiffer 1968, p. 32.

by the material conditions (e.g. the possibilities that clay could afford and the realizable marks that men and women could impress on a small surface) and the previous cognitive layer. But the resulting issue was also constrained by the *symbolic space* of Mesopotamian cultures. Thus, given the different status accorded to urban and non-urban commercial products, one was led to distinguish between the goods coming from each area. Complex tokens, for instance, were used to count and account urban, manufactured products like textiles, garments, vessels and processed foods, for example, perfume, metal and jewelry (Schmandt-Besserat 1992, p. 168).



(From Schmandt-Besserat 1992, p. 51 and 53)

Figure 1. Plain Tokens and Complex Tokens

The mediated nature of activity leads us to see perception in a different way. In fact, the "crude" and "real" object is inevitably inaccessible to us, for it is always mediated by the historically crystallized signs and cultural conceptions and tools that culture interposes between us and the object. Indeed, upon entering the world, a child already finds well organized surroundings in place; furthermore, they appear neither "as the world appertaining to a specific institutional context", nor as one of the possible worlds, but "as *the world*" (Berger and Luckmann 1967, p. 141). Through (active) observation and participation (see Rogoff 1990), the child learns what is to be highlighted and what is not, what is interesting and what is not, according to her cultural setting. The objects that inhabit the child's world can only be *mediately* perceived. Culture, so to speak, carves the forms that the world wears in front of its individuals. Luria, for instance, has shown how the young child's perception, from the beginning, finds support not only in language but through the whole complex system of cultural mediating signs (gestures, colors and so on). "The first peculiarity of perception", he says, "consists of its mediated active character." (Luria 1984, p. 62).

In the case of mathematics, an eloquent example of how perception is culturally shaped is given by the Babylonian conception of a square. J. Høyrup taught us that, in fact, a square, for the surveyors of the Old Babylonian period, was not primarily an equilateral rectangular figure having first an area and only derivatively a side but, conversely, a

quadrilateral rectangular figure *being* one of its equal sides and having only derivatively an area<sup>9</sup>. This difference is not a superficial matter: it underlies the fundamental conception of algebra that the Babylonians developed<sup>10</sup>.

### 3. Signs and Concept Formation

Let us now turn to the epistemological problem of concept formation and the role of signs from our sociocultural perspective.

Instead of being exclusively an intra-cerebral process accomplished by the individual, in the intimacy of her private cognitive world—a process eventually ‘oriented’ by her milieu, as Radical Constructivism concedes—, concept formation, in our perspective, is a *sociocultural process*, whose products are the individuals' theoretical and practical reflections about their world.

As any cultural process, concept formation is culturally mediated through signs. What we need to explore, then, are the particular mediating roles that signs may play in the course of the construction of conceptual objects. We shall focus our attention on two complementary forms of concept formation: (1) *schematization* and (2) *objectivation* and shall examine them through the case study of the historical example of tokens in the Ancient Near-East, to which we have been referring.

In order to do so, let us notice that the token, as a practical and palpable tool to count animals (e.g. cows, sheep), textiles, grains and other goods, appears, in fact, as a *sign*. Token-signs create a kind of "microcosm", a *schematization* of objects in a miniature world (Nicolopoulou 1997, p. 219).

The schematic microcosm appears as a sensory conception directly intertwined with practical activity carrying out a "primary" idealization of the activity.

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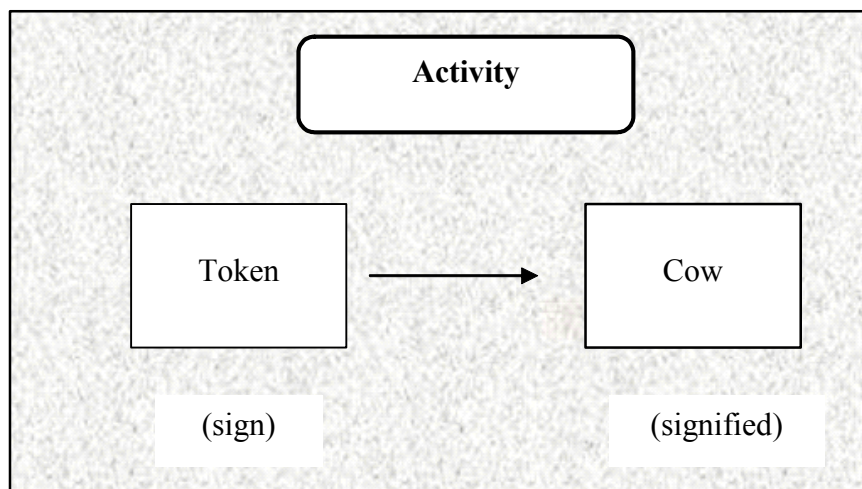
<sup>9</sup>"We think of a square as *being* (e.g.) 4 square feet and *having* the side 2 feet (knowing that, strictly speaking, the square is a complex configuration which can be equally well characterized by any of these parameters). The Babylonians, on their part, thought of the square as *being* 2 feet and *having* an area 4 square feet<sup>[5]</sup>." (Høyrup 1993, p. 4; emphasis as in the original). This point was previously elaborated in connection with the Greek term *dynamis* in Høyrup 1990a. See in particular p. 208 ff.

<sup>10</sup> As a contemporary case, let us mention the ongoing Research Project conducted by Pallascio et al. (1996, 1997) at the CIRADE, in Montreal, about the learning of geometrical concepts in Inuit communities in northern Quebec. One of the results is that the Inuit spacial skills differ from people living in southern Quebec. As a matter of fact, the perception and representation of certain geometric forms seem to be couched in the particular cultural activities of each group.



In the conceptual formation of the microcosm, the token-sign results not of the mere *act of representation* of the substituted object (e.g. the cow), but of the counting needs ensuing the economic activity. The sign is the product of a semiotic, psychological act, couched in human activity and in the cultural web of signifying practices. This is why a sign is a cognitive and a cultural phenomenon as well. Left to itself, in the exterior of human activity, the token (and the sign, in general) means nothing.

Instead of the classic triad *sign-object-signified* (see e.g. Eco 1988, p. 35 ff), the basic structure of a sign, within our cultural framework, appears as follows:



Semiotic map of the sign

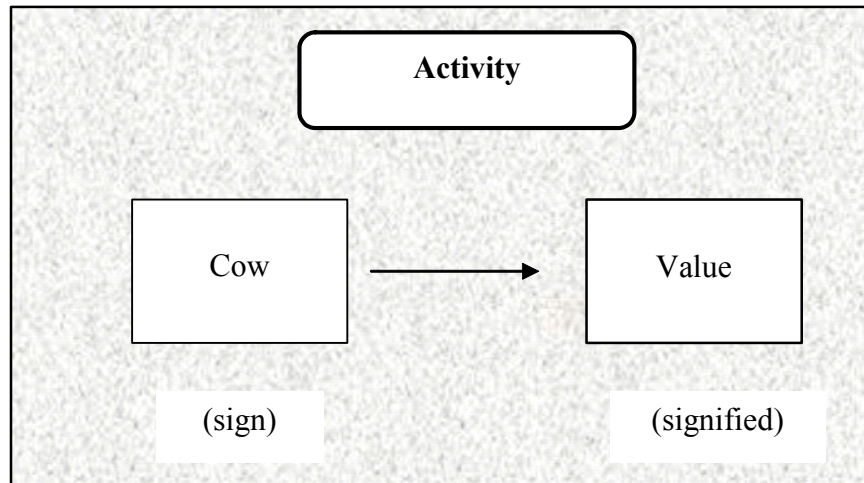
where the arrow linking the signifier to the signified does not refer to an associative term between them but to a specific *semiotic-conceptual signifying relationship*, rooted in the particularities of the activity that we have placed in *front* of the cultural scene and not *behind* it as a decorative background.

The peculiarity of a sign resides in the possibility that it offers us to move beyond the object under consideration through its signified. Without signs, we are left with the crude object, and reflection cannot go very far.

Besides providing us with a *reproduced* schematic microcosm, signs function as pivotal intellectual artifacts in the processes of *conceptual objectivation* that the individuals undergo through activity —conceptual objectivation being, as said before, the second form of concept formation.

In order to explain this point, let us further pursue our previous example.

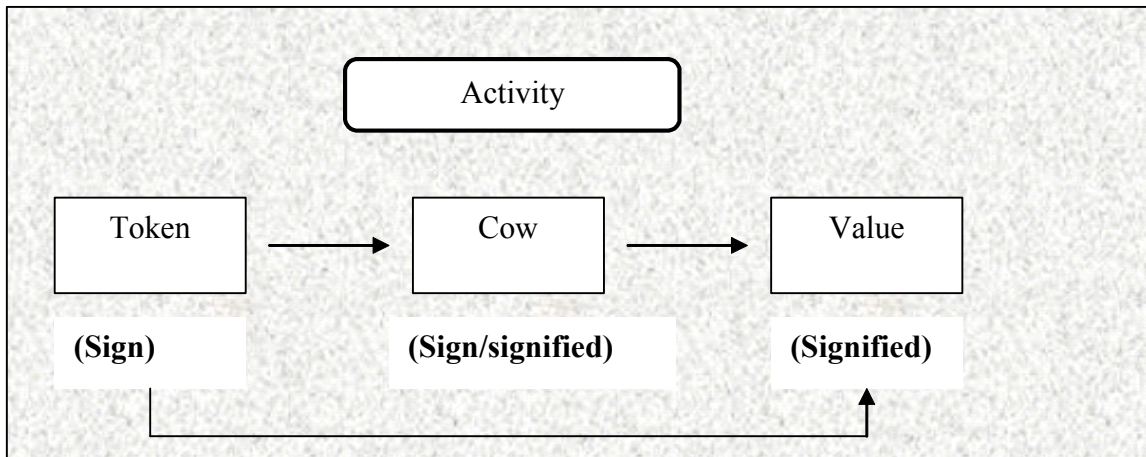
Cattle breeding, one of the main economic activities of the first urban settlements, led to the exchange of some animals for others (e.g. cows for sheep). Such exchange can only occur if some kind of equality may be acknowledged, for instance, two sheep for one cow. In this case, the equality is underlain by the *value* of the merchandise. Two sheep have the same value as one cow. The value, this non-palpable concept, that which is common to different things and which shows up in the exchange relation, as Marx insisted in *The Capital*, appears as a complex *signified* of the actual cow, which now plays the role of the signifier of another sign.



Semiotic map of the sign'

The concept of value, then, is a cultural entity produced by economic activity. And its objectivation is ensured by signs, through a *semiotic* shift from the actual merchandise to its value.

This semiotic shift gives rise to a 'composition of signs'. In fact, tokens now intervene as the sign of *another* sign, making it possible to achieve calculations in terms of an abstract object—the value of the cow or the sheep.





Semiotic map of the sign of a sign

This does not mean, however, that the signifier is dismissed. On the contrary, our discussion suggests that, for instance, the cow, as merchandize, cannot exist without its signified (its value), and conversely, without the cow its value has no sense. In other terms—*non obstante* idealism—the one cannot exist without the other and such a coexistence is ensured by the social activity from where they emerge.

The objectivating role of signs is not a sporadic phenomenon. Signs are always objectivating human actions. For instance, the pictographic signs of the proto-cuneiform texts from Uruk and Jemdet Nasr were obtained by carving the clay tablets with the help of thick and thin round styluses. These hand-made signs formed various interesting semiotic systems for counting and measuring objects such as lengths and surfaces of fields, grain production and so on. In the system to measure lengths, known as  $S(pc)$ <sup>11</sup>, the sign **D** was

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
<sup>11</sup> See Friberg 1995.

used to represent the unit of length, called ninda; ten of those signs were equal to a sign of this form , called éš, and six éš were equal to one uš, for which the sign  (i.e. a bigger ninda sign) was used. The ninda seems to have been used to measure shorter lengths, while the uš (ca. 60 m) was probably used to measure fields and éš, whose meaning in the context was "side" or "length", was probably used to measure the sides of bigger fields. The highest Sumerian unit of length measurement was the danna (about 10 km) and, as Friberg (1995) suggests, bears the literal meaning of "long way".

The  $S(pc)$ , is shown in the next figure:



Thus, the third sign from right to left, i.e. the uš, is equal to 6 éš, and to 60 nindas.

The sign-formation rules of the pictographic tablets bear the traces of practical activity<sup>12</sup> and again show the perceptual origins of early conceptualizations. In turn, once established, those rules come to influence the idealized (i.e. theoretical) space in which the individuals move. Indeed, by introducing names and signs in the surveying, accounting or any other practical activity, the mind introduces marks in the world and the latter acquires a new shape. Thus, the signs , , etc. mark qualitative and quantitative limits of the Mesopotamian micro-, meso- and macro-spatial paysage. But there is something more. The fourth and sixth signs in the  $S(pc)$  system, in contrast to the other signs of this system, at a morphological level, attest to a new feature of sign production: new signs are formed from the combination of previous ones. Those signs are, indeed, "composed signs" or "signs within signs" (Friberg 1995, p. 4) formed from two previous signs. This point is important from a semiotic and epistemological point of view, for such an action could not be done in the realm of the 'impressed tablets', given their own technology of production (see fig. 2)<sup>13</sup>. Thus, the 'natural paysage' depicted through the pictographic signs is no longer a schematic copy of the external world but a culturally modified version of it.

<sup>12</sup>See e.g. Damerow 1996, p. 337 ff.

<sup>13</sup>The 'impressed tablets' are those tablets which replaced the tokens and preceded the pictographic tablets. Their signs were formed by the mark that the token left on the tablet when the former was pressed against the still soft clay surface of the latter.

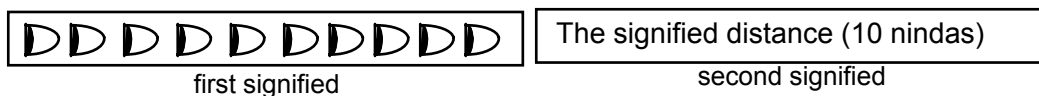
Sign-formation rules witness of an important conceptual *gap*. Indeed, through them, a new theoretical link has been forged between the new sign and the set of previous signs that the former is now replacing.

DDDDDDDDDD = ○



Fig. 2. Impressed tablet  
(From Schmandt-Besserat 1992, p. 130)

This gap may be grasped by noticing that the new sign (○) now has, properly speaking, two related signifieds:



The first signified corresponds to the set of previous signs. The second signified to the length of 10 nindas.

The introduction of the third sign in the sequence (something related to the surveying needs) enlarges the web of relationships between signs. As a result, the new sign has still more signifieds, and so on.

Simple rules of substitution, one may retort, do not bring us far in the conceptual field. Of course not. Fortunately, rules of substitution are but the result of *one* of the diverse features of human activity. J. Høyrup (1990b) succeeded in showing us that Babylonian arithmetic covered different contextual conceptions of number operations<sup>14</sup>—something that can only be understood, I think, by the original specific activities where different kinds of additions, subtractions, multiplications, etc. were needed in Mesopotamia. Thus, according to the archaeological evidence, the problems from where algebra emerged were problems inspired by the surveying practices of the scribes of the Old Babylonian period (ca. 2000 BC to 1600 BC). The rectangles on which those problems were based had gained, at the time, the necessary conceptuality, to become the core of the algebraic problems (Radford 1996, in print). This was only possible through an active reflection on representations of grain

<sup>14</sup>For instance, the term *wasābum*, that is translated as "to append" and which is an "identity-conserving addition" has to be distinguished from the term *kamārum* which refers to entities accumulated into one "heap". Besides these additive terms related to "sum" actions, there are many terms referring to "multiplicative" actions that, like in the case of "sum" actions, cannot be reduced to a single concept. (See Høyrup 1990b, p. 45 ff).

fields, through the unlimited stock of representations, and not on the concrete fields of grain themselves.

## 5. Representations

We have already used the term *representation* in the previous section. And, to some extent, we use it as a synonym for sign. In fact, as suggested by Derrida (1973), a sign, in its indefinite possibility of repetition, is already re-presenting the signified and, hence, is already a representation of it. Peirce, too, seems to have seen things in similar terms. In his own definition of sign, he says "A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity." (Peirce 1955, p. 99, emphasis as in the original).

The term representation, however, is very often seen as something more complex than the sign. In the classic theory of representations, it is frequently associated with "mental images".

What could a "mental image" be in a sociocultural approach to the mind?

Certainly not those pristaline "copies" of the external —encoded in propositional or figural format— that the mind, as a central processor of information, is uncontaminately processing in a hidden central plant within the brain. Bakhurst (1988) has shown that underneath such a view lies the problematic Cartesian metaphor of self-contained representations of the external world projecting themselves through a transparent screen from behind which the individual is surveying the private representations presented to him.

Indeed, under closer examination, "mental images" are not the product of a contemplating individual nor the "internal image" produced by our visual organs but the result of a complex interrelation of sensory activity (an activity that includes all our senses). Furthermore, since the sensory activity is not produced in a vacuum but is culturally situated, as was said previously when we mentioned the mediated nature of perception, a "mental image" is framed by the historicity of the setting in which it is produced and bears the marks of the biography of the individual who is actually producing it. By this we do not mean that "mental images" are culturally pre-determined: mental images are simply culturally *constrained*.

The cultural embodiment of mental images (and internal representations in general) is reinforced, at another level, by their functional nature: the representation, like the sign, is a conceptual tool used to interact with our culture, its particularity residing on the local, situated re-constructive feature of previous experiences.

This point is clearly made by John-Steiner (1996, p. 5), who, after mentioning Damasio's concept of mental images —a concept that the latter presents as "momentary constructions, *attempts at replications* of patterns that were once experienced"—, suggests instead the less mentalistic term *representational activity* by which she stresses the fact that any representation is embedded in social practice and is instantiated in constructions of the human mind.

Davydov sees internal images as one of the features of the central concept of the *ideal*:

The ideal is a reflection of object reality in the forms of the subjective activity of man in society (in his internal images, incentives, and goals), who is reproducing this world of objects. (1990, p. 237).

Ilyenkov uses the word "image" in a suggestive context of representations. He puts the matter as follows:

The ideal [object] is the subjective image of objective reality, i. e. reflection of the external world in the forms of man's activity, in the forms of his consciousness and will. (1977, p. 252).

As it is clear from those quotations, first of all, an internal representation is a mental *re-construction* of a previous activity from the viewpoint of the individual. This does not mean that an internal representation is necessarily a frozen activity resulting from the past experience of the individual. On the contrary, internal representations have an internal mobility that is a key feature of theoretical thought (Davydov 1990, p. 249 ff). What this does mean is that the mental re-constructions, as well as new mental constructions —which make up an internal representation— are embedded in specific cultural frames of thought. One of the most important theoretical points about creativity is precisely to provide accounts about the formation of new objects within a given frame of thought. I cannot further develop this point given the limits of this article; suffice to say that one of our previous works is devoted to a case study: the emergence of the second algebraic unknown in Mediaeval Italian algebra (Radford 1997).

Getting back to our main discussion, let us submit that the 'similar texture' that "mental images" (as mental, cultural constructs) seem to bear with their corresponding objects is due not to our cognitive capacity of internally reproducing or mirroring the external world but to the way in which the world is internalized by the individual<sup>15</sup>. Particularly

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<sup>15</sup>Some recent discussions about the Vygotskian concept of internalization may be found in John-Steiner & Mahn (1996) and Lawrence & Valsiner (1993).

interesting in the processes of internalization is the role played by signs. V. P. Zinchenko (1985) has suggested that, just as external material activity involves the mind as an essential component (without which we could not carry out any action—to drive a car for example), in the same way internal activity (even autonomous mental activity) retains genetic connections with the external activity. This ‘materialization’ of the mind and ‘mindization’ of the concrete leads us to see signs and other cultural artifacts as extensions or protheses of the mind.

In his enterprise of anthropologization of the mind, Geertz develops a similar point:

the accepted view that mental functioning is essentially an intracerebral process, which can only be secondarily assisted or amplified by the various artificial devices which that process has enabled man to invent, appears to be quite wrong. On the contrary, a fully specified, adaptatively sufficient definition of regnant neural processes in terms of intrinsic parameters being impossible, the human brain is thoroughly dependent upon cultural resources for its very operation; and those resources are, consequently, not adjuncts to, but constituents of, mental activity. (1973, p. 76).

In this line of thought, the written numerical calculations as well as algebraic signs that a student writes when solving a problem do not merely appear as a robe or carrier of internal thought, but a materialized extension of thought *itself*. The calculations through positional numerals is the most evident example. "I wonder", says Leibniz, in his *Dialogus de conexione inter res et verba*, "whether you could make any mathematical calculations without numerals."<sup>16</sup>

In this context, "mental images" could be considered as materialized pieces of thought, in the external sphere of the individual's activity, colliding with the task at hand, and hence externally situated.

## 6. Concluding Remarks

Signs—given the cultural nature in which they find themselves enveloped, a cultural nature that we attempted to point out in the previous sections—cannot be reduced to formal calculations as Positivists tried to do in the first half of this century. Signs have a pragmatic and a cultural symbolic substance that cannot be grasped in formal terms.

A. A. Leontiev wrote:

"the sign (...) emerges as a constituent part of the system of conventional signifying forms and means for external expressions and the consolidation of ideal phenomena". (1981, p. 244).

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<sup>16</sup>Quoted by Martin 1964, p. 60.



It took us 4 sections (from section 2 to 5) to disentangle Leontiev's dense formulation and we have not finished yet. Indeed, belonging to the conventional signifying forms of their culture, signs do not function in isolation; they function in the intersection of different semiotic systems. Bakhtin's literary analyses show that this happens even within the confines of a same semiotic space, for example, that of the Novel (Bakhtin 1981). A sign is never a sign in itself. To say it with a Bakhtian flavour, a sign is a point of contact where several voices meet. In this sense, a sign is *always* a shared sign, living two lives *at the same time*: the public and the private, cognitive one.

This is why Kaput's category of "shared signs" and his "working assumption" (1987, p. 171) concerning the "dual but intimately related" life of shared signs, namely, the public for communication and the other for the execution of private cognitive processes, seem unnecessary.

Such a distinction begs for a particular conception of cognition in which the individual, while using a culturally inherited system of signs (something that Kaput himself acknowledges), succeeds in isolating herself from the uproarious torrent of activities of cultural life and in placing herself in the encircling fortification within which the "private cognitive domain" would be situated and the cognitive act finally accomplished.

Confronted with the idea of such an intimacy we found ourselves faced with the following dilemma: either we maintain this variant of the traditional dualism presented here as a diachronic interplay between the cognitive sanctuary and the external cultural world or we abandon it.

To maintain such an interplay between both domains, the private and the public, requires the former to have some windows to communicate with the latter. But Lerman (1996) has already shown that the smallest of the holes in the enceinte of the fortification of private cognition is the most dangerous of the menaces: how, in fact, can the individual control what is allowed to pass through the hole or not? How, for instance, to sort out the words of language? To close the window would lead us back to the Cartesian screen—an impossible move given Bakhurst's threatening cold Bishop awaiting in its fianchetto position.

Lerman's version of the Trojan horse led us to the second option, that of abandoning the cognitive private domain, something that we attempted within the framework of the post-Vygotskian semiotics and Activity theory, a theory which provides a non-dualistic account of the individual and her local, ecological setting.

At first sight, the loss of the cognitive private domain, given our cultural background, is a perturbing thought, insofar as it may carry out the monstrous idea of our individuality being violated by the intromission of others in our utmost intimacy. This seems to be an inevitable consequence of being—to borrow Aristotle's expression—a social animal; a

fortunate condition without which, under closer examination, we would still be trying to light some fire in front of a dark cavern, as the French biologist and philosopher Henri Laborit, once suggested.

One may argue with Glasersfeld that

no analysis of social phenomena can be successful if it does not fully take into account that the mind that constructs viable concepts and schemes is under all circumstances an individual mind. (1995, p. 121).

Except that, as Davydov says:

An individual person's thought is the functioning of historically developed forms of society's activity which have been *conferred* on him. (1990, p. 232).

thereby requiring us to pay further attention to the situatedness of cognition and the *interaction* of the individual and its milieu.

Beyond this epistemological discussion, it is clear that the repercussions in mathematics education of considering signs from a sociocultural viewpoint may have an important impact—ranging from our own evaluation and understanding of children's use of signs in some mathematical tasks to the design of teaching settings. Although a couple of works have started to pave the way, for example, Nemirovsky (1994) and Meira (1996, in print), a lot of interesting things remain to be understood.

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